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CONTENTS

NUMBER I.

NUMBER II.

NUMBER III.

NUMBER IV.

NUMBER V.

PLATE I

PIETRO TACCHINI

THE

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AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXII JULY 1905 NUMBER 1

PIETRO TACCHINI

By A. RICCÒ

On March 24, 1905, at Spilamberto (Modena), there ended the life of a man to whom Italian astronomy, meteorology and geodynamics are indebted for important progress and great developments.

Pietro Tacchini was born at Modena, on March 21, 1838. In 1857 he carried on studies in theoretical engineering with great success at the university of his native city. In 1858 the ducal government of Modena, influenced by Tacchini's special tastes, sent him to the Observatory of Padua, in order that he might study theoretical and practical astronomy under the direction of Santini and Trettenero. In 1859, when only twenty-one years of age, he was appointed temporary director of the Observatory of Modena, which Bianchi had left on the occasion of the change of government.

In 1863 he obtained the position of assistant astronomer at the Observatory of Palermo, which he had sought in order that he might enjoy better means of research and a more favorable sky than at Modena. At Palermo he devoted himself with great zeal to solar investigations, and initiated the beautiful and important series of direct and spectroscopic observations which he continued nearly thirty years and which gave him a great reputation.

In conjunction with Father Secchi, he founded in 1872 the *Società degli Spettroscopisti Italiani*, which he directed with the most steadfast devotion up to his last days. After the death of Father Secchi, the Italian government having taken possession of the Observatory of the Roman College, Tacchini was appointed its director in 1879, and also director of the Central Meteorological Bureau. He organized both of these institutions with great energy and skill; he established the daily meteorological bulletin, the forecasting and storm service, and the studies for a magnetic chart of Italy. In 1887 a section of geodynamics was added to this bureau. The Copernican Museum, annexed to the observatory, was greatly improved and developed by Tacchini. In 1895 he founded the *Società Sismologica Italiana* and the bulletin of this society.

The marked initiative of Tacchini is also illustrated outside of the institutions which he directed. In 1880 he succeeded in starting, and later in completing, the construction of an observatory on Monte Cimone at an altitude of 2160 meters, and in the same year he obtained the means required for the construction of the observatory on Etna at an altitude of 2950 meters. In 1885 he also secured means to erect an observatory at Catania in connection with the one on Etna; and he furthermore succeeded in causing the establishment of a chair of astrophysics, the only one in Italy, at the University of Catania.

Having persuaded the Italian government to take part in the international photographic catalogue and chart of the heavens, Tacchini proposed that the Observatory of Catania be the Italian station. In 1892 the photographic building was erected and the photographic telescope placed in it.

Tacchini's scientific activity was remarkable. At Palermo, in addition to the daily solar observations, he determined the difference in longitude between Palermo and Naples; observed with the meridian circle 1001 southern stars, which were reduced and catalogued by Father Hagen; and made studies of the climate of Palermo. In 1870 he went to Terranova (Sicily) to observe the total eclipse of the Sun; in 1874, to Muddapur (India) to observe the transit of *Venus*. Then he undertook a series of expeditions to distant countries for the purpose of observing solar eclipses: to Kamorta (Nicobar) in 1875, to Sohag (Egypt) in 1882, to Caroline Island (in the Pacific)

in 1883, to Grenada (Lesser Antilles) in 1886, to Surriscaja (Russia) in 1887, to Menérville (Algiers) in 1900; he also expected to go to Spain to observe the eclipse of August 30 next. On these expeditions he obtained many important results, of which I may mention only the discovery of the *white prominences* of the Sun.

The greater part of the numerous investigations of Tacchini are published in the *Memorie della Società degli Spettroscopisti Italiana*, the *Memorie del R. Osservatorio del Collegio Romano*, the *Atti della R. Accademia dei Lincei*, the *Comptes Rendus*, etc. He directed the editorial work of the *Memorie della Società degli Spettroscopisti Italiana* with the greatest devotion during about thirty years.

In recognition of his merits he was elected to membership in various scientific academies and societies in Italy and abroad, such as the Accademia dei XL, the Accademia reale dei Lincei, and the Royal Society of London, which awarded him the Rumford medal in 1888; the Paris Academy awarded him the Janssen medal in 1892. He was a collaborator of the *Astrophysical Journal*. The Italian government appointed him Commander of the Crown of Italy, and Chevalier of the Order of Merit of Savoy.

After forty years of very active service, he was retired at his own request in 1899, to the great regret of his friends and admirers who saw our country deprived of the work of such a man. But he enjoyed little opportunity for rest in the country and among his relatives. A serious attack of pneumonia, complicated by an affection of the liver, carried him off at the moderate age of sixty-seven years.

Tacchini was the object of much friendship and regard in Italy and abroad because of his frankness and loyalty, his cheerful and amiable character, and his great kindness of heart. The heavy loss experienced by our country and by science will always be deeply deplored.

OSSERVATORIO DI CATANIA
May 5, 1905.

RESEARCHES IN THE SUN-SPOT SPECTRUM, REGION F TO *a*

By WALTER M. MITCHELL

INTRODUCTION

The results embodied in this paper are the outcome of a detailed study of the F–*a* region of the sun-spot spectrum, made at Princeton in the year March 1904–1905.

The purpose of the investigations was to obtain as complete a table of lines as possible in this portion of the sun-spot spectrum, and to secure spectroscopic evidence on which to base a discussion of the various sun-spot theories.

INSTRUMENT AND METHODS

The instrument used was the spectroscope of the Halsted Observatory. The telescope itself is a refractor of 23 inches aperture and 30 feet focal length, made by the Clarks. A stiff frame of four steel rods is carried by two rings which fit over the tailpiece of the telescope.[1] This frame carries the spectroscope. The collimator is mounted centrally in it, in such a way that it can be adjusted with respect to the optical axis of the telescope, and can also be moved longitudinally into the focal plane for rays of any wave-length. The view telescope and collimator have objectives of 2½ inches diameter and of 30 inches focal length. They are fixed at a permanent angle. A Rowland plane grating of $4 \times 2\frac{1}{2}$ inches ruled surface, 20,000 lines to the inch, was used in all the observations. It was found that the third-order spectrum on the more dispersive side was the most satisfactory, and it was used in most instances. The resolving power was sufficient to divide such lines as λ5264.4, etc. For observations below λ6600 the second-order spectrum was used, as it was more brilliant. Absorbing screens were placed in front of the eye-piece when the overlapping orders of the spectrum interfered.

In making the observations, which were all visual, the whole region was hastily surveyed for anything particularly noticeable; then

[1] *Astronomy and Astro-Physics*, **11**, 292, 1892.

a portion was selected for detailed examination. The method was to compare the affected lines in the spot-spectrum directly with Rowland's *Photographic Map of the Solar Spectrum* (*second series*, 1888) and from it to read off the wave-lengths as accurately as possible. For a few lines below B, Thollon's map was used, as Rowland's does not extend sufficiently low in the spectrum. The process of examining each line in the spectrum is very tedious, and progress is necessarily very slow, two or three hours being required in going from C to D.

After the observations were finished, the approximate wave-lengths were corrected to two decimal places with the aid of Rowland's *Preliminary Table of Solar Spectrum Wave-Lengths*. In many cases, however, this was impossible, since many of the most prominent lines in the spot-spectrum are very faint in the spectrum of the photosphere, and are difficult to identify surely with the lines of Rowland's map. In these instances the wave-length was determined by differential measurements with the micrometer from the nearest lines that were surely identified.

The spectrum of sun-spots, as is well known, is composite, consisting of essentially two parts:

1. The nearly continuous spectrum of general absorption.
2. The superposed array of affected Fraunhofer lines.

The absorption spectrum, at times, is resolved into a countless number of fine, closely packed lines. The resolution is most frequently seen in the region from the *b*'s to λ5100. It was first seen here by Professor Young in 1883,[1] and afterwards confirmed by Dunér and others. The spectrum has also been resolved into fine lines in the region λλ6380–6400 by Professor Young, and also by the writer.[2] In the great spot of February 3, this year, the whole spot-spectrum from C nearly to F was thus resolved, and in the same spot on its return appearance the region λ6770 to B was similarly affected. The lines are most closely crowded in the region λλ5000–5160; in the lower portions of the spectrum, particularly below D, the lines form groups, rather than a uniform succession of lines as above the *b*'s. The writer doubts whether the greater part of these "band-lines" are lines ordinarily exceedingly faint in the photospheric

[1] *The Sun*, p. 132. [2] *Astrophysical Journal*, **19**, 359, 1904.

spectrum and brought into prominence by the vapors of the spot, but is inclined to the opinion that they are lines not present in the photospheric spectrum at all. This view is supported by the fact that many of these band-lines are very wide (0.5 tenth-meter, in some cases), and fade out at the borders of the spectrum of the umbra (short lines), instead of being pointed, as would be the case of a fine line much widened. They are not nearly so intensely black as the ordinary Fraunhofer lines, but appear more like a wide, fuzzy shade. Of course, there are numerous band-lines that are fine and sharp, extending into and sometimes beyond the spectrum of the penumbra (long lines). These exceptional lines are undoubtedly faint lines in the ordinary spectrum.

There are several bright streaks, or rather partial interruptions in the absorption-spectrum, the most noticeable of these being at $\lambda 5163.7$. Here the spot-spectrum is almost as brilliant as that of the neighboring photosphere. This streak would seem to indicate that the carbon band beginning at $\lambda 5165$, although present in the chromosphere, takes no part in the spectrum of the sun-spot. The band-lines above this particular wave-length show no regularity of arrangement whatever; also the bright streak mentioned above is situated just above the head of the carbon band, a region which should be dark instead of bright.

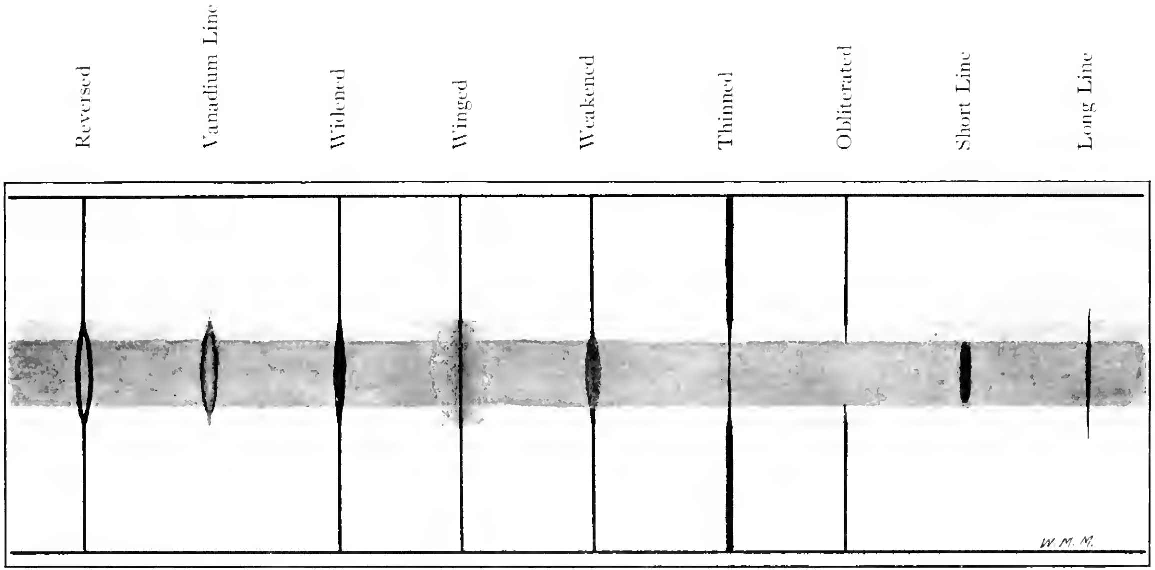
It has been noticed on several occasions that the absorption-spectrum has divided itself into certain dark regions or bands. Nine of these bands, situated below D, were observed early in 1885 at Stonyhurst.[1] At Greenwich in 1880–1883, seventeen were seen in the more refrangible part of the spectrum. One proved identical with a fluting drawn by Young in 1872.[2] Hale in 1902[3] secured some photographs of the spot-spectrum showing similar bands. The wave-lengths of these indicate that they are probably identical with the band-lines in the region above the *b*'s. Taylor Reed at Princeton in 1892 also attempted to photograph the band-lines in the *b* region, and was partially successful.

The array of affected Fraunhofer lines includes the lines widened, reversed, weakened, winged, darkened, and thinned. The figure (Plate

[1] *Monthly Notices*, **47**, 19. [2] *Nature*, Dec. 12, 1872.

[3] *Astrophysical Journal*, **16**, 219, 1902.

PLATE II



TYPES OF LINES IN SPECTRUM OF SUN-SPOTS

II) is an attempt to indicate the appearance of these. The widened lines are the most numerous. Sometimes it is extremely difficult to tell whether a line is actually, or only apparently, widened. Lines which appear considerably widened and winged under low dispersion, when viewed with a more powerful instrument are found not to be widened at all, and sometimes not even winged. It is impossible to say whether the widening under the low dispersion is only subjective, and hence disappears with the high, or whether the widening of the lines is real, and is made imperceptible by the general widening of the line itself under the higher dispersion (as the naked-eye markings on the Moon disappear when viewed with the telescope). Evidence seems to favor the view that the widening under lower dispersion is often subjective, due to the apparent narrowing of the other portions of the line by irradiation, the effect of the brightness of the spectrum outside the spot. A method of testing this has been used which consists in moving the micrometer-wire parallel to the line in question, until the bright space between the wire and the line becomes quite narrow. If this narrow bright space has the same width throughout its extent, the line is certainly not widened. With many lines apparently widened it was found, on applying this test, that the widening disappeared on the side of the line nearest the wire, while the other side of the line still appeared widened, i. e., the line appeared unsymmetrical. A similar effect was produced if the micrometer-wire was moved to the other side of the line. A line that would not stand the test, and yet appeared widened, was called "darkened."

There is considerable difficulty in selecting a suitable scale to record the widening of the lines. Father Cortie's method of estimating the widening of a line in tenths of its normal width was found too uncertain in its application, particularly for the very faint lines. In the table a scale has been used, from 1 to 10, indicating the amount that the line is affected, regardless as to whether it is widened, reversed, etc. The writer heartily endorses Professor Fowler's suggestion[1] that the most satisfactory method of recording the lines is to note the actual intensities of the spot-lines by comparison with neighboring solar lines outside the spot-spectrum. The advantage of this method

[1] *Monthly Notices*, **65**, 206, 1905.

was not appreciated by the writer until the greater part of the observations were made, and then it was too late for any change.

The reversed lines are perhaps the most interesting. Here the widened line is split in two through the spectrum of the spot, the central portion being bright. The fainter lines are usually the most strongly reversed. The widening of these lines is considerable, being sometimes ten times the normal width of the line.

Many of the reversals have been seen not only over the umbra itself, but even more distinctly over the penumbra, and over the photosphere in the immediate vicinity, as instanced by the lines λλ5225.97, 5250.39, 6082.93, 6137.21, 6173.55. It is to be understood that these reversals outside the umbra were not over "bridges," or bright portions of photospheric matter projected into the spot, but appeared more as if caused by a mass of gas of greater radiating power or greater density, similar to the calcium flocculi which have been so successfully photographed by Hale at the Yerkes Observatory. It would be extremely interesting if spectroheliograms of the Sun could be taken through some of the lines mentioned above.

Of the lines which are always reversed, iron claims eleven, chromium three, and nickel, titanium, and unknown, each one.

The weakened lines may be considered as a sort of "degenerate" reversal. The appearance is similar to that of a widened line, except that the widened part is not so dark as the normal line. With low dispersion these lines often disappear completely in the spot-spectrum, but with high dispersion the widening and the weakening are distinctly seen. It has been found that lines plainly reversed in the region around the spot are frequently weakened when the umbra is directly on the slit of the spectroscope. The weakened appearance is characteristic of the silicon lines in the green portion of the spectrum. A few prominent chromospheric lines in the red are also weakened in the spot.

The D lines in the spot-spectrum, under low dispersion, are the best examples of the winged lines. These wings generally disappear (unless the spot is very large) with the dispersion that the writer has used, no doubt because of lack of contrast, due to their being drawn out to a great extent. It has been noticed that lines which are narrowed, or thinned, without losing intensity, across the spot-

spectrum, are always accompanied by the wings. This is particularly noticeable of lines in the green region, of which the b_3 line is an instance.

Taken as a whole, the most productive portion of the sun-spot spectrum is the region λλ5700–6600. Here the reversals are most numerous, and the most widened lines are found. Below wave-length 6600 observations are difficult on account of the faintness of the solar spectrum itself; also there are few affected lines. From λ5700 to the *b*'s there are numerous widened lines, it is true, but the widening in general is not very great, the lines being principally winged; the reversals in this region are as a rule narrow, and often scarcely visible. Above the *b*'s the band-lines are exceedingly troublesome, disguising the Fraunhofer lines in such a manner that the detection of their widening is very difficult. Above λ5000 the affected lines are mostly "darkened," while near F the spot-spectrum is frequently so solidly black that scarcely any detail can be made out.

EXPLANATION OF THE TABLE

The first column contains the wave-lengths of the affected lines, as given in Rowland's table. Italics indicate that the line is of special importance. "R" signifies that the line has been seen reversed by the writer.

The second column gives the element, with a few exceptions, as determined by Rowland. Vanadium lines due to Hasselberg are indicated by "VH," lines due to Kayser and Runge are indicated by "KR."

The third column gives the number of times that the line was seen affected in the spot-spectrum. A — indicates that no observations of the line have been recorded by the writer. As the whole region from F–*a* was examined nearly seven times, on the average, the ratio of this number to seven will give a fair estimate of the frequency of the line.

The fourth column gives the intensity of the line in the normal solar spectrum, as determined by Rowland.

The fifth column indicates the conspicuousness of the line in the spot-spectrum, and represents the amount that the line is affected. The scale, as already mentioned, ranges from 10 to −5; a line marked

10 is strongly affected; 1 indicates that the line is only slightly affected. The negative numbers indicate that the line is less conspicuous than in the photosphere, −5 indicating that the line is obliterated.

The sixth column contains remarks on the appearance of the line, and the manner in which it is affected. No remark denotes that the line is simply widened.

When possible, the observations of others have also been incorporated in the table. These are represented by abbreviations as follows:

Ba and Bb = *Bothkamper Beobachtungen*, Vols. 1 and 2, respectively. These contain observations made during 1870, 1871, and 1872, by H. C. Vogel.

Ys = observations by C. A. Young, made at Sherman, Wyo., *U. S. Coast Survey Report for* 1873.

In both of these the wave-lengths of Ångström's atlas were used, and the identification of some of these lines with the lines on Rowland's map is doubtful.

Y = observations by C. A. Young and Taylor Reed, mostly unpublished. These were made at Princeton during the early summer of 1892, with the same instrument that the writer has used, except that the second-order spectrum on the less dispersive side was employed.

L = observations made at the Solar Physics Observatory, South Kensington, under the direction of Sir Norman Lockyer. The number in parentheses indicates the number of times the line was among the "most widened" lines during 1894. This particular period was chosen as most nearly corresponding, in regard to solar activity, to the time during which the writer's observations were made.

C = observations by A. L. Cortie, *Astrophysical Journal*, **20**, 253, 1904; *Memoirs Royal Astronomical Society*, **50**, 29, 1890.

F = observations by A. Fowler, made at the Royal College of Science, South Kensington, *Monthly Notices*, **65**, 205, 1905.

Quotations and remarks following the abbreviations indicate the behavior of the lines as recorded by the various observers.

It will be found, on comparing these lists of lines reported by previous observers with the table, that various lines have been omitted. This was done only when the evidence of the existence of the line in the spot-spectrum was very slight. In the Bothkamp Observations

TABLE OF AFFECTED LINES

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
F**4861.53	*H*	3	30	..	Occasionally reversed. Ba. Bb. See notes.
4862.03	*Cr*	6	0	9	
4863.83	*Fe*	5	2	6	Darkened.
4864.51	*Ni?*	1	1	10	Bb. Lockyer gives enhanced *Cr* line here.
4864.92	*V*	6	0	9	Bb?
4866.46	*Ni*	3	2	8	Winged. Bb.
4868.45	*Ti*	4	0	6	Darkened. Bb.
4870.32	*Ti*	5	1	7	Darkened.
4870.99	*Ni, Cr*	2	3	4	
4871.51	*Fe*	4	5	5	Winged.
4872.33	*Fe*	4	4	5	Winged.
4875.67	*V*	6	1	10	
4876.06	*Fe*	4	2	6	
4876.59		1	1	−5	
4881.74	*V*	5	1	6	Darkened.
4882.34	*Fe*	2	3	3	Winged.
4883.87	*Yt*	2	2	4	Thickened and hazy.
4885.26	*Ti*	6	2	6	Sometimes widened and sometimes darkened.
4885.96	*Cr*	3	0	4	
4886.13	*Cr*	2	00	5	
4888.71	*Cr*	1	00	5	
4890.95	*Fe*	5	6	8	Winged.
4891.68	*Fe*	5	8	8	Winged.
4893.03	*Fe*	1	1	−1	Widened and winged.
4900.09	*Ti, La*	5	2	4	Darkened.
4904.60	*Ni?*	1	3	2	Hazy and widened.
4907.92	*Fe*	2	2	5	Darkened.
4913.80	*Ti*	5	2	5	Darkened.
4915.41	*Ti*	5	000	4	Short line.
4918.19	*Fe*	1	1	−1	Hazy.
4918.54	*Ni*	1	2	−1	Hazy.
**4919.17	*Fe*	2	6	3	Winged. Ba.
4920.68	*Fe*	2	10	4	Winged.
4921.15	*La*	2	0	2	
4921.96	*La, Ti*	5	1	7	Darkened.
4925.75	*Ni*	2	1	−1	Weakened and hazy.
4926.33		6	000	8	
4928.51	*Ti*	3	0	5	Darkened, long line.
4930.98	*Ni*	2	00	2	

TABLE OF AFFECTED LINES—*Continued.*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
4935.05		1	0000	4	Darkened.
4937.90	*Ti*	5	000	5	
4938.99	*Fe*	2	4	7	Long line.
4939.87	*Fe*	2	3	8	Long line.
4942.66	*Cr*	2	2	4	Hazy.
4952.46	*Fe*	2	1	4	Weakened once.
4952.82	*Fe*	5	2	5	Weakened twice.
4953.39	*Ni*	2	2	4	Weakened once.
4954.99	*Cr*	2	2	4	Darkened.
4957.48	*Fe*	2	5	4	Winged. Ba.
4957.79	*Fe*	2	8	4	Winged. Ba.
4958.43	*Ti*	2	00	5	
4965.11	*Cr*	1	1	10	
4966.04	*Mn*	2	00	4	
4968.08	*Fe*	2	3	7	Hazy.
4968.77	*Ti*	2	0	3	
4970.10	*Fe*	3	3	8	Hazy.
4975.53	*Ti*	2	00	4	Darkened.
4978.37	*Ti*	5	00	7	Widened and darkened.
4978.78	*Fe*	2	3	3	Hazy.
4980.35	*Ni*	2	4	3	Winged.
4981.91	*Ti*	3	4	4	Winged. Bb.
4982.99		2	2	8	
4986.16	*Cr*	4	00	5	
4989.33	*Ti*	3	00	4	Darkened, long line.
4991.25	*Ti*	1	3	9	
4997.28	*Ti*	4	0	9	On one occasion was strongest line in this region.
4998.41	*Ni*	1	1	−5	
4999.69	*Ti–La*	1	3	−3	Also winged.
5000.53	*Ni*	2	2	4	Widened and hazy.
5002.04	*Fe*	2	5	4	
5007.91		1	00	6	
5009.83	*Ti–Co*	6	00	10	Darkened, long line.
5013.48	*Cr–Ti*	2	2	5	
5016.34	*Ti*	1	2	5	
5016.66		1	00	4	
5017.76	*Ni*	2	3	4	Winged.
**5018.5	*Ni–Fe*	1	1–4	5	Enhanced line of *Fe*.
5020.21	*Ti*	4	2	7	
5021.78	*Fe*	1	0	6	Hazy.
5022.11	*Cr*	2	000	5	Darkened.
5023.05	*Ti*	3	2	8	Darkened.
5025.03	*Ti*	3	3	8	Darkened.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5025.75	*Ti*	3	1	7	
5027.31	*Fe*	2	3	5	Winged.
5027.94	*Fe*	2	1	6	Almost obliterated once.
5036.65	*Ti*	3	2	7	Darkened.
5038.58	*Ti*	4	2	6	Darkened.
5039.54	*Ni*	3	00	-4	Always much weaker and hazy.
5040.14	*Ti*	2	3	9	Darkened.
5040.79	*Ti*	3	00	6	Darkened.
5043.76	*Ti*	3	00	2	
5044.39	*Ni-Co-Fe*	1	3	5	Winged.
5045.58	*Ti*	3	00	3	L (10).
5048.61	*Fe*	1	3	5	Darkened.
5052.08	*Cr*	4	0	5	
5053.06	*Ti*	2	0	1	
5060.26	*Fe*	2	3	5	Darkened.
5062.29	*Ti*	1	0	5	L (37).
5064.84	*Ti*	1	3	4	Widened.
5066.17	*Ti*	3	000	5	Darkened.
5071.66	*Ti*	2	0	3	
5073.11	*Cr*	2	1	8	
5080.71	*Ni*	2	4	-3	Hazy.
5081.76		1	000	4	
5081.94		1	000	4	
5082.53	*Ni*	4	2	-4	Once obliterated.
5087.24	*Ti*	4	0	6	L (33).
5109.83	*Fe*	1	2	-3	
5113.30	*Cr*	2	00	3	
5113.62	*Ti*	3	0	3	Darkened.
5120.59	*Ti*	2	0	-2	Widened.
5121.75	*Ni-Fe*	1	2-0	2	Thinned.
5122.16	*Cr*	2	000	7	
5122.30	*Cr*	2	000	3	Hazy.
5122.61		2	000	7	
5122.97	*Co*	3	000	5	
5123.39	*Y*	1	0	8	
5123.64	*Cr*	2	000	4	
5127.86		2	00	3	
*5129.55	*Ni*	1	2	4	Darkened.
5129.81	*Fe*	4	1	-5	Always obliterated.
*5131.64	*Fe*	1	2	-3	Widened and hazy.
5131.94	*Ni*	2	4	3	
5132.84		1	00	4	
5134.70?		—	000	..	L (123). "Band-line."

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5136.27	*Fe*	1	00	5	Darkened. L (106)
5137.25	*Ni*	2	3	6	Winged.
5138.7		1	....[1]	5	L (130).
5139.4		1	....	5	Band-line.
5139.5	*Fe*	1	4-4	3	Winged. Bb "einseitig nach dem Violett verbreitert."
5140.33		3	0000	10	Band-line.
5141.38		3	000	9	Band-line.
5141.92	*Fe*	3	3	2	Winged. Bb. L (2).
5142.45		3	0000	9	Band-line.
5143.90		3	0000	10	Band-line.
5144.20		3	0000	10	Band-line.
5144.85	*Cr*	3	00	8	
5145.27	*Fe*	3	1	9	Much widened, hazy. Ys.
5145.64	*Ti*	4	0	6	Darkened, long line. Bb.
5146.0		1	....	9	Band-line.
5146.66	*Ni*	2	3	5	Bb.
5147.65	*Ti*	4	0	8	Darkened, long line. Bb. widened toward blue.
5148.0		2	....	7	Band-line.
5148.41	*Fe*	3	3	6	Darkened.
5149–5150		..	....	..	Three heavy shades here.
5150.4		1	....	6	L (9).
5150.74		1	000	5	Band-line.
5152.36	*Ti*	4	0	9	Darkened, long line. Bb. and Ys give the *Fe* line.
5155.30	*Ni*	1	1	4	
5155.94	*Ni*	2	2	5	
5156.24		1	000	8	Band-line.
5156.78		1	0000	9	Band-line.
5156.82		1	00	10	Band-line. L (76).
5159.23	*Fe*	1	2	-4	Bb "stark nach dem Violett verbreitert."
5163.07		1	000	2	Band-line. L (36).
5163.7		6	....	..	Bright streak.
5164.73	*Fe?*	1	1	-3	
5165.59	*Fe*	3	2	-3	Usually weakened.
5166.45	*Cr–Fe*	1	3	4	Bb.
5166.9		1	....	9	Band-line. No corresponding dark line.
b_4**5167.50	*Mg*	—	15	..	Ba. Bb. Ys.
5168.3		1	....	5	Band-line.
5168.83	*Ni*	1	1	-3	

[1] Lines not present in the photospheric spectrum are indicated by (....).

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
b_3**5169.22	*Fe*	5	4	−4	The lower component of b_3, always thinned; the line appears notched on red side. Ys.
b_2**5172.86	*Mg*	—	20	..	Ba. Bb. Ys.
5176.95	*V*	5	000	7	
5177.41	*Fe*	2	0	4	Darkened. Bb?
5178.97		2	000	4	Band-line.
b_1**5183.79	*Mg*	—	30	..	Ba. Bb. Ys.
5184.45	*Fe*	2	2	5	Hazy.
*5186.07	*Ti*	1	2	−3	
*5188.08	*Fe*	1	1	−3	
5188.3		1	....	4	Band-line.
5188.86	*Ti*	1	2	2	Bb "nach dem Violett verwaschen." Ys.
5191.63	*Fe*	2	4	3	Reversed? Bb. Ys.
*5195.11	*Fe*	1	4	5	Winged.
5195.65	*Fe*	1	2	4	Winged.
5196.23	*Fe*	4	1	−3	Usually weakened.
5196.61	*Cr.*	3	0	5	Darkened.
5198.11		2	0	5	
5198.7		2	....	8	Band-line. No corresponding dark line.
*5198.89	*Fe*	3	3	6	Widened and hazy.
*5200.36	*Cr*	1	00	3	
5200.59	*V*	2	0	4	Darkened. Bb?
*5202.52	*Fe*	4	4	5	Winged. Bb. Ys.
*5204.77	*Fe–Cr*	4	3–5	5	Thinned and winged. Bb. Ys. Y.
*5205.90	*V*	1	0	−4	
*5206.22	*Cr–Ti*	4	5	5	Thinned and winged. Bb. Ys. Y.
*5208.60	*Cr*	4	5	5	Thinned and winged.
*5210.56	*Ti*	2	3	5	Darkened. Bb "sehr stark verbreitert." Ys.
5218.08	*Fe*	3	0	5	
5218.37	*Fe*	3	1	5	
5219.88	*Ti*	5	0	10	Long line. L (8).
5220.25	*Cu* KR	4	000	3	L gives (9) on λ5220.
5221.93	*Cr*	2	0	4	
5222.56	*Cr*	1	00	3	
5222.85	*Ti–Cr*	3	00	3	
5223.35	*Fe*	2	0	3	Darkened.
5223.53	*Ti*	2	0000	5	Darkened.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5223.79	*Ti?*	1	000	3	Ys?
5224.47	*Ti*	3	0	5	Darkened. Bb "sehr breit nach dem Violett verwaschen."
5224.71	*Ti–Cr*	2	00	4	
5225.20	*Cr-Ti-Fe*	4	00	7	Bb?
5225.70R	*Fe*	3	2	6	Reversed twice. Ys.
5225.97R	*Cr*	2	000	5	Reversed beyond umbra once.
**5226.71	*Ti*	2	2	..	Obliterated? once, reversed? once.
*5227.04	*Fe–Cr*	4	3	−4	Weakened three times.
5228.55		2	1	3	Bb.
5230.38	*Co–Cr*	4	00	8	Bb and Ys both give the *Fe* line.
*5233.12	*Fe*	2	7	5	Winged.
**5234.79		2	2	−3	Thinned and winged.
5235.35	*Co*	2	000	4	Bb?
*5237.49	*Cr?*	2	1	−3	Bb.
5238.74	*Ti*	6	000	7	Bb?
5239.14	*Cr*	6	00	6	
5241.04	*V?*	2	000	2	
5242.66	*Fe*	1	2	6	Bb.
5243.95	*Fe*	1	1	4	Bb.
5247.23R	*Fe*	2	1	4	Narrow reversal once.
*5247.74R	*Cr*	4	2	5	Reversed twice.
5248.09	*Co*	2	000	3	
5249.28	*Fe*	2	00	3	Hazy.
5250.39R	*Fe*	7	2	8	Reversed three times, once in region preceding spot.
5252.28	*Ti*	4	000	7	Bb.
5253.42		2	0000	4	Bb.
5255.30	*Cr*	1	0	4	Bb?
5255.49	*Mn*	3	0	6	
5257.81	*Ti?*	2	0	4	Ys?
5260.14		2	000	5	
**5260.56R*	*Ca*	6	0	5	Reversed three times. Bb.
5264.42	*Ca*	2	3	3	Darkened.
5265.32	*Cr*	2	00	4	
5266.14	*Ti*	4	0	5	
E_2**5269.72	*Fe*	2	8	5	Winged. Bb. Y.
*5275.34	*Cr*	3	00	6	Bb? Ys? Y "triplet, lower line int. 2, and other two disappear."

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5275.93R	*Cr*	5	1	6	Reversed once, reversed? twice.
**5276.17	*Fe*	1	3	4	Generally not affected. Bb. Ys.
5280.54	*Fe*	1	1	5	Bb "nach dem Violett verwaschen."
5280.80	*Co*	2	00	4	
5282.58	*Ti*	3	00	4	
5284.60	*Fe–Ti*	2	00	3	
5295.95	*Ti?*	6	00	8	Y.
5296.87	*Cr*	3	3	7	Long line. Bb. Y.
5297.41	*Cr–Ti*	4	000	6	Long line. Hazy.
5298.46	*Cr*	2	4	−2	Ys?
5298.67	*Ti*	2	0	−2	Weakened, or reversed? Bb.
5300.15		1	00	3	Y.
5300.93R	*Cr*	3	2	4	Reversed once.
5301.09		2	0000	4	Bb.
5302.48	*Fe*	1	5	4	Bb.
5307.54	*Fe*	1	3	3	
5321.29	*Fe*	2	2	4	Long line.
5324.37	*Fe*	2	7	6	Winged. Y "heavily winged."
*5328.24	*Fe*	2	8	5	Winged. Y "heavily winged."
*5329.33	*Cr*	1	3	5	
5329.96	*Cr*	3	0	5	Hazy.
5340.12	*Fe*	1	6	−2	Winged. Ys.
*5341.34	*Fe–Mn*	2	7–1	−2	Winged. Ys.
5343.15		2	0000	5	Band-line.
*5345.99	*Cr*	2	5	−2	Winged. Ys. Y.
5348.51	*Cr*	2	4	−2	Narrower and winged. Ys. Y.
5349.65	*Ca*	2	4	−2	Narrower and winged. Ys. Y.
5351.26	*Ti*	3	00	4	Darkened.
*5365.07	*Fe*	1	5	7	Winged.
5366.83		3	000	5	
5369.78	*Co–Ti*	2	1	6	Darkened.
**5371.70	*Fe–Cr?*	2	4	8	Winged and thinned. Ys.
5373.91	*Fe–Cr*	3	2	6	Hazy, reversed? once.
5383.57	*Fe*	2	6	5	Winged.
5387.17	*Cr–Fe*	2	0	4	
5387.77	*Cr*	2	00	4	
5389.37		5	000	5	

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5390.05		2	00	4	
*5393.37	*Fe*	2	5	5	Winged. Y.
5394.85R	*Mn*	6	1	7	Reversed once, partially reversed once, line appeared "twisted." Y.
5396.78	*Ni*	2	000	6	Shaded toward red once
*5397.34	*Fe*	2	7	8	Winged. Ys. Y winged.
5399.67R	*Mn*	2	1	9	Reversed twice.
5401.47		1	0	9	Wide and hazy, reversed?
5404.36	*Fe*	1	5	4	Winged. Ys.
*5405.99	*Fe*	2	6	8	Winged. Ys. Y "heavily winged."
5407.69	*Mn*	3	1	5	"Twisted" once.
5409.02		3	000	8	Band-line. Y.
5409.34R	*Fe*	1	2	3	Narrow reversal.
*5410.00	*Cr*	2	4	4	Winged. Y.
5412.99	*Mn*	1	00	4	
5413.89	*Mn*	2	00	5	
*5415.42	*Fe–V*	1	5	3	Winged. Ys. Y.
5420.55	*Mn*	4	0	7	"Twisted" once. Ys. Y.
5424.29	*Fe*	2	6	4	Winged. Ys. Y.
**5425.46		1	1	−4	Y "weakened."
5426.47R		7	00	10	Always one of the most prominent lines, reversed once. L (108)
5429.35	*Ti?*	3	00	3	
**5429.91	*Fe*	3	6	4	Winged. Y.
***5432.75R*	*Mn*	6	1	9	Reversed three times. Ys. Y.
*5434.74	*Fe*	2	5	4	Winged. Ys. Y winged.
5436.51	*Fe*	1	1	−2	Winged.
5436.80R	*Fe*	3	1	5	Reversed twice.
5442.63	*Cr*	1	00	7	
**5447.13	*Fe*	2	6	4	Winged. Ys. Y winged.
5453.86		2	000	4	Darkened. Y.
5460.72		6	000	9	Ys. L (20).
5461.76R		4	00	5	Reversed three times.
5462.71R	*Ni*	4	1	9	Reversed four times. Y reversed.
5464.18	*Cr*	1	000	4	
5466.61	*Fe*	2	3	3	Winged.
5467.20	*Fe*	2	1	5	Reversed? once.
5470.80R	*Mn*	5	0	10	Reversed twice, usually very wide and hazy. Y.
5471.41	*Ti*	5	000	8	Darkened. Y.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5474.44	*Ti*	3	00	5	
5477.90	*Ti*	4	00	7	Darkened, short line
*5481.5	*Fe–Ti*	1	1	5	Y "duplicity vanishes without apparent widening."
5482.08	*Ti?*	3	00	8	Y.
5483.31R	*Fe*	1	1	4	Narrow reversal.
5483.56	*Co*	4	1	5	Darkened. Y.
5488.37	*Ti?*	2	00	2	Y.
5490.37	*Ti*	5	0	6	Darkened. Ys. L (4).
5493.71R	*Fe*	2	1	5	Reversed twice.
5494.68R	*Fe*	2	0	5	Reversed twice.
5495.10R	*Ni*	1	00	8	
*5497.74	*Fe*	4	5	4	Reversed? once, generally winged. Y.
*5501.68	*Fe*	2	5	8	Winged. Ys. Y.
5504.12	*Ti*	3	0	4	Darkened. Ys. Y.
5506.09R	*Mn*	3	1	8	Reversed once, generally much widened. Ys. Y.
*5507.00	*Fe*	1	5	7	Winged. Ys. Y.
5511.87R	*Fe*	1	0000	8	
5512.74	*Ti*	2	2	5	
5513.20	*Ca*	2	4	5	Winged.
5514.56	*Ti*	3	2	4	Darkened. Ys.
5514.75	*Ti*	3	2	4	Darkened.
5516.95	*Mn*	2	0	6	Hazy. Ys.
5522.66	*Fe*	1	2	4	
5525.77	*Fe*	2	2	5	Hazy.
5530.99	*Ti*	2	00	3	Darkened. Y.
5537.93R	*Mn*	6	00	8	Reversed three times. Y reversed. Ys.
5538.74R	*Fe*	5	1	9	Reversed four times. Y reversed. Ys.
5544.16R	*Fe*	2	2	5	Narrow reversal twice.
5546.73R	*Fe*	3	2	4	Reversed twice.
5547.22	*Fe–V*	3	1	5	Darkened. Ys. Y.
5556.0		1	....[1]	6	Band-line.
5565.70	*Ti*	5	00	5	Darkened. Y.
5573.08	*Fe*	2	6	5	Winged. Ys. Y.
5573.33		1	1	3	
5582.20	*Ca*	2	4	5	Winged. Y.
5582.9		1	....	5	Band-line.
5583.1		1	....	4	Band-line.
5584.53R	*V?*	4	000	4	Reversed once. Y reversed. Ys.

[1] Lines not present in the photospheric spectrum are indicated by (....).

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
*5586.99	*Fe*	2	7	4	Winged. Bb. Ys.
*5588.99	*Ca*	3	6	4	Widened and winged. Bb. Ys.
5590.34	*Ca*	3	3	4	Widened and winged. Y.
5594.69	*Ca*	1	4	4	Darkened and winged. Ys.
5598.71	*Ca*	1	4	4	Darkened and winged. Bb. Ys.
5600.45	*Fe*	2	3	2	
5619.82R		2	0	4	Narrow reversal once.
5620.72R	*Fe*	2	0	4	Narrow reversal once.
5625.09		4	000	5	Y.
5626.26		3	....	4	Y. No corresponding dark line.
5627.86	*V*	6	00	8	Ys. Y. L (14).
5628.87	*Cr*	3	00	3	Y. L (3).
5636.93R	*Fe*	2	0	4	Reversed once.
5637.63R	*Fe*	1	1	5	Reversed once.
5641.21R		2	1	5	Reversed twice.
5644.37	*Ti*	4	0	5	Darkened. Y.
5645.83	*Si*	—	1	−4	Y "almost disappears."
5646.04	*V?*	3	00	4	
5648.79	*Ti*	3	00	4	Y.
5654.09R	*Fe*	1	1	3	Narrow reversal.
5657.66	*V?*	5	000	5	Reversed ? twice. Y.
**5658.10R	*Y-*	2	2	5	Reversed twice. Y.
5662.37	*Ti*	2	0	4	Ba. Y.
5663.16	*Ti-Fe-Y*	2	1	4	Hazy. Ys. Y.
5667.37R		1	0	9	Wide reversal.
*5667.74R	*Fe*	1	2	9	Wide reversal.
5668.59R	*V*	6	000	6	Reversed once. Y.
*5669.26R		1	1	3	Narrow reversal. Y "almost disappears."
5671.07	*V*	6	0	9	Ys. Y. L (161).
5672.05	*Sc*	6	0	8	Ys. Y. L (156).
5680.15		3	000	4	Y.
5682.87	*Na*	3	5	3	Slightly widened and winged. Ba. Ys. Y.
5684.42		1	1	−3	Hazy.
5684.71	*Si*	2	3	−4	
5684.95		1	0000	8	
5687.06		4	000	7	Reversed ? once. Y.
5688.44	*Na*	1	6	3	Winged. Ba. Ys.
5689.69	*Ti*	5	0	6	Y. L (2).
5690.65	*Si*	2	3	−4	
5694.96	*Cr*	4	0	3	Y.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5698.55	*Fe–Cr*	2	1	4	Ys? Y.
5698.75	*V*	6	1	8	Darkened. Y.
5700.51	*Cu*KR	7	00	4	Always darkened. Y. L (5).
5701.32	*Si*	1	1	−4	Y "cut out."
5702.54	*Cr*	4	0	4	Reversed? once.
5702.87	*Ti*	2	000	4	Y.
5703.80	*V*	6	1	8	Darkened. Y. L (12).
5707.20	*V*	5	0	7	Darkened. Y. L (5).
5708.32	*Fe*	3	1	2	Ba.
5708.62	*Si*	2	3	−4	
5712.10	*Fe*	5	3	4	Widened on blue side twice. Y.
5712.36R	*Fe*	5	2	5	Reversed five times. Y.
5712.99	*Cr*	2	0	2	Y.
5714.12		2	000	3	Y.
5716.67	*Ti*	7	00	3	Y.
5720.66	*Ti*	4	0	2	Y.
5727.27	*Ti–V*	4	2	8	Not always affected. Y.
5727.87R	*Cr?*	7	00	10	Always much affected. Ba. Ys. Y. L (120). Reversed once.
5731.44R	*V*H	7	00	10	Reversed three times. Ys. Y. L (106).
5731.98	*Fe*	1	4	−2	Hazy.
5737.29	*V*H	6	0	7	Reversed? once. Ys. Y. L (10).
5737.90	*Mn*	1	1	3	Darkened. L gives (107) on λ5737.8.
5739.87R		5	000	4	Reversed once.
5740.37		5	0000	3	
5742.07R	*Fe*	1	2	4	Narrow reversal. Ys. L(1).
5743.18		2	0	5	L (76).
5743.65		4	00	5	Reversed? once. Ys. Y.
5746.64	*A*-	2	000	6	Reversed? twice.
5748.17R	*Fe*	3	2	5	Reversed twice.
5748.57R	*Ni*	5	2	7	Reversed four times. Y reversed.
5752.25	*Fe*	1	4	3	Hazy.
5753.34	*Fe*	1	5	3	Hazy.
5754.88	*Ni*	2	5	2	Hazy. Ba. Y.
5760.57	*Fe*	2	1	−2	Hazy.
5761.05	*Ni*	1	2	−2	Hazy.
5762.49	*Ti*	5	000	4	Y.
5766.55	*Ti*	6	0	4	Y.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
5771.82		1	00	3	Y.
5774.25	*Ti*	6	0	5	Y.
5778.68	*Fe*	2	1	3	Y.
5781.13R	*Cr–Ti*	3	00	5	Reversed twice.
5781.40R	*Cr*	6	0	10	Generally widely reversed, but difficult on account of faintness of line. Y reversed.
5781.97R	*Cr*	6	0	10	Similar to λ5781.40. Y reversed.
5783.29R	*Cr*	5	2	7	Reversed five times. Y reversed.
5784.08R	*Cr*	4	3	5	Reversed three times. Y reversed.
5784.88R	*Fe*	4	1	3	Reversed four times. Y reversed.
5785.19R	*Cr*	5	2	5	Reversed four times. Y reversed.
5785.50R	*Fe*	4	3	4	Reversed twice.
5785.95	*Cr*	3	1	4	Y.
5786.19R	*Ti–Cr*	3	0	3	Reversed twice. Y.
5794.14	*Fe*	2	2	3	Y.
5798.08		4	3	4	Ba.
5804.48	*Ti*	2	0	5	Y.
5804.68	*Fe*	—	0	−5	Y "vanishes in spot."
5823.91		1	00	2	Y.
5828.10		1	0	3	Hazy.
5847.22	*Ni*	4	1	3	Reversed? once.
5848.34	*Fe*	2	3	2	
5852.44	*Fe*	2	3	2	
5853.54		5	000	3	
5856.31	*Fe*	2	2	3	
5857.67	*Ca*	—	8	..	Ba. Bb. Ys. Y. C. [Not recorded by the writer.
5859.81	*Fe*	1	5	3	Winged. Bb. C.
5862.58	*Fe*	1	6	3	Winged. Bb. C.
5866.68	*Ti*	6	3	4	Darkened. Bb. Ys. Y. L (2). C.
5867.79	*Ca*	3	2	2	Ys. Y. L. (18).
5873.44		1	1	1	Ba. L (3).
D_3**5875.98	*He*	3	....	3	Faint shade. See notes.
5880.49R		7	000	10	Faint line, always widely reversed.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
D_2**5890.19	*Na*	4	30	6	Winged. See notes.
D_1**5896.16	*Na*	4	20	6	Winged. See notes.
5899.52	*Ti*	5	1	6	Darkened. C. F.
5902.69R	*Fe*	1	0	8	
5910.20R	*Fe*	3	1	7	Reversed three times. Y.
5916.47	*Fe*	3	3	3	Reversed ? once. F.
5918.77R	*Ti*	3	0	5	Reversed once. Y. F.
5922.33	*Ti*	4	0	5	Darkened. Y. F.
5929.90	*Fe*	1	2	4	Hazy.
5938.04R		6	000	6	Reversed three times. Y.
5941.98	*Ti*	7	00	7	Darkened. Y. F.
5944.95	A (wv)	1	1	5	Darkened. Y. C. F.
5949.56	*Fe*	1	1	4	F.
5952.94	*Fe*	2	4	3	C.
5953.39	*Ti*	5	1	6	C. F.
5956.92	*Fe*	3	4	5	Y. C. F.
5958.46R	*Fe* KR	1	1	8	Reversed once.
5966.05	*Ti*	6	2	7	Ys. Y. C. F.
5978.77	*Ti*	6	1	8	Ys. C. F.
5984.81		3	0000	4	
5989.51	A (wv)	2	0	3	Hazy. Y. C.
5999.92R	*Ti*	7	0	6	Reversed twice. C. F.
6002.97R		4	0000	4	Reversed twice.
6005.77R	*Fe*	6	1	5	Always reversed. C. F.
6007.54	*Ni*	2	1	3	
6008.19R	*Fe*	7	4	6	Always reversed. Ba. Ys. C. F.
6012.45R	*Ni*	5	1	5	Always reversed. Y reversed. C. F.
6013.72R	*Mn*	7	6	5	Reversed four times. C. F.
6016.86R	*Mn*	7	6	5	Reversed four times. C. F.
*6022.02	*Mn*	4	6	3	Reversed ? once. C. F.
*6024.28	*Fe*	1	7	3	Y. C.
6032.0?R		1	....	4	Reversed once. No dark line in spectrum.
6039.95R	*V*	6	0	7	Reversed four times. Ys. C. F.
6053.91R	*Ni*	2	0	5	Reversed once. C. F.
6058.3		4	....	6	Hazy line. Y. C.? F. No corresponding dark line.
6063.08		4	.0	5	Ys. C. F.
6064.85R	*Ti*	6	00	9	Always widely reversed. Y. C. F.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
6078.71	*Fe*	2	5	3	Darkened.
6079.23R	*Fe*	7	2	10	Always strongly reversed. C combines both *Fe* lines.
6081.67R	*V*	7	0	9	Reversed three times. Ys. Y. C. F.
6082.93R	*Fe*	7	1	10	Always strongly reversed, reversed in region following spot once. Y reversed. C. F.
6085.47R	*Ti–Fe*	5	2	6	Reversed once. Ys. C. F.
6089.79R	*Fe*	5	1	4	Reversed twice. C.
6090.43R	*Ti–V*	7	2	7	Reversed twice. Y. C. F.
6091.40R	*Ti*	5	0	5	Reversed once. Y. C "obliterated once." F.
6093.03R	*Ti?*	6	000	3	Reversed twice. Y.
6093.37R	*Mn?*	6	00	4	Reversed once. Y.
6096.88R	*Fe*	3	3	4	Reversed once. Y. C. F.
6098.46R		2	0	3	Reversed twice. C. F.
6098.87	*Ti?*	7	00	3	Y.
*6102.94R	*Ca*	7	9	7	Reversed three times. Ba. Ys. Y. C. F.
6111.87	*V*	7	0	8	Reversed? C does not give it! F. Y.
6119.74	*V*	6	1	8	Y. C. F.
**6122.43	*Ca*	4	10	5	Winged. Ba. Bb. Ys. Y. C. F.
6126.44R	*Ti*	6	1	6	Reversed twice. Y. C. F.
6127.85R		3	0000	5	Reversed once.
6129.19R	*Ni*	7	1	3	Reversed twice. C. F.
6131.79R		1	0	4	Reversed once. C.
6132.07R		1	0	5	Reversed once.
6134.81		3	000	4	
6135.58	*V*	7	00	4	Darkened. Y. C. F.
6135.98	*Cr*	3	00	4	
*6136.83	*Fe*	2	8	3	Winged. C.
6137.21R	*Fe*	9	3	15	The most strongly reversed line in the spot; always reversed; reversal generally extends into and beyond the penumbra, though sometimes on the following side only.
*6137.92	*Fe*	2	7	3	Winged. C.
6143.39		2	0000	3	

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
6145.23		—	2	..	Y "almost vanishes." C "obliterated once."
6146.45		3	000	4	Y.
6150.36R	V	7	0	8	Reversed once. Ys. Y. C. F.
6151.83R	*Fe*	7	4	10	Always reversed. Y reversed. C.
**6154.44R*	*Na*	7	2	7	Reversed three times. Bb. Ys. Y. C. F.
6156.24R		2	00	3	Reversed once. C. F.
*6160.96	*Na*	4	3	6	Winged. Ys. Y. C. F.
6161.50	*Ca*	4	4	5	Ba. Y. C. F.
**6162.39	*Ca*	3	15	4	Winged. Ba. Bb. Ys. Y. C.
6163.97	*Ca*	5	3	7	Reversed? twice. Y. C.
6170.42	V	8	0000	4	Never strongly affected. Y. C.
6170.73R	*Fe–Ni*	2	6	5	Reversed twice.
**6173.55R*	*Fe*	7	5	12	Always widely reversed, frequently beyond penumbra. Ba. C. Y.
6186.93	*Ni*	2	2	5	Y. C.
6188.21R	*Fe*	6	4	8	Always reversed. C. F.
6191.39R	*Ni*	7	6	4	Reversed three times. C.
6199.40R	V	8	0	8	Hazy reversal twice. Y. C. F.
*6200.53R	*Fe*	4	6	4	Reversed twice. Ys. C.
6204.83R	*Ni*	2	1	3	Reversed twice. C.
6210.90R		7	00	7	Reversed once. C. F gives origin as *Sc*.
6213.64R	*Fe*	7	6	10	Always reversed. C.
6214.08R	V	7	000	4	Hazy reversal twice. C does not give it. F.
**6216.57R*	V	7	1	12	Reversed five times. Y. C. F.
*6219.49	*Fe*	5	6	5	C.
*6221.55	*Fe*	4	00	3	F.
6224.71R	V	7	000	8	Hazy reversal twice. Ys. Y. C. F.
6226.95R	*Fe*	3	1	3	Reversed twice. C "obliterated once."
6229.44	*Fe*	3	1	4	C "obliterated twice."
***6232.86R*	*Fe*	7	3	9	Always reversed. C. F.
6233.1?R		5	....	4	Strongly reversed twice, apparently no dark line at this point. F.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
6233.41R	V?	3	000	7	Reversed once. Y.
**6238.60		1	2	−4	Y "weakened." C "obliterated once."
6240.17		3	00	4	C. F.
6240.86	*Fe*	5	3	6	Reversed? twice. C.
6243.06R	V	6	000	4	Hazy reversal twice. Y. C. F.
6243.32R	V	7	1	10	Hazy reversal three times. Y. C. F.
6244.03		2	2	−3	C.
6244.68		2	2	−3	C.
6246.54R	*Fe*	1	8	5	Reversed once. C.
**6247.77		2	2	−4	Y "much weaker." C.
6252.05R	V	8	00	5	Wide hazy reversal twice. Ys. Y. C. F.
6257.09	V?	6	000	3	Y.
6258.32	*Ti*	5	2	5	Darkened. C. F.
6258.93	*Ti*	5	3	5	Darkened. C. F.
6261.32	*Ti*	3	1	4	C. F.
6261.50	V	1	0000	2	Y. F.
6265.35	*Fe*	2	5	3	C.
6266.55R	V	6	000	3	Hazy reversal once. Y reversed. C. F.
6269.08R	V	7	000	4	Hazy reversal once. Y reversed. C "obliterated." F.
6271.48	*Fe*	3	0	3	Hazy. Y. C. F.
6274.87R	*V*	7	00	8	Hazy reversal twice. Y. C. F.
6280.83	*Fe*	2	3	7	Strongly darkened and widened once. Ba. C. F.
6285.38R	V	7	00	7	Hazy reversal twice. Y. C "hazy once." F.
6293.03R	V	6	000	5	Hazy reversal once. Y. C "darkened once." F. Ba.
6296.58R	V	7	0000	7	Hazy reversal three times. Y. C not mentioned. F.
6298.00	*Fe*	3	5	4	Fuzzy. C.
*6301.72	*Fe*	3	7	4	Fuzzy. Displaced toward violet once, also by C. F.
*6302.71R	*Fe*	5	5	7	Reversed without widening three times. Y. C.
6304.1		3	....	6	No dark line here. Y.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
6305.88		6	0000	7	Y.
6311.72	*Fe*	2	1	4	Y reversed. C.
6312.46		4	00	4	C.
6319.46		1	0	−3	C.
6322.91	*Fe*	1	4	9	C.
6327.82R	*Ni*	3	2	5	Reversed once. Y. C. F.
6330.31R	*Cr*	6	1	9	Reversed five times. Y reversed. C. F.
6331.06R	*Fe*	4	2	5	Reversed twice. C.
*6335.55	*Fe*	5	6	4	Y. C.
6336.33	*Ti*	6	000	4	Y.
*6337.05R	*Fe*	5	7	5	Reversed twice. Y. C.
6344.37	*Fe*	4	4	4	Ys C.
**6347.31		1	2	−4	Hazy. Ba. Ys. C.
6355.25	*Fe*	2	4	5	C.
6358.90	*Fe*	3	6	5	Reversed ? twice. Ys. C.
6363.09R	*Cr–Fe*	7	2	10	Widely reversed three times. Y. C. F.
6366.56	*Ti*	3	000	5	Darkened. F.
6370.57	*Ni*	—	00	−5	Y "extinguished in spot-spectrum."
6392.75		2	0	2	Hazy. C. F.
6400.54R	*Fe*	4	2	4	Reversed once, reversed ? twice. Y weakened. C "weakened once, almost reversed once." F.
6415.20		1	1	−4	Y "obliterated." C "less dark twice."
**6417.13	*Fe* KR	—	1	−5	Y "disappears in spot." C "obliterated six times."
**6432.89	*Fe* KR	3	1	−4	Y "disappears in spot." C.
6439.29	*Ca*	2	8	3	Winged. Bb. Ys. C. F.
6441.16	*Mn ?*	2	000	3	Hazy.
6450.03R	*Ca*	2	6	5	Reversed once. Ba. Bb. Ys. C. F.
6452.54		2	00	4	C.
6455.82R	*Ca*	5	2	6	Widely reversed once. Ba. Y. C. F.
**6456.60		3	3	−4	Y "almost disappears." C "darkened."
*6462.78	*Ca*	3	5	4	Ba. Bb. C. F.
6462.96	*Fe*	3	3	4	Ba. Bb. C. F.
6464.90		4	00	3	Y. C. F.
6471.89	*Ca*	3	5	4	Y. C. F.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
6475.85		4	2	8	Widened and darkened. C.
6482.10R		3	3	5	Reversed without widening once, line "twisted."
6483.03	*Ni*	2	1	5	C.
6494.00	*Ca*	2	6	3	Winged. Bb. Ys? C. F.
*6495.21	*Fe*	2	8	3	Winged. C. F.
6496.69	*Fe*	3	2	−4	
6499.17R	*Fe*	6	1	8	Reversed four times.
6499.88	*Ca*	4	4	6	Winged. Bb. Y. C. F.
6532.0	*V?*	7	0000	5	Ba. Y.
6533.11	*Ni?*	3	0	2	
6538.5		3	....	3	Faint shading.
6546.48	*Ti–Fe*	1	6	−3	Y "weakened." C. F.
6554.47	*Ti*	7	0	9	Ys. F. Not given by C!
6556.31	*Ti*	7	1	9	F. Not given by C!
C***6563.05*	*H*	8	40	..	See notes.
6569.46	*Fe*	3	5	2	Reversed? once. Y. C.
6573.03	*Ca?*	9	1	12	Bb. Ys. Y. C.
6574.47R		9	1	10	Reversed twice. Y. Not given by C!
6575.27	*Fe*	4	2	4	Darkened and winged. C.
6581.45R		5	0	4	Reversed once. Y. C.
6586.55	*Ni*	3	1	3	Darkened. C.
6593.16	*Fe*	4	6	5	Winged. Bb. C.
6594.12	*Fe*	4	4	4	Winged. C.
6597.81	*Cr*	2	1	4	
6599.35	*Ti*	7	00	8	Y.
6605.81		1	000	2	Hazy. Y.
6606.16		1	000	4	
6608.28		2	0	5	C.
6609.36	*Fe*	4	3	5	Darkened. C.
6609.82		2	00	3	
6625.28		7	0	8	Y. C.
6630.27	*Cr*	7	000	5	
6632.71	*Co?*	2	00	2	
6633.99R	*Fe*	3	2	5	Reversed once. C.
6640.00		2	0	2	C.
6661.32	*Cr*	2	00	2	
6663.70	*Fe*	3	3	2	Dark and hazy. Y. C.
6678.24	*Fe*	1	5	4	Darkened. See notes for *He* line.
6696.27		4	1	7	Y.
6698.91		3	0	3	
6703.82		5	1	4	C.

TABLE OF AFFECTED LINES—*Continued*

Wave-Length	Element	No. Obs.	Solar Inten.	Spot-Inten.	Remarks and Other Observations
6705.35		4	1	4	C.
6707.69	*Ti?*	3	000	5	Y.
6710.57		2	0	5	
6717.94	*Ca*	3	5	5	Ys. Y. C.
6726.93	*Fe*	1	2	3	C.
6743.38	*Ti*	6	1	8	Strongly darkened. Y.
6750.41	*Fe*	3	3	4	Darkened. Y.
6752.97	*Fe*	2	1	2	Hazy. Y.
6771.31	*Co*	4	0	4	Y.
6820.63	*Fe*	1	2	4	Reversed? C.
6828.85	*Fe*	1	2	4	Reversed? C.
6840.09		1	1	2	Y.
6842.95R		1	1	5	Reversed once.
6881.98R	*Cr*	3	0	4	Reversed once.
6882.78R	*Cr*	3	1	4	Reversed once.
6883.33R	*Cr*	3	1	4	Reversed once.
6925.13	*Cr?*	1	0000	5	
7068.68		1	2	2	
7107.74		1	0	5	
7111.18		1	1	5	
7122.48		1	4	6	
7131.20		1	3	6	
7148.44		1	3	6	

lines have been given which have not been recorded by any other observer. These, with a few from the other lists, have been omitted. In Cortie's list numerous lines are given whose origin, as given by Rowland, is water-vapor in the Earth's atmosphere. The writer has never observed these lines affected either before or after their publication, although they have been given a careful examination. In regard to the few water-vapor lines in the table, the writer doubts whether they are due solely to water-vapor. They may possibly be solar lines, unidentified as yet, due to elements having lines accidentally coincident with the water-vapor lines, the widening being due to the solar line. An instance is the line $\lambda 5958.46$, given as water-vapor by Rowland, and as *Fe* by Kayser and Runge.

Chromospheric lines are indicated by an asterisk (*); double ** indicates that the line has a chromospheric frequency of 5 per cent. or greater.

This table of 680 lines may be summarized, showing the number of affected lines of each element, as follows: In this summary lines due to more than one element are entered as due to each of the elements. Lines whose assignment to any particular element is doubtful are nevertheless considered as due to the element. The numbers of column 2 are not included in column 3.

Element	Total No. Lines Affected	Always Reversed	Occasionally Reversed	Doubtfully Reversed	Weakened or Obliterated
Iron	210	11	38	8	21
Unknown	136	1	21	4	5
Titanium	121	1	9	1	5
Chromium	79	3	13	2	5
Nickel	47	1	11	1	7
Vanadium	43	..	19	3	..
Calcium	24	..	4	1	1
Manganese	20	..	9	1	1
Cobalt	11	..	..	..	..
Sodium	6	..	1	..	..
Silicon	5	..	..	..	5

Yttrium, 5; lanthanum, 4; magnesium, 3; hydrogen, 2; copper, 2; helium, 1; scandium, 1; lines, attributed to more than one element, 40; total number reversed, 138.

NOTES ON THE BEHAVIOR OF METALLIC LINES IN THE SPECTRUM OF SUN-SPOTS

Iron.—The total number of iron lines seen affected is 210, about 31 per cent. of the total number of lines. Of these, 49, or 23 per cent., have been seen reversed by the writer.

Sir Norman Lockyer, after an extended study of the widened lines in sun-spots, has been led to the view that the selection of lines to be widened and darkened varies from epoch to epoch of solar activity. In 1886,[1] from a study of sun-spot observations made on a fixed plan during six years, he was led to the conclusion that as we pass from minimum to maximum the lines of known chemical elements gradually disappear from among those widened, their places being taken by lines which are unidentified.

[1] *Proc. R. S.*, **51**, 256.

To quote Miss Clerke:[1]

The evidence for the progressive change was indeed slight, except as regarded iron; and iron alone was taken account of in the confirmatory Stonyhurst observations.[2] So far as they went, however, they were decisive, and all the more so that they covered a different spectral range (B to D) from that (D to F) examined at South Kensington. They showed demonstratively that, throughout the disturbed interval between January 1884 and October 1886, iron lines were all but completely replaced by "unknown" lines in the list of those affected in spots, while they duly reappeared upon the restoration of photospheric tranquility.

Lockyer, after an investigation of the widened lines in sun-spots during an additional period of eight years, has reiterated his conclusions as follows:[3]

The period embraced by the observations practically enables us to study what has taken place at two successive sun-spot minima and maxima . . . . At the minima the iron lines are prominent among the most widened lines, at the maxima we find only lines about which nothing is known.

Cortie in his earlier observations, as quoted above, confirmed these conclusions. However, his views have since changed, for he states:[4]

My observations afford no evidence of crossing points when faint lines of vanadium and titanium give way to lines of iron at a period between the sun-spot maximum and minimum.

One important distinction has nevertheless been established by Cortie, namely, that the iron lines, while not displacing other faint lines, are more affected in tranquil spots than in the torn and ragged type. Since the former predominate at periods of minimum disturbance, and the latter at maximum periods, the statistical outcome is that the spectral variations depend only upon the individual spot, and do not indicate any periodic change in the general solar temperature.

The writer's observations hardly confirm those of Lockyer. More lines of iron have been observed affected during the period of observation than those of any other element, including lines of unknown origin. The period of the writer's observations, although

[1] *Problems in Astrophysics*, p. 90. [2] *Memoirs R. A. S.*, **50**, 43.

[3] *Proc. R. S.*, **57**, 200. [4] *Astrophysical Journal*, **20**, 264, 1904.

short, is near the date of the sun-spot maximum, when, according to Lockyer, there should be very few iron lines affected. Even in the region investigated by him (D to F) the iron lines predominate.

The writer's observations agree with Cortie's conclusions in regard to the widened lines. The reversed lines are equally prominent in both types of spots. Of course, from the short period of observation no definite conclusion can be drawn, but no change in the number of reversed lines was noticed in connection with the type of spot.

From the fact that the reversals are equally prominent in each type of spots, while the widening in general is not, one concludes that the widening of many of the lines in the minimum type of spot may be subjective; probably due to the fact that the minimum spot presents a larger umbra in proportion to the whole area of the spot, and hence the dark spectrum of the umbra is more prominent, making it very difficult to differentiate the real widening from the subjective.

Titanium and chromium.—The manner in which the lines of these two elements are affected can best be discussed by treating them together. Titanium has 121 lines affected, chromium 79, but with the reversed lines chromium leads with 16, while titanium has 10. The titanium lines are more conspicuous in the spot-spectrum, by reason of their being, in the large majority of instances, darkened without much widening, while the chromium lines, although equally faint in the solar spectrum, are more widened and less darkened, i.e., are hazy, in the spot-spectrum. Three chromium lines, $\lambda\lambda$5781.40, 5781.97, and 5783.29, are always reversed. The titanium reversals are less frequent, only one line, λ6064.85, being always reversed.

Nickel.—Of the lines of this element in the solar spectrum, 47 have been affected in that of the spot. Twelve of these have been reversed; the line λ6012.45 is always so affected. A fair proportion of the nickel lines are weakened in the spot-spectrum, the line λ5039.54 particularly.

Vanadium.—The importance of the vanadium lines in the spectrum of sun-spots was first shown by Professor Young in 1892.[1] Cortie in 1898 also noted the lines of this element.[2] The behavior of these lines is perhaps more striking than that of any others. They are, with few exceptions, exceedingly faint in the spectrum of the photo

[1] *Princeton College Bulletin*, **4**, 58. [2] *Monthly Notices*, **58**, 370.

sphere, yet in the spot they are relatively more conspicuous than other lines. The total number of lines affected is 43, which is fully four-fifths of the vanadium lines in this region. Of these, 44 per cent. have been seen reversed. The lines $\lambda\lambda 6224.71$, 6243.06, 6243.32, and 6252.05 have been seen so widely reversed at times as to give the effect of a pair of hazy lines, rather than a single line split in two. The above lines have all been given by Cortie as very much widened. The reversing of the vanadium lines seems to show no preference for any particular type of spot. Possibly the reversals are more distinct when the spot is near the limb, but the evidence of this is so slight as to warrant no definite conclusion.

Calcium.—The calcium lines present no striking peculiarities, the total number affected being 24, while only 4 have been seen reversed. The calcium lines with low dispersion are nearly all widened, but with high dispersion the widening disappears and the lines appear merely winged. The H and K lines were first noted as always reversed by Professor Young at Sherman, in 1872; this has been confirmed by Hale, Deslandres, and others. These lines are beyond the spectral region examined by the writer, and no attention was given to them.

Manganese.—This element, strange to say, has the greatest proportion of reversed lines, 45 per cent. of its lines being thus affected. The lines reverse occasionally, showing no preference for any particular type of spot. One very interesting change, however, has been noted in connection with the great spot of February 1905. At the first observations of this spot, on February 3 and 4, the lines $\lambda\lambda 5394.85$, 5399.67, 5432.75, 5470.80, and 5506.09 were all noted "strongly reversed;" on the return of the spot, observations on March 3 showed that these lines were no longer reversed, but instead were all extravagantly widened and very hazy. It is to be regretted that this change could not have been noted in the red lines (below $\lambda 6000$) also, but, unfortunately, the region of the spectrum observed on the earlier date did not include those lines. Observations in March showed them as widened only. No similar change has been noted in the lines of any other element, probably because, owing to the extent of the whole region investigated, it was rarely that more than one observation could be made on a spot in any given portion of the spectrum.

Silicon.—These lines have always been weakened, when affected in the spot-spectrum.

Sodium.—The sodium lines behave in various ways. The red pair at λ6154.44 and λ6190.96 are usually affected; the former line has been reversed on several occasions. Both are chromospheric lines. The D lines are always both affected alike. They are probably the first lines ever to have been seen reversed in the spectrum of a sun-spot. This observation was by Professor Young in 1870; since then there have been many other reversals observed, of which no account need be given. It is, however, sufficient to say the reversal of the D lines is by no means an unusual occurrence, although the writer himself has never seen them distinctly reversed. The nearest approach to a reversal was in the great spot of February 1905, when the D's were each broadened to about three times their normal width, the widened part being somewhat less dark than the normal line. The whole was surrounded by wings extending a considerable distance on each side of the lines. On October 14, 1904, the D's were seen with a faint streak through the middle of each line. The lines were not widened, however, at that time, except for the usual wings. The weakening of the central portion of the lines in neither of these instances was of such a character that the lines could be called reversed. Of the green pair at λ5682.87 and λ5688.44, the former is the more affected; both are usually winged.

Magnesium.—The *b*'s are the only lines of this element which have been recorded as affected in spots. There are records of their being seen reversed by Young, Naegamvala, Crew, and others. Their reversal seems to be less frequent than that of the D's.

Helium.—The writer has never seen the red line at λ6678.37 affected by a spot, but it has been seen on several occasions by other observers. Observations of the yellow line D_3 are more numerous. Professor Young recorded it as a dark shade in 1870. Cortie gives two observations of it as a bright line in 1883 and 1885.[1] The Greenwich observers give several instances of its visibility during 1882, and it has been seen numerous times since then. The general behavior of the line is somewhat uncertain. It seems to show itself from time to time, independent of the particular type of spot. The

[1] *Memoirs R. A. S.*, **50**, 55, 1890.

great spot of February 1905, at its first appearance, did not show it at all, but on the reappearance of the spot, while still very near the limb, D_3 was distinctly seen extending over its whole area. The line has been seen on several other occasions by the writer, but always as a faint shade. The rare visibility of the line may be an indication that the spots lie well down in the solar atmosphere and, unless there is some unusual activity, have no great effect on the upper solar atmosphere. One such instance was noted and will be given under hydrogen.

Hydrogen.—Only two hydrogen lines lie in the region of the spectrum investigated, C and F. Of the two, C is more affected. F is generally a reproduction, on a small scale, of C. The lines are not widened as the metallic lines are, but are, nevertheless, frequently reversed. The reversals are different from those of the metallic lines, which are simply widened and split in two by a bright streak through the middle of the line. The hydrogen reversals take place in all portions of the spot, umbra, penumbra, and over bridges; they are usually caused by overlying prominences. These can often be seen by opening the slit of the spectroscope, although it is difficult to distinguish more than the general outlines, on account of the great brilliancy of the spectrum under these circumstances.

Distortions in the C line due to motion in the region of the spot are very frequent. Many observations have been recorded. But distortions are very seldom seen directly over the umbra, such signs of activity being almost always in the penumbral regions. As an instance of this may be mentioned the observations recorded by Crew and by Hale in the spot of February 1892.[1]

In some remarks on observations of the spot of September 1898, Fényi states[2] that quiescent prominences do not occur over areas of spots, which are the scene of transitory eruptive phenomena. The writer does not agree with him in excluding quiescent prominences from sun-spots. It is true that the eruptive kind are more numerous, but both the C and F lines have been seen reversed over spots by the writer many times without showing any distortion whatever. Between spots and prominences there seems certainly to be some connec-

[1] *Astronomy and Astro-Physics*, **11**, 308 and 310, 1892.

[2] *Astrophysical Journal*, **10**, 333, 1899.

tion, but just what, it is difficult to say. An instance of this connection was observed on October 31, 1904, during the observations on a spot near the Sun's limb. At about 12 M. the writer was examining the spectrum of the spot in the region near C, and noticed that the line had become much distorted near the southern edge of the spot-umbra. By moving the spectroscope, this distortion could be traced to the limb and beyond it. On making the slit tangential and opening it, a large zigzag prominence was seen, extending, as measured, 70,000 miles beyond the limb. The displacement of the C line indicated a motion toward the observer of approximately 250 miles per second. In less than five minutes the whole prominence had faded away, and the region was again quiet. During the time that the prominence was visible, a hasty glance was taken at the D_3 line, which was seen as a faint shade in the region of the spot from which the prominence started. This instance is noteworthy from the fact that the prominence could be seen arising almost out of the umbra.

Unknown.—Next in number of affected lines to those of iron are the lines which have not been identified with any terrestrial element. The total number of these is 136, or approximately 20 per cent. of all the lines observed. These lines are affected in every variety of way, widened, reversed, weakened, etc., indiscriminately, with the exception that as in the case of the iron lines the percentage of reversals is greater in the red end of the spectrum. Only one line of this class at $\lambda 5880.49$, is always reversed.

In regard to the closely packed band-lines, those given in the table are only a few of the most prominent ones, no attempt whatever being made to give a complete list.

NOTES ON THE PROBABLE LEVEL AND CONSTITUTION OF SUN-SPOTS

A comparison of the chromospheric lines with the lines affected in the spot-spectrum affords the following conclusions:

a) Lines of considerable frequency (5 per cent. or more) in the chromosphere are, with two exceptions, very little affected in spots.

b) "High-level" chromospheric lines are not affected in spots.

c) Lines most affected in spots are either absent from, or of low frequency (less than 5 per cent.) in the chromosphere.

These conclusions favor the view that spots are at least below the chromosphere.

The interesting question arises why some lines of a given element are affected and others not. The lines most affected may be enhanced lines or short lines in the arc, but, as the writer was unable to find a table of such lines extending below the green region, it was impossible to make a comparison. It has been stated by Jewell[1] that many of the lines in the solar spectrum have their origin at different levels. In this, the writer believes, is a solution of the question. That the lines most affected are caused by vapors at a low level is apparently indicated by the fact that they are not chromospheric lines, and it is manifest that the spots are at the level which produces the lines that are most affected.

The vapors situated low down in the photosphere, and consequently under greater pressure and at a higher temperature, would give rise (the bright background of the photosphere being absent) to an emission-spectrum; this, in conjunction with the cooler and less dense layer above, would produce a dark line with bright center, i. e., a reversed line. It has been noted that the reversed lines are usually the fainter Fraunhofer lines, the lines H, K, F, and C being excluded, as it is fairly well established that the reversals of these lines are due to overlying prominences, flocculi, etc., and not gases low down in the spot.

The fact that the most widely reversed lines are faint may be explained by assuming that the vapors that produce these lines are intimately mingled with the photospheric clouds, and do not extend to a great elevation above them. The probably continuous spectrum of the photosphere is not subject to so great an absorption by this thin layer, and the resulting dark lines are thus not very intense. A striking example of this is afforded by the vanadium lines. Observations would seem to indicate that the vanadium vapors are situated in and among the photospheric clouds, as evidenced by the fact that the vanadium lines are nearly all faint in the solar spectrum. This shows that there may not be a sufficiently deep layer of the vapor to cause strong absorption; also only one line (frequency 3 per cent.) is visible in the chromosphere. The appearance in the

[1] *Astrophysical Journal*, **4**, 138, 1896.

spot-spectrum is precisely what might be expected. Nearly every vanadium line is affected, and presents the appearance of a wide, hazy reversal.

The behavior of the titanium lines might at first glance seem to contradict the above theory, in that they are faint lines in the solar spectrum, are very prominent in the spot, and likewise prominent in the chromosphere. These objections may be removed, however, by assuming that the titanium vapors are at a slightly higher level than those of vanadium. This seems in general to be indicated by the fact that the faintest titanium lines in the spectrum are the most affected in the spot, while the stronger lines are less prominent in the spot and more prominent in the chromosphere. Also there are few reversed lines, and those are not chromospheric. The titanium lines are nearly always darkened without much widening, indicating a slightly stronger absorption in the spot.

These views of the writer agree in the main with those of Cortie, who has suggested[1] that the level of sun-spots may be that of the vapors of such elements as have an atomic weight of about 50.

The purely visual observations of sun-spots indicate that the spot is a rent or perforation in the photosphere. Whether they are actual depressions or not, the observations of the apparent widths of the penumbra at various distances from the limb are too uncertain to determine. It can hardly be believed that the umbra is at a higher elevation than the penumbra, for it is unquestionable that the penumbral filaments overlie the umbra, and often unite forming bridges across it.

Whether the spot is caused by up-rushes, according to the earlier theories of Faye and of Secchi, or by down-rushes, as suggested by Lockyer and by Oppolzer, is not as yet determined. Line-shifts in the spot-spectrum, with the exception of those due to hydrogen, are very rare, and those given in the table are regarded by the writer as mostly spurious, due to neighboring faint lines. In only one instance has a distinct shifting been noticed, which would favor either of the above theories. This was on February 24, 1905, when every line in the spectrum was shifted toward the blue by about 0.05 tenth-meters. The shifting was not over the umbra of the spot,

[1] *Monthly Notices*, **58**, 373.

but in the region of a facula overlying a rift at one end of the spot. The following day it was noticed that a small umbra had developed in exactly the region of the shifting, indicating, in this instance, that the small spot was formed by an upheaval. Also the behavior of the manganese lines, already noted, may be considered as evidence that the heated vapors in the spot had moved upward, and, becoming cooler, produced stronger absorption, and the much-widened lines.

From the radiometric investigations of Langley, Frost, and Wilson, we learn that the photospheric radiation decreases as we approach the limb, while that of the spot changes but slightly. This can be explained in one of two ways: either the spots are high above the photosphere, as maintained by Howlett and others, and hence subject to less absorption; or the total radiation from the spots is of a different type from that of the photosphere, and therefore the absorption of the solar envelopes exerted on the photosphere would be different for the spot.

In regard to the first hypothesis, the spectroscope seems to indicate that the level of the spots is below the chromosphere; hence it may be assumed that the absorption of the upper solar envelopes is exerted on the spots as well as on the photosphere.

The second hypothesis has been suggested by Professor Young. The photosphere is rich in radiation of short wave-lengths, while the spots are noticeably deficient in radiation of this nature, as shown by the lack of detail in the upper regions of the spot-spectrum. It has been shown by Vogel and others that the violet light of the photosphere becomes relatively much more feeble as we approach the Sun's limb, than does the red. The solar atmosphere then absorbs to a considerable degree the short wave-length radiations of the photosphere, while the total radiation of the spot, not possessing these short waves, is not subject to this great absorption, and passes through the solar atmosphere nearly undiminished in intensity. It is thus possible that the total radiating power of the spot may be as high as or higher than that of the surrounding photosphere, and so indicate by the thermoscope a higher "temperature" for the spot.

The writer is inclined to the opinion that sun-spots are probably caused by the heated vapors of the interior slowly oozing through and vaporizing the clouds of the photosphere. The vapors from

below, at first hot, would become cooler through expansion and exposure, resulting in the reformation of the photospheric clouds in the shape of veils and bridges, which are generally heralds of spot-decay.

That the spots are regions of relatively high temperature has been suggested by Wilson,[1] and is borne out by the reversed lines. Moreover, if the spots were a cooler region, condensation would take place tending to destroy the character of the spot.

In conclusion, the writer wishes to thank most heartily Professor Young, whose advice and suggestions have been of the greatest value in carrying out this research.

[1] *Monthly Notices*, **65**, 224.

THE OBSERVATORY,
Princeton, N. J.,
March 30, 1905.

POSTSCRIPT

As this is going through the press, detailed observations of the great February spot by Fowler have come to hand (*Monthly Notices*, **65**, 513). He states:

> It appears that the high-level lines (chromospheric) were not among those intensified in the spot, while the common lines were chiefly those of iron, chromium, and calcium, which appear as strong Fraunhofer lines.

This is in complete agreement with the writer's observations.

A comparison of the chromospheric lines given with the spot lines recorded at Princeton shows that, excluding the hydrogen and helium lines, among the long lines (high-level) given by Fowler, four are recorded by the writer as widened: the D sodium lines, the lines $\lambda 5018.63$ and $\lambda 5276.17$, both due to enhanced iron. Seven others are recorded as weakened or obliterated; two of these, $\lambda 5169.22$ and $\lambda 6456.60$, are due to enhanced iron; the others, $\lambda 5425.46$, $\lambda 6238.60$, $\lambda 6247.77$, and $\lambda 6347.31$, are still unidentified; $\lambda 6432.89$ is *Fe* KR.

The line $\lambda 6232.86$ given as a chromospheric line of frequency 5 per cent. by Young is not recorded by Fowler. This line is one of the strongest spot lines, being always reversed. Similarly the manganese line $\lambda 5432.75$ is given by Young with chromospheric frequency

8 per cent., and is not recorded by Fowler. This line is also strong in the spot, being reversed three times.

To those contemplating investigations on the spectra of sun-spots the writer would like to suggest that a comparison of observations on the same spot at different periods of its development will be more fruitful than will the comparison of observations on different spots. The behavior of the manganese lines noted above is an example of what perhaps may be expected.

June 14, 1905.

SYNCHRONOUS VARIATIONS IN SOLAR AND TERRESTRIAL PHENOMENA

By H. W. CLOUGH

I. THE THIRTY-SIX-YEAR CYCLE IN TERRESTRIAL PHENOMENA

Numerous attempts have been made to discover definite periods of recurrence of meteorological phenomena, and many so-called weather cycles have been announced, ranging in length from a few days to a hundred or more years. The cycle discovered by Dr. Brückner is, however, the only one which has gained general acceptance among meteorologists. Brückner in 1890 published an elaborate monograph[1] in which he seemed to demonstrate the existence of a cycle of about 35 years in terrestrial climates, utilizing not only all available meteorological observations from about 1700, but in addition a vast amount of material affording indirect indications of climatic changes, including records of the advance and retreat of glaciers, the time of grape harvest, the opening and closing of navigation by ice, and the occurrence of severe winters, by which he was enabled to trace back the period nearly 1000 years. Briefly, his conclusion is that the whole Earth undergoes climatic variations or oscillations, cold and wet periods alternating with warm and dry periods. The mean dates or epochs of the former are 1700, 1740, 1780, 1815, 1850, and 1880; and of the latter, 1720, 1760, 1795, 1830, and 1860.

A careful examination has been made of Brückner's results, and where the author has given in general terms intervals of time, 10 to 20 years in length, embracing periods during which the values of the meteorological elements were above or below the average, I have attempted to assign in place of these relatively long periods single lustrums or years, representing as nearly as possible the average date or the epoch of each extreme. Comparison of the epochs of the meteorological elements as regards their sequence is thus facilitated. Besides utilizing the data which Brückner published, exten-

[1] *Klimaschwankungen seit 1700*, Vienna, 1890.

sive use has been made of data derived from other sources, and corresponding epochs have been determined in like manner for several additional meteorological phenomena.

Table I contains the meteorological epochs. The series of epochs for barometric pressure from 1740 to 1830, pressure-gradient, variability of temperature, frequency of easterly winds, frequency of West Indian hurricanes, frequency of thunderstorms, and grain prices are my own additions to Brückner's data. The remaining portions of the table are reproduced in all essential respects from Brückner's work, the main difference being, as above stated, that the mean epochs have been more accurately determined.

1. Temperature

Instrumental records of temperature are available from about 1730 in Europe, and from about 1780 in the United States. Brückner regarded this element as the one upon which all other elements depend, either directly or indirectly, and I also find that the epochs of temperature almost invariably precede those of precipitation and pressure. He concluded that the variations in temperature in his 35-year cycle are synchronous over the entire globe, but a careful examination of his lustrum means leads me to the conclusion that a slight retardation of the epochs occurs in southern as compared with northern Europe. This retardation is quite clearly shown by a comparison of the lustrum means of Scandinavia and northwestern Russia with those of southern Europe (p. 227).

The fluctuations are more regular and of greater amplitude in high latitudes, the extreme range of variation, considering the unsmoothed lustrum means, being 1°.0 to 1°.5 C. in northern Europe. In the tropics the fluctuations are somewhat irregular and of small amplitude.

The retardation and decreased amplitude of the oscillations in low latitudes is probably due to the fact that the circulatory activity of the atmosphere decreases toward the tropics, and the waves of high and low pressure, with their attendant temperature variations, which traverse the atmosphere in middle and high latitudes, penetrate into low latitudes slowly and with diminished intensity. These conditions are shown on the daily weather map in winter, the cold

waves appearing first in high latitudes and gradually extending southward with a diminution of intensity. By analogy we infer that the long-period atmospheric oscillations appear earliest in high latitudes, and ultimately extend into low latitudes with diminished intensity.

2. Precipitation

European rainfall records extend back to 1688, when observations were begun at Paris. From 1725 they are sufficiently numerous for accurate determination of epochs. In the United States, although isolated and fragmentary records date from 1738, it is not until about 1810 that sufficient records are available, and accordingly historical accounts of great floods in the Mississippi and Ohio Rivers were utilized to determine the epochs in the eighteenth century. The epochs in the table refer to the interior of Europe and the United States, and the variations in these two regions appear to be synchronous. Brückner finds that, unlike temperature, the epochs of precipitation are not synchronous over the whole globe, but that there are oceanic regions where the variations are the reverse of those over the interior of the continents. These regions he characterizes as "temporary" and "permanent" exceptions. Examples of these exceptional regions are found along the Atlantic coast of the United States, the coast of Ireland, and on some of the islands of the Atlantic Ocean. He considers that the oceanic areas experience rainfall variations opposite to those of the continental areas, so that a compensatory relation between continent and ocean seems to exist as regards rainfall. He also shows that the amplitude of the oscillation increases with the continentality of the region, the greatest range being in western Siberia, where 2.3 times as much rain falls in the rainy period as in the dry period.

While Brückner inferred that no progressive retardation in the epochs of rainfall occurs, with change either of longitude or latitude, yet a careful inspection of his rainfall data seems to lead to the conclusion that, as with temperature, the extremes occur earlier in high latitudes, the retardation averaging probably five years at the tropics. In the equatorial regions the epochs of rainfall are probably synchronous with those in high latitudes. This is shown by the fluctuations of the Nile, which synchronize closely with those of rainfall in northern

Europe. Brückner's curves of rainfall variations (pp. 181, 182), showing the changes of the secular oscillations from north to south in the Old and New Worlds, illustrate this fact of retardation in low latitudes.

Comparison of the epochs of temperature and precipitation shows that cold periods are attended by an excess and warm periods by a deficiency of precipitation over the continental areas. A tendency, however, toward a retardation in the epochs of precipitation is clearly evident, the average amount being about six years.

3. Height of Water Surface of Lakes and Rivers

Supplementing and confirming the series of rainfall epochs are those obtained from records of fluctuations of the water surface of inland seas, lakes, and rivers. One series of epochs in the table refers to the oscillations of lakes without outlets in various regions of the globe, the data being largely derived from historical and traditional accounts of high and low water. Another series comprises epochs derived mainly from records of mean stages of European rivers and lakes in river courses. Both series are reproduced without change from Brückner. The general agreement of these epochs with those of rainfall is readily apparent.

4*a*. Barometric Pressure

The records which Brückner employed in his investigation of pressure variations in Europe and surrounding regions were those compiled by Dr. Hann in his work on the pressure distribution in Europe.[1] These records begin in 1826, and the epochs in the table, beginning with 1831–1835, are those determined by Brückner. The epochs prior to 1825 were determined by me from the records compiled by Buys-Ballot,[2] and the entire series of epochs relate to pressure variations in Europe only.

Correlation of pressure and rainfall variations.—Brückner found that the curves of rainfall and pressure for Europe are nearly exact counterparts of each other as regards synchronism of phase and amplitude of variation, excessive rainfall and low pressure being coincident. This relation between pressure and rainfall which he

[1] *Die Vertheilung des Luftdruckes über Mittel-und Süd-Europa*, Vienna, 1887.

[2] *Met. Jahrbuch*, 1870.

found for Europe during the period from 1826 to 1885 is confirmed by the epochs of pressure from 1740 to 1825. He did not investigate pressure variations in America, but it is found that a relation, the reverse of that for Europe, prevails in the northeastern portion of the United States, particularly in winter. In this region relatively high pressure and excessive precipitation prevailed about 1850 and 1882, and low pressure and deficient precipitation prevailed about 1835, 1865, and 1900. This relation was derived from investigation of rainfall variations in the upper Ohio valley and pressure variations at Toronto. A similar relation exists for the region of low pressure about Iceland, as will be shown below. The explanation of this inversion in pressure between Europe and northeastern United States evidently lies in the fact that the belt of maximum storm frequency attains its most southerly position in America, extending over the region of the Great Lakes and the St. Lawrence valley; while in Europe the path of greatest frequency lies far to the northwestward, being traced over the Atlantic midway between Iceland and the Faroe Islands, thence over extreme northern Scandinavia.

In his investigation of pressure and rainfall variations over the whole Earth, Brückner discovered that during the continental wet periods relatively high pressure, or pressure above the normal, prevails over the North Atlantic Ocean in the vicinity of Iceland and the Faroe Islands, also over the equatorial belt of low pressure in the northern part of the Indian Ocean and the China Sea. At the same time the pressure is below the normal throughout the permanent belt of high pressure which extends from the Azores northeastward through central Europe to the interior of Russia and in winter over Siberia. The reverse is true for the dry period. The pressure variations in winter and summer for the wet and dry periods are shown in the following table by Brückner, presenting variations from the normal for the season:

Period	North Atlantic		Western and Central Europe		Eastern Europe and Siberia	
	Winter	Summer	Winter	Summer	Winter	Summer
Wet	Above	Below	Below	Below	Below	Above
Dry	Below	Above	Above	Above	Above	Below

From the foregoing facts Brückner deduced the generalization that wet periods are characterized by a diminution of local and seasonal differences of air-pressure, or a weakening of sea-level gradients. This implies a decrease in the annual pressure-gradient over middle latitudes, a decrease in the seasonal gradient between continent and ocean, and a decrease in the amplitude of the seasonal variation at any given locality. In other words, the gradient between the North Atlantic Low at Iceland and the North Atlantic High at the Azores is less during the wet periods than during the dry periods. The gradient between Iceland and central Europe also varies in like manner. Furthermore, the pressure over the interior of the continents during the wet periods is lower in winter and higher in summer than during the dry periods.

Variation in the activity of the general circulation.—In his discussion of these relations Brückner concluded that the increase in pressure in the polar and equatorial regions, the simultaneous decrease over middle latitudes, and the decrease in the amplitude of the seasonal variation, during cold, wet periods, imply a decreased activity of the general circulation. He attributed the cold periods to a decrease in solar radiation, and accordingly assumed that during such periods the temperature-gradient between equator and pole should be less than during warm periods, resulting in a diminished circulatory activity. Since, however, his table of temperature fluctuations for different latitudes showed that greater amplitudes prevailed in high latitudes, he concluded, in order to account for this discrepancy between theory and fact, that the slight fluctuations in the tropics were caused by discontinuity in the records and to a masking effect of the eleven-year period of Köppen. It will be shown below, however, that a weakened pressure-gradient in middle latitudes, as between Iceland and the Azores, and a decreased amplitude of the seasonal variation—conditions occurring during the cold, wet periods—probably denote greater circulatory activity. *A priori*, a decrease in solar radiation would result in a diminished temperature-gradient between equator and pole, so that a paradox is apparently involved in attributing the cold periods to a decrease in solar radiation.

From a consideration of some phenomena of the general circulation

it appears probable that changes in pressure, similar to those which characterize cold periods, result from an increase in circulatory activity. The distribution of air-pressure over the surface of the Earth, considered as a rotating globe, is mainly the resultant of two factors. The first factor is the temperature-gradient between the equator and the poles caused by their varying insolation. The resulting circulation tends to form a belt of relatively low pressure in the equatorial region, a belt of high pressure near latitude 35°, a belt of low pressure near latitude 65°, and a region of relatively high pressure around the poles. This is the pressure distribution resulting from the circulation of the atmosphere over an ideal water surface. The second factor is the distribution of land surface over the Earth which distorts the ideal courses of the isotherms parallel with the equator, resulting in a seasonal temperature-gradient between continent and ocean. The effect of this factor is to modify the ideal distribution by the formation of high-pressure areas during winter over the continents in the northern hemisphere, and of corresponding low-pressure areas during summer. This interaction of land and water, summer and winter, results in large seasonal inequalities of pressure. The seasonal charts of pressure in the northern hemisphere, therefore, show isobars greatly distorted from the ideal courses, parallel with the equator, which largely prevail in high latitudes in the southern hemisphere with their preponderance of water surface, and in the free air above the irregularities of land surface that offer great resistance to pressure readjustments in the lower atmosphere.

If the general atmospheric circulation be accelerated, thereby overcoming to a greater extent the inertia of the lower atmosphere, we should expect, in the first place, the influence of the second factor, which distorts the ideal isobars, to be weakened and the resulting distribution to be more uniform over the Earth, the differences between the pressure on land and water being thereby lessened. This tendency toward more uniform pressure distribution with increased circulatory activity implies a decrease in pressure over the regions of high pressure which prevail in winter over the continents and in summer over the oceans, and a corresponding increase over the regions of low pressure, or a general diminution of the seasonal control

which results from the interaction of land and water. A further result of an acceleration of the general circulation is an increased centrifugal force of the great circumpolar whirls, crowding the subtropical belts of high pressure nearer the equator and causing the belt of average storm-tracks to descend to lower latitudes. The pressure over middle latitudes to the southward of the path of average storm tracks will therefore decrease, while that over the polar and equatorial regions will increase. These changes in pressure caused by greater circulatory activity are apparently the same as those found by Brückner to prevail during the wet periods as compared with the dry periods. The conclusion, therefore, is that the cold, wet periods are characterized by an increase in the rapidity of the atmospheric circulation, attended by greater decrease in the temperature of the polar as compared with the equatorial regions, and consequently an increase in the temperature-gradient between pole and equator.

Dr. Hann in a recent paper[1] discussed the abnormal variations in the pressure-gradient between the Azores and Iceland, and concluded that an increase in the gradient is a consequence of an increased intensity of the atmospheric circulation. But it is possible that even with increased gradients at sea-level the gradients in the upper atmosphere may at the same time be diminished, so that the intensity of the general circulation is decreased. In this investigation the instances of abnormal variations during certain months may be misleading, since the mean distribution of pressure over the Earth in any given month may be the result of so many factors that the effect of variations in the intensity of the general circulation is largely masked. It is conceivable that there may be other causes, aside from the thermal gradients between pole and equator, continent and ocean, that bring about pressure variations. Hence only annual or lustrum means in which the effect of other factors is probably eliminated, should be considered. By reducing upward, we obtain the pressure distribution in the upper atmosphere which is immediately related to the general circulation. It may easily be shown

[1] "Die Anomalien der Witterung auf Island in dem Zeitraume 1851 bis 1900 und deren Beziehungen zu den gleichzeitigen Witterungsanomalien in Nordwesteuropa." *Sitzungsberichte der Akad. der Wiss. in Wien.*, Jan. 1904.

that the higher the level to which the reduction is made, the less becomes the influence of variations in the sea-level pressure, so that at great heights the pressure is almost entirely a function of the surface temperatures and the direction of the isobars approximates closely to that of the isotherms at sea-level. For example, at 30,000 feet a variation of 0.01 inch in the sea-level pressure becomes a variation of 0.003 inch, while a variation of 1° F. becomes a variation of 0.04 inch, showing that variations in the sea-level pressure become almost negligible at this elevation. During the cold periods, therefore, the diminished pressure-gradient at sea-level will have but slight effect on the pressure-gradient at high levels, while the increased temperature-gradient will be reflected in an increased pressure-gradient; consequently an increase in the intensity of the general circulation will result.

4*b*. Pressure-Gradient

A series of epochs which confirm the relation above stated is afforded by the variations in the annual pressure-gradient over the region from the high-pressure belt of central and western Europe northwestward to Iceland. They are: maximum gradient in 1790, 1831–35, 1860, and 1895; minimum gradient in 1815, 1840, and 1875. During cold periods, therefore, a decrease in the pressure-gradient occurs over western Europe. This is true also for the gradient between the Azores and Iceland.

5. Frequency of Easterly Winds

Since the direction of the pressure-gradient over western Europe implies a prevailing southwesterly surface current, a weakening of the average gradient would seem to indicate an increase in the frequency of easterly to northerly winds; and observations show this to be the case. Numerous records of wind-direction frequency have been examined, and at stations north of the permanent high-pressure belt a variation in the frequency of easterly winds was disclosed, corresponding with the variations of pressure-gradient, such that a decrease in the gradient and an increase in the frequency of easterly winds coincide. This relation is found to exist also in the United States, and is probably universally true over the northern hemisphere where the gradient involves a maximum frequency of westerly winds.

Tracks of storm-centers.—Since the main track of low-pressure areas over the North Atlantic is from mid-ocean northeastward, skirting the coast of Norway, the prevailing winds over Europe, north of the ridge of high pressure, are westerly to southerly. Hence an increase in the frequency of easterly winds at any point would imply that more storm-centers pass to the southward of the locality. In other words, the average latitude of storm-tracks is lower than usual. A diminished pressure-gradient likewise implies a lower average latitude of storm-tracks. The conclusion therefore is that in cold periods the storm-tracks lie farther south than in warm periods. This shifting to the southward of the belt of average storm-tracks was shown above to be a probable result of the increased circulatory activity which was assumed to characterize cold periods.

Additional confirmation of this relation results from a count of storms passing north and south of Chicago during the period 1873–1900, disclosing a maximum ratio of southern to northern storms about 1878, the center of a cold period, and a minimum ratio about 1895, the center of a warm period. The number of storms passing north and south for each year, with the ratio of southern to northern storms, is shown in the following table:

Year	N.	S.	Ratio S.:N.	Date	N.	S.	Ratio S.:N.
1873.......	80	40	0.50	1887......	68	34	0.50
1874.......	73	38	0.52	1888......	61	32	0.52
1875.......	66	44	0.67	1889......	74	31	0.42
1876.......	70	35	0.50	1890......	80	36	0.45
1877.......	68	40	0.59	1891......	72	35	0.49
1878.......	55	43	0.78	1892......	81	25	0.31
1879.......	70	38	0.54	1893	76	33	0.43
1880.......	85	32	0.38	1894......	77	26	0.34
1881.......	53	31	0.58	1895......	73	30	0.41
1882.......	64	26	0.41	1896......	75	27	0.36
1883.......	62	38	0.61	1897. . . .	62	28	0.46
1884.......	67	35	0.52	1898......	71	26	0.37
1885.......	56	32	0.57	1899......	72	25	0.35
1886.......	65	42	0.65	1900.....	81	36	0.44

6. Variability of Temperature

An important deduction has been drawn from a study of the variability of mean daily temperature, or the average change in mean temperature from one day to the next, considered without regard to sign. The variations of the annual means of this quantity from

year to year at any place are due to varying meteorological conditions, such as amount of cloudiness, distance from storm-centers passing north or south, intensity of storm-development, velocity of storm-movement, etc. It is found, however, that the latter element chiefly influences the variability, so that at stations situated within or near the belt of average storm-tracks, girdling the Earth north of the forty-fifth parallel, the changes in this element are an index to the varying velocity of movement of storm-centers. This relation is based upon a study of the variations in the two phenomena in the United States. Yearly values of this element are available for several Russian stations,[1] enabling the variations to be traced from about 1750, and show a tendency to fluctuate with mean epochs as given in the table. The close synchronism of these epochs with those of temperature is readily apparent.

The inference drawn from this series of epochs is that cold periods are characterized by increased variability of temperature, which probably implies an increase in the velocity of storm-movement, and consequently an increase in the circulatory activity of the atmosphere during these periods.

Velocity of storm-movement.—Further confirmation of this deduction has resulted from an investigation of the average velocity of movement of storms in the United States. The average velocity for each year from 1872 to 1901 has been computed from the monthly means given in the *Monthly Weather Review*, and the resulting values, when smoothed, show a decrease from a maximum about 1882 to a minimum about 1895. The average yearly velocities are shown in the following table:

1872...	26.2	1878...	22.4	1884...	32.7	1890...	30.8	1896...	26.7
1873...	25.2	1879...	31.7	1885...	28.7	1891...	27.1	1897...	25.8
1874...	26.8	1880...	30.5	1886...	27.7	1892...	29.6	1898...	26.0
1875...	28.2	1881..	33.6	1887...	28.6	1893...	29.8	1899...	27.1
1876...	27.2	1882...	28.8	1888...	30.0	1894...	24.2	1900...	29.5
1877...	25.7	1883...	32.2	1889...	28.2	1895...	26.1	1901...	27.8

Thus the conclusion derived from a consideration of the general atmospheric circulation, that an increase in activity occurs during the cold periods, is confirmed by two independent investigations.

[1] Wahlen, *Tägliche Variation der Temperatur an 18 Stationen des Russischen Reiches.* St. Petersburg, 1886.

7. Frequency of West Indian Hurricanes

The frequency of tropical hurricanes has been shown by Poëy and Meldrum to vary in a period approximately 11 years, or that of the solar spots, and it appears to be fairly well established that they reach a maximum frequency shortly after the sun-spot maximum. An examination of Poëy's table of West Indian hurricanes from 1750 to 1873[1] discloses in addition a long-period variation with well-defined maxima in 1786, 1817, and 1838, and minima in 1762, 1798, 1823, and 1864. The remaining epochs in this series were derived from his catalogue of hurricanes from 1493 to 1855, and from the records of the Weather Bureau, beginning with 1873. The epochs of maximum frequency coincide with the wet periods of the Brückner cycle, particularly with the corresponding epochs of precipitation in the United States.

The conditions favorable to the development of tropical hurricanes are thus probably connected with the general circulation, and cannot be regarded as of purely local origin. Years in which the movement of storms of middle latitudes is most rapid and their paths extend far southward, indicating an increased activity of the general circulation, are signalized by frequent hurricanes in low latitudes. Furthermore, observation of the daily weather map shows that the development of West Indian hurricanes is usually coincident with an increase in the velocity of movement of high- and low-pressure areas in the United States.

8. Frequency of Thunderstorms

Von Bezold,[2] in his investigation of thunderstorm frequency, arrived at the conclusion that periods of maximum thunderstorm frequency are conditioned upon high temperature as well as a solar surface free from spots. His table of relative numbers for Europe yields epochs approximately as follows: maxima in 1768, 1797, 1822, and 1852; minima in 1783, 1814, and 1837. Comparison of these epochs with those of temperature in Europe shows that thunderstorms are least frequent during cold periods, thus confirming v. Bezold's conclusion in regard to temperature conditions.

[1] *Comptes Rendus*, **77**, 1223, 1873.

[2] "Ueber gesetzmässige Schwankungen in der Häufigkeit der Gewitter," *Sitzungsberichte der math.-phys. Klasse der B. Akad.*, **4**, 1874.

9. Frequency of Severe Winters

Pilgram's catalogue of severe winters was used by Brückner to extend his cycle back nearly 1000 years, and confirmatory evidence is at hand to prove the validity of the method he employed. For every fifth year, as 800, 805, 810, etc., he gives a number which represents the total number of severe winters recorded in the 20-year period of which it is the center. The epochs in the table were derived by me from Brückner's table (p. 268), and are intended to represent as nearly as possible the centers of periods of maximum and minimum frequency of severe winters. They approximately coincide with the general epochs of low and high temperature. These variations in temperature are shown graphically on Chart 2, the epochs being plotted with a uniform amplitude of variation.

This series of epochs is the result of careful consideration of all data available and comparison with the mean epochs derived from other climatic records; it departs from Brückner's table of warm and cold periods from 1020 to 1890, and omits in the sixteenth century one oscillation which should be regarded as secondary. The third and fourth columns give the intervals, derived from the epochs of maximum and minimum frequency of severe winters, embracing each three successive periods. Thus in column 3, the first number, 120, is the interval between the two epochs 1000 and 1120. The longest three-period interval in the series is 125, and the shortest is 90. If there is an additional oscillation in the sixteenth century, the three-period intervals in that century would be reduced to 75 years, which is highly improbable, since the latter interval is the length of two average periods, and there is no other three-period interval less than 90 years in the entire series.

The average length of this cycle was computed by Brückner from his table of cold and warm periods derived from his table of the frequency of severe winters. He writes (p. 270): "The table embraces the years 1020 to 1890. Within this period of time of 870 years we enumerate twenty-five cold periods and twenty-five warm periods, hence twenty-five complete oscillations. We find therefore the average length of one oscillation to be 34.8 years." But, as shown above, the number of complete oscillations should be reduced by one, making the length of the cycle 36.25 years.

10. Duration of the Season of Navigation

The dates of the opening and closing of navigation on rivers, lakes, and harbors in Russia have been recorded for 150 years at many localities and in the vicinity of St. Petersburg from the middle of the sixteenth century. Brückner derived his data from Rykatschef's memoir on ice conditions in Russian waters.[1] The epochs of this series were derived from Brückner's tables and are well defined, affording a most valuable indirect method of exhibiting climatic variations. They average about 6 years later than those of severe winters.

11. Time of Grape Harvest

The time of beginning of the grape harvest in France and southern Germany has been recorded in some localities for many hundred years, and these records were utilized by Brückner to determine secular climatic variations. In this series of epochs, as with that of the frequency of severe winters, a rearrangement of dates in the sixteenth century was necessary in order to eliminate one oscillation in Brückner's table. The epochs synchronize well with those of the severity of winters, occurring on the average five years later. Late harvests, therefore, characterize periods of excessive precipitation.

12. Grain Prices

In his discussion of climatic oscillations, Brückner did not refer to fluctuations in grain prices as an index to these changes, but in a later paper[2] he compares prices and climatic variations in Europe during the past 200 years, and finds a relation such that high prices occur during or shortly after periods of maximum rainfall.

In order to confirm this relation and extend the comparison as far back as possible, Rogers' *History of Prices and Agriculture in England* was consulted. These volumes contain an exceedingly valuable collection of prices of grain and commodities, beginning with 1265. Rogers incidentally mentioned the possibility of a seasonal cycle being discovered in the fluctuations of grain prices. He writes:[3]

[1] *Ueber den Auf- und Zugang der Gewässer des Russischen Reiches.* St. Petersburg, 1887.

[2] "Der Einfluss der Klimaschwankungen auf der Ernteerträge und Getreidepreise in Europa," *Geographische Zeitschrift*, 1895.

[3] Vol. I, Pref., p. xi.

"Lastly, as there were no regular means for supplying deficiencies in the produce of the home market by foreign importation, the prices of necessaries such as corn give no small insight into the course of the seasons, if, as I do not dare to assert, such a cycle can yet be found."

Examination of Rogers' statistics of grain prices discloses fluctuations in the prices of wheat, rye, barley, etc., corresponding with those of temperature, periods of high prices occurring shortly after periods of low temperature, with an average retardation of about seven years. Thus the series of epochs of grain prices serve to confirm the epochs of severe winters. This is especially the case in the earlier centuries. In the last two centuries disturbing influences have contributed to mask the fluctuations, and accordingly from about 1700 onward, grain prices in continental countries were also used in determining the epochs.

These epochs synchronize very well with the epochs of the time of grape harvest, and the conclusion is that the same stress of weather which tended to retard the maturity of the vine in France caused deficient grain harvests in England.

II. THE THIRTY-SIX-YEAR CYCLE IN SOLAR PHENOMENA

Brückner discussed at considerable length the origin of the secular climatic variations which he discovered, and concluded that it must be referred to a cosmical source. He examined the sun-spot relative-numbers of Wolf, but found no evidence of his 35-year cycle in their variations. Nevertheless, he stated it as his conviction that such a variation must exist in solar phenomena, and that the climatic oscillations on the Earth point to a solar cycle, to be discovered later. He thought it probable that the cycle would be shown in the variations in the intensity of solar radiation.

Dr. W. J. S. Lockyer[1] pointed out that a cycle of about 35 years exists in the variations of the interval from one sun-spot minimum to the succeeding maximum. He writes: "There is some law at work which introduces a secular variation by retarding the sun-spot maxima in relation to the preceding minima." He considers this as the source of the Brückner cycle.

[1] "The Solar Activity, 1833–1900," *Proc. R. S.*, **68**, 285, 1901.

Professor A. Wolfer,[1] on the other hand, discusses Dr. Lockyer's results, and concludes from examination of the relative-numbers and epochs from 1750 that no regular periodicity exists, and that "the continued existence of a 35-year cycle is not yet demonstrated." It will be shown, however, in this paper that a 36-year cycle in the variations of solar phenomena undoubtedly exists, and also that a much longer cycle exists, underlying the 11 and 36-year cycles.

Variations in the length of the eleven-year cycle.—In Table II, "Sun-Spot Epochs" (Wolfer), the epochs of sun-spot maxima and minima, determined by Wolf and revised by Wolfer, are shown in the first two columns. The third and fourth columns contain the successive intervals, maximum to maximum and minimum to minimum. It is well known that the so-called 11-year cycle is only an average of these varying intervals. Uniting these intervals into one column, and smoothing them by the formula $\frac{1}{3}(a+b+c)$, we have a series of numbers, column 5, which vary more or less regularly. These numbers are plotted to form the first curve in Chart 1. By inspection of the data in this and in the preceding columns, the dates in columns 6 and 7 are obtained. These epochs represent the centers of periods of maximum and minimum intensity of the processes which result in the 11-year cycle of solar activity, a rapid completion of this cycle indicating a maximum intensity of solar activity, as will be shown below. The mean length of this cycle of variation in the length of the 11-year period, based on the epochs from 1615 to 1880, is 35.7 years. This cycle of solar activity is thus derived from nearly 300 years' observations of sun-spots, since the invention of the telescope, during which period the successive 11-year epochs of maxima and minima can be relied upon as approximately correct.

Fritz[2] has compiled a list of all recorded observations of sun-spots previous to 1610, when their regular observation by the telescope began. Nearly all of these observations are derived from ancient

[1] "Revision of Wolf's Sun-Spot Relative-Numbers," *Monthly Weather Review*, **30**, 171, 1902.

[2] "Die Perioden solarer und terrestrischer Erscheinungen," *Vierteljahrsschrift der Naturforschenden Gesellschaft*, Zürich, 1893.

Chinese annals, beginning about 300 A. D.; a few records are from European sources. From this list Fritz deduced approximate epochs of sun-spot maxima, where sufficient observations were available, and showed that a period averaging about eleven years has existed during the entire interval. In the same paper he gives a list of years during which auroras have been recorded, with the number of displays observed each year, and derives therefrom a series of probable epochs of auroral maxima. From these two independently determined series, supplemented by early records of great hailfalls, he deduces a series of probable sun-spot maxima from 301 A. D. to 1616, there being only twenty-seven epochs missing for which no adequate data exist. From 1057 to 1616 there are only six epochs missing from his list, which is reproduced in column 1 of Table III —"Epochs of Sun-Spot Maxima" (Fritz). The required epochs have been supplied and are designated by an asterisk. One change was made in the epochs, namely that of 1603, which is obviously too early, and in the table 1605 was substituted. The second column of the table contains the intervals between these epochs of maxima. From the data in these two columns the epochs in columns 3 and 4 were derived, being a continuation backward of the epochs in columns 6 and 7 in Table II. They are less exact, however, having been derived from epochs of maxima only, which are subject to considerable uncertainty. Nevertheless, the mean interval between these epochs during the period 1050 to 1600 is 36.6 years, which is very nearly identical with that obtained from the table of Wolfer's epochs, thus confirming in a remarkable manner the general accuracy of these "probable maxima" of Fritz.

The epochs of maxima and minima of the 36-year solar cycle from 1000 to 1900 are shown graphically on Chart 2.

The missing epochs of Fritz's list of "probable maxima" from 301 A. D. to 1057 have been approximately determined, and probable epochs of the 36-year cycle derived. Table III contains the 36-year epochs from 295 to 1100. The average length of the 11-year cycle from 301 to 1104 is 11.00 years, while from 1104 to 1894 it is 11.13 years. The mean length, therefore, during 144 periods from 301 to 1894 is 11.063 years. The average length of the 36-year cycle from 300 to 1900 is 36.5 years, the mean from 300 to 1100 being practically the same as from 1100 to 1900.

The sun may therefore be regarded as a variable star, whose mean period of variation undergoes a cyclical variation in length. Chandler has shown that this phenomenon is characteristic of many variable stars.

Lockyer's conclusion that a 35-year period exists which alters the time of occurrence of the maxima in relation to the preceding minima, is evidently only partially true, since the interval maximum to minimum likewise undergoes a similar variation. The solar-spot activity is periodically accelerated and retarded, and this action is primarily manifest in the varying length of the 11-year spot cycle, since it operates continuously throughout the entire interval to accelerate or retard the occurrence of the two phases.

Variations of the relative-numbers.—In Table II, column 8, are given the relative-numbers at the time of each maximum, beginning with 1685. Prior to 1750 the average relative-number for the year in which the phase occurred is

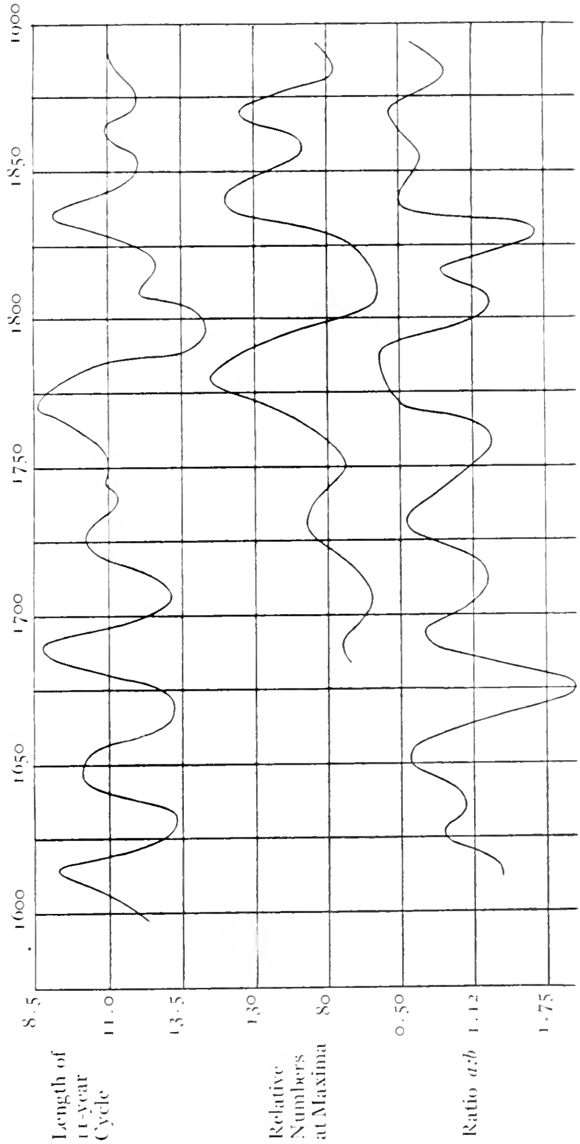


CHART I.—The 36-Year Solar Cycle.

given; subsequently, the highest value contained in Wolfer's table of smoothed numbers is placed opposite the corresponding epoch. Comparing the variations in these numbers with the variations of the 11-year period, shown in column 5, the conclusion is evident that periods of rapid development of the cycle of changes averaging 11 years are also those of increased intensity of solar activity, as evidenced by the increased frequency of spots. That is to say, when the period is shorter the sun-spot number is larger. The second curve on Chart 1 displays graphically these variations in the relative-numbers.

Wolf[1] showed that the shortest periods brought the most acute crises. This relation for the 11-year period, first stated by Wolf, was confirmed by Dr. Halm,[2] who also found that "in the individual spot-periods the maximum occurs earlier in proportion as the development of the spots is more rapid." This inverse relation between the intensity at a maximum and the interval from the preceding minimum follows as a deduction from the general law, which may be expressed as follows: The solar spottedness varies inversely with the length of the cycle of activity. It will be shown below that this law applies to the 36-year cycle as well as to the 11-year cycle.

One oscillation in the series of relative-numbers at maxima, column 8, is absent, although existing in the variations of the 11-year interval, thus resulting in a 59-year interval from the maximum of 1778 to that of 1837. The abnormally low numbers for the maxima of 1805 and 1816 correspond with the unusually long sun-spot periods between 1788 and 1830.

The average relative-number for each 11-year period, minimum to minimum, is shown in column 9, opposite the corresponding maximum epoch. These numbers are from the paper by Fritz quoted above. The variations of this series are synchronous with those of the maximum relative-numbers in the preceding column.

Comparing the variations in the length of the 11-year cycle with those of the relative-numbers, a retardation of the epochs of the latter is evident, averaging 5 years.

Variations of the ratio a:b.—In columns 10 and 11 of Table II are given the intervals of the 11-year cycle, minimum to maximum and maximum to minimum. Representing the former by a and the latter by b, the successive ratios a:b were computed and are shown

[1] Wolf, *Astronomische Mittheilungen*, No. 12, 1861. [2] *A. N.*, **156**, 33, 1901.

in column 12, opposite the corresponding maximum epochs. Chart 1 exhibits in graphical form the variations of these ratios. Inspection of this series of ratios discloses variations parallel with the 36-year variation in the length of the 11-year period; the ratio varies directly with the length of the period. The epochs of these variations of the ratio occur about 7 years later than the corresponding epochs of the variations of the length of the period.

Comparison of the variations of the relative-numbers with the variations of the ratios *a:b* shows that the two series of numbers vary inversely, with epochs of variation nearly coincident.

Column 13 of the table contains the ratios *a:b*, smoothed by taking the mean of each five successive values. These smoothed ratios disclose a long-period variation, with a maximum about 1685, and a minimum about 1865. In other words, the sun-spot curve flattens out about 1685, so that the intervals *a* and *b* approach equality, or *a* is even greater than *b*.

Referring to the series of epochs of the 36-year cycle, columns 6 and 7, it is apparent that the intervals between the epochs are not uniform in length, being relatively long about 1700 and short about 1850. Thus the long-period variation in the ratio *a:b* corresponds with a similar long-period variation in the length of the 36-year interval, the ratio varying directly with the length both of the 11- and the 36-year cycles.

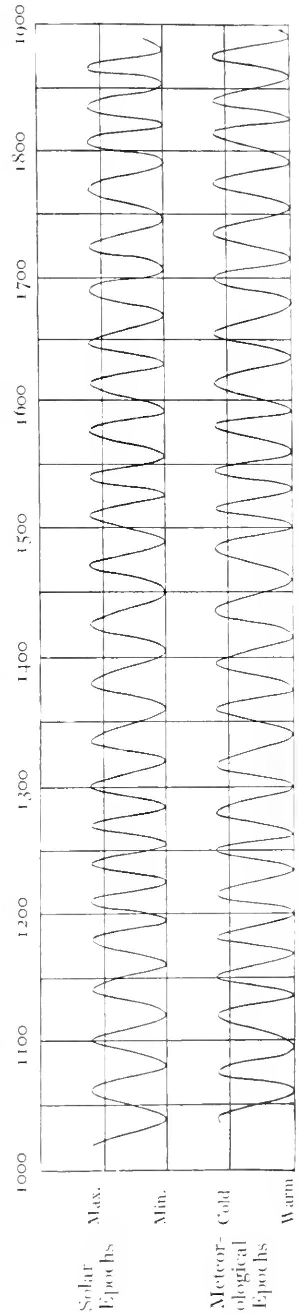


CHART 2.—Epochs of the Thirty-six-Year Cycle.

The long-period variations in the ratio *a:b* are generally synchronous with the secular variations of spottedness, columns 8 and 9, varying inversely with the latter, in conformity with a similar relation shown above to exist in connection with the 36-year cycle.

Reliability of Wolfer's epochs.—The accuracy of the sun-spot epochs in the seventeenth and eighteenth centuries, particularly those from 1788 to 1805, has been questioned by some investigators. It has been assumed that a uniform period exists, and that the irregularities which are shown by Wolfer's epochs arise from imperfections of the records. But the records of magnetic declination which are available from about 1780 show that variations in the range exist with epochs practically coinciding with Wolfer's epochs. The epochs of auroral frequency from 1700 also confirm the sun-spot epochs. Furthermore, the evidence for the 36-year cycle, cited above, proves that variations in the length of the 11-year period really exist and are of a periodic nature. The normal period is eleven years, subject to alternate acceleration and retardation during a cycle averaging 36 years. The synchronism between the variations of the ratio *a:b* and the variations of the 11-year interval furnishes additional evidence of the substantial accuracy of the epochs of Wolfer.

Epochs of magnetic declination range.—These epochs, given in Table IV, columns 1 and 2, were derived from the table of smoothed means of declination range in a paper by Fritz,[1] with the exception of those subsequent to 1878, which are those determined by Ellis in his discussion of the Greenwich magnetic observations. The epochs of the 36-year cycle, columns 6 and 7, were derived in the same manner as those of the sun-spots in Table II, and the two series of epochs are almost exactly synchronous. The table is particularly instructive as affording indirect confirmation of the accuracy of the sun-spot epochs of Wolfer, especially those in the latter part of the eighteenth century.

The average range at each epoch of maximum and minimum is given in columns 8 and 9.

Epochs of auroral frequency.—The 11-year auroral epochs, given in Table V, columns 1 and 2, are those determined by Fritz,[2] and

[1] *Viertel. d. Natur. Gesell.*, Zürich, 1884.

[2] *Die Beziehungen der Sonnenflecken zu den magnetischen und meteorologischen Erscheinungen der Erde*, Haarlem, 1878.

have been treated similarly to the sun-spot epochs. The resulting 36-year epochs, columns 6 and 7, seem to occur about 5 years later than the corresponding 36-year sun-spot epochs.

The secular epochs of maximum and minimum visibility of the aurora are given in columns 8 and 9 of the table, and apparently occur about 8 years later than the epochs in columns 6 and 7. The variations in the visibility of the aurora are thus shown to lag behind the variations in solar activity in the 36-year period by about 10 or 15 years. This probably indicates a dependence upon terrestrial as well as solar conditions. The belt of maximum frequency of auroras descends to lower latitudes during increasing solar activity in the 11-year and the 36-year cycles. It was shown in the first part of this paper that a similar change of position occurs in the belt of maximum storm-frequency during cold periods, which will be shown to follow closely periods of maximum solar activity. The intimate relation between auroral and meteorological phenomena is thus apparent.

Correlation of solar and terrestrial variations.—As shown above, terrestrial and solar phenomena undergo cyclical variations in recurring intervals of about 36 years, and these variations have been traced back to about 1050 A. D. in each instance. The epochs of maximum and minimum severity of winters, Table I, series 9, which are practically coincident with those of temperature, will be considered as the primary series of meteorological epochs. Comparing these epochs with those of solar activity, Table II, columns 6 and 7, it is readily apparent that the two series synchronize closely, with an average retardation of about 7 years in the meteorological epochs.

When the two series of epochs, previous to 1610, are compared, the synchronism is less exact, as might be expected. It seems remarkable that from the epochs of sun-spot maxima, determined by Fritz, the epochs of the 36-year cycle should be derived in such a regular sequence, considering the source and character of the data he utilized. Although the lag of the meteorological epochs is somewhat greater and more variable than that since 1610, still the correspondence is much closer than one would anticipate. The great number of coincidences shown in the comparison of the two series of epochs from 1050 to 1895 makes the conclusion irresistible that our meteoro-

logical variations are conditioned upon variations in solar activity. (Compare Chart 2.)

It was stated above that the epochs of maximum and minimum spottedness in the 36-year cycle show a slight retardation when compared with the epochs with which the meteorological epochs were compared. Hence the latter differ by less than 5 years from the epochs of variation of solar spottedness.

Summarizing, therefore, the foregoing results, we conclude that periods of maximum solar activity, characterized by a minimum length of the 11-year cycle, are followed 7 to 10 years later by terrestrial temperature minima, and 6 years thereafter by rainfall maxima; and that, coincidently with the low temperature, the activity of the general circulation reaches a maximum, and storm-centers move with increased velocity and in lower latitudes.

III. SHORT CYCLES OF SOLAR AND METEOROLOGICAL PHENOMENA

A brief reference will now be made to the shorter cycles of solar activity and the corresponding meteorological variations.

The evidence for the existence of an 11-year variation in meteorological phenomena is very conflicting and inconclusive, but on the whole it points to greater activity of atmospheric circulation, lower temperature, and excessive precipitation shortly after the sun-spot maximum.

A study of the short cycle of solar activity, evidenced by variations in the frequency of solar prominences, yields far more satisfactory results. Sir Norman Lockyer and Dr. W. J. S. Lockyer[1] first announced a period of about $3\frac{1}{2}$ years in the prominence frequency, and traced synchronous variations in pressure and rainfall. Professor F. H. Bigelow[2] previously had shown that a 3-year variation existed in meteorological phenomena in the United States and found similar fluctuations in the terrestrial magnetic field. In a recent paper[3] he showed that the pressure over the Indo-Oceanic and

[1] "On Some Phenomena which Suggest a Short Period of Solar and Meteorological Changes," *Proc. R. S.*, **70**, 500, June, 1902.

[2] "Inversion of Temperatures in the 26.68-Day Solar Magnetic Period," *Am. Jour. Science* (4), **18**, Dec., 1894.

[3] "Synchronism of the Variations of the Solar Prominences with the Terrestrial Barometric Pressures and the Temperatures," *Monthly Weather Review*, **31**, 509, 1903.

Arctic regions varies directly with the prominence frequency, while over the Azores and Hawaii it varies inversely; also that the temperature over Iceland, northern Europe, and the northern United States varies inversely with the prominence frequency

In order to trace synchronous variations in the $3\frac{1}{2}$-year cycle, analogous to those shown above to exist in the 36-year cycle, I have made a careful comparison of the variations in the prominence frequency with those of various meteorological phenomena during the period 1873–1903, and the following conclusions appear to be justified. Coinciding with the maxima of the prominence curve, indicating secondary maxima of solar activity, are:

1. Increased activity of atmospheric circulation, shown by—
   a) Greater velocity of storm movement in longitude.
   b) Lower latitude of storm-tracks.
2. Higher pressure over arctic and tropical regions.
3. Lower pressure over middle latitudes, shown most clearly by the pressure at the Azores and Hawaii.
4. Weaker gradient between the Azores and Iceland.
5. Lower temperature in Iceland, northern Europe, and the northern United States.

These conditions, prevailing at or shortly after the secondary maxima of solar activity in the $3\frac{1}{2}$-year cycle are identical with those shown to exist in connection with the solar maxima of the 36-year cycle, and the two results are mutually confirmatory.

The effect of an increase in solar activity upon the Earth's atmosphere, shown by both short- and long-period variations, is immediate, and results in increased activity of the polar whirls, forcing equatorward masses of cold air, and causing both highs and lows to traverse paths in lower latitudes and with increased velocity.

Speculation as to the manner in which the solar influence is exerted seems unprofitable in the light of our present knowledge of the manifestations of solar energy. Whether variations in solar radiation exist, sufficient to produce such variations in climate, is a problem still undetermined. The paradox involved in attributing the cold periods to diminished solar radiation, apparently precludes variations in the latter as the efficient cause, or at least renders it probable that they are of secondary importance. The fact that our meteoro-

logical variations are greater and occur earlier in high latitudes seems to indicate that the polar and not the equatorial regions are mainly influenced by the varying manifestation of solar energy, in which case some action involving variations in the magnetic field of the earth must be taken into consideration.

IV. THE THREE-HUNDRED-YEAR CYCLE

The tendency of the ratio *a:b* to decrease from about 1685 to 1860 suggests a long-period variation in solar activity, since, as shown above, this ratio varies inversely with the relative-numbers in the 36-year cycle. Furthermore, the length of the 36-year interval is not uniform, but is least about 1850, averaging 30 years, greatest during the early part of the eighteenth century, averaging 40 years, and decreases again in the early part of the seventeenth century.

Regarding variations in solar activity since 1600, the records indicate that a chief minimum occurred in the latter part of the seventeenth century and a maximum about the middle of the nineteenth century. Miss Clerke[1] writes: "Spoerer's researches showed that the law of zones was in abeyance during some 70 years previous to 1716, during which period sun-spots remained persistently scarce, and auroral displays were feeble and infrequent even in high latitudes. An unaccountable suspension of solar activity is, in fact, indicated." Young[2] writes: "From 1672 to 1704 absolutely no spots were recorded in the northern hemisphere."

Thus considering the period 1600 to 1900, a minimum of solar activity prevailed about 1680, associated with a maximum value of the ratio *a:b* and a maximum length of the 36-year interval; the reverse conditions prevailed about 1860. The maximum of 1778 and the minimum of 1810 appear to be phases of a secondary variation.

For the centuries previous to 1600 we have the catalogue of early observations of sun-spots and auroras, compiled by Fritz, which enable us to trace this secular variation back for nearly 1500 years. The following table gives for each hundred-year interval the number of years when sun-spots and auroras were recorded.

[1] *History of Astronomy*, p. 148.

[2] *The Sun*, p. 149.

Interval. A. D.	Sun-Spots	Auroras
100– 200...........	1	1
200– 300...........	1	0
300– 400...........	24	2
400– 500...........	3	8
500– 600...........	8	25
600– 700...........	1	11
700– 800...........	1	12
800– 900...........	10	19
900–1000...........	1	21
1000–1100...........	6	13
1100–1200...........	19	36
1200–1300...........	7	12
1300–1400...........	10	18
1400–1500...........	0	6
1500–1600...........	7	56

Curves showing this secular variation in the frequency of sun-spots and auroras may be found on Chart 3.

Fritz[1] asserts that the sixth, ninth, twelfth, sixteenth, and nineteenth centuries have been distinguished by great and frequent auroral displays; the above table of frequency of auroras in each century serves to illustrate this statement. The table of sun-spot frequency shows that sun-spots have also been more frequently observed in these centuries. It was shown above that the periods of maximum visibility of the aurora during the last 200 years have been preceded by periods of increased solar activity, and it may therefore be considered as probable that unusual outbursts of solar energy occurred during the centuries above mentioned.

Approximate epochs for these long-period variations of solar activity and auroral frequency are given in Table VI, from which the average length of this cycle is found to be about 300 years.

Since the length of the 36-year cycle has varied parallel with variations of solar activity during the last 300 years, a similar relation would be expected to prevail in the preceding centuries. The series of 36-year epochs in Table III show that variations in the length of this cycle have indeed occurred. The epochs of maxima and minima with the mean length at each epoch are given in Table VI, columns 5 and 6. These epochs correspond very well with the epochs of sun-spot and auroral maxima and minima. The conclusion is that

[1] *Die Beziehungen der Sonnenflecken zu den magnetischen und meteorologischen Erscheinungen der Erde*, p. 41.

variations in solar activity in the 300-year cycle are associated with variations in the length of the 36-year solar cycle, ranging from 30 to 45 years, the period-length decreasing with increasing solar activity. A smoothed curve of these variations in the length of the 36-year solar cycle is shown in Chart 3. Similar variations in the length of the 11-year cycle exist, and the approximate epochs of minimum and maximum length with the average interval at each epoch are shown in Table VI, columns 11 and 12.

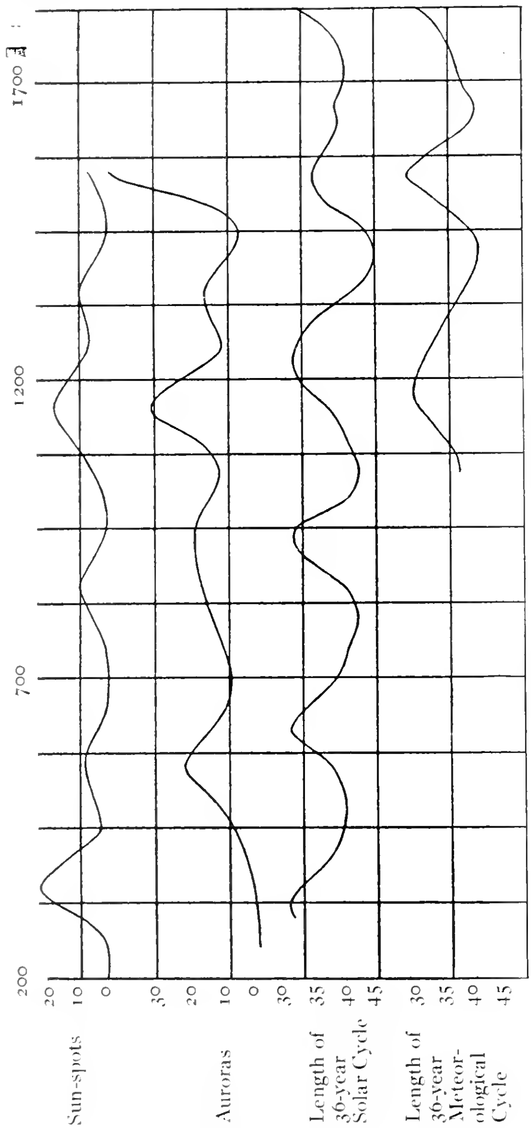


CHART 3.—The 300-Year Cycle.

With regard to meteorological variations in a cycle of 300 years, the best evidence at hand is the nearly continuous record of the time of grape harvest at Dijon, France, since 1400. The average date of beginning of the harvest for each half-century is shown in the following table:

Period	Average Date	Period	Average Date
1400–1450	September 24	1650–1700	September 23
1450–1500	September 28	1700–1750	September 27
1500–1550	September 27	1750–1800	September 30
1550–1600	September 29	1800–1850	October 2
1600–1650	September 26		

There is clear evidence of periods of high temperature about 1425 and 1675, while low temperatures prevailed about 1550 and 1825.

The average date of opening of navigation at Riga has varied as follows:

1530–1623	March 28.2	1751–1802	March 25.3
1626–1750	March 24.4	1803–1852	March 27.5

The variations in temperature shown by this table accord very well with those shown by the average time of grape harvest.

Referring to the series of epochs of the severity of winters, Table I, series 9, an inspection of columns 3 and 4 discloses variations in the length of the 36-year interval, minima occurring about 1200, 1525, and 1850, and maxima about 1050, 1415, and 1675. Chart 3 contains a smoothed curve of these variations. The secular variations in the time of grape harvest at Dijon agree closely with these variations in the length of the 36-year cycle, periods of low temperature corresponding with periods during which the average length of the cycle is 30 to 32 years, while periods of high temperature coincide with an average length of 40 to 42 years.

These epochs of maxima and minima in the length of the 36-year cycle in meteorological phenomena correspond closely with those found above for the 36-year solar cycle, thus furnishing additional evidence of a close connection between the two phenomena.

Chart 2 exhibits graphically this 300-year variation in the length of the 36-year solar and meteorological cycles.

NOTE.—The increased retardation of the meteorological epochs at the minima of the 300-year solar cycle, shown by these curves, is significant as indicating that the relation between the two phenomena is one of cause and effect.

TABLE I

Meteorological Epochs

1						2				3			
Temperature						Precipitation				Height of Water-Surface			
Europe		United States		Whole Earth		Europe		United States		(a) Closed Lakes		(b) Rivers	
Cold	Warm	Cold	Warm	Cold	W'rm	Wet	Dry	Wet	Dry	High	Low	High	Low
										1600			
										1638?			
											1656?		
										1674?			
											1683?		
						1701–05				1710			
							1726–30				1720		
1736–40				1738		1741–45		1740		1740		1740	
	1750				1750		1761–65		1762?		1760		1760
1766–70		1784?		1770		1771–75		1775		1780		1775	
	1786–90		1796		1790		1796–00		1795		1800		1795
1811–15		1815		1813		1811–15		1815		1820		1820	
	1821–25		1826		1825		1831–35		1836		1835		1831–35
1836–40		1837		1838		1846–50		1848		1850		1850	
	1860		1860		1860		1861–65		1864		1865		1861–65
1876–80		1880		1878		1880		1882		1880		1876–80	
	1895		1892				1898		1895				1895

4				5		6		7		8	
Barometric Pressure (a)		Pressure-Gradient (b)		Frequency of Easterly Winds		Variability of Temperature		Freq'cy of West Indian Hurricanes		Frequency of Thunderstorms	
Low	High	Min.	Max.	Max.	Min.	Max.	Min.	Max.	Min.	Min.	Max.
								1590?			
								1625?			
								1655			
									1685		
								1710			
									1725		
1741–45								1745			
	1760				1755		1755		1762		1768
1771–75				1775		1780		1786		1783	
	1795		1790		1790		1795		1798		1797
1811–15		1815		1815		1810		1817		1814	
	1831–35		1831–35		1830		1821–25		1823		1822
1846–50		1840		1845		1836–40		1838		1837	
	1861–65		1860		1865		1855		1864		1852
1876–80		1875		1880		1876–80		1885			
	1895		1895		1895		1895		1895		

TABLE I—*Continued*

9				10		11		12	
FREQUENCY OF SEVERE WINTERS				SEASON OF NAVIGATION		TIME OF GRAPE HARVEST		GRAIN PRICES	
		INTERVAL OF THREE PERIODS							
Max. or Cold	Min. or Warm	Max. to Max.	Min. to Min.	Short	Long	Late	Early	High	Low
1000									
	1025								
1045									
	1060	120							
1075			110						
	1095	105							
1120			110						
	1135	105							
1150			100						
	1170	95							
1180			100						
	1195	100							
1215			90						
	1235	100							
1250			105						
	1260	100							1265
1280			105					1290	
	1300	110							1305
1315			115					1320	
	1340	115							1340
1360			115					1370	
	1375	120							1385
1395			120			1405		1405	
	1415	125					1421–25		1420
1435			125			1446–50		1438	
	1460	120					1466–70		1465
1485			115			1481–85		1482	
	1500	110					1501–05		1500
1515			95			1511–15		1527	
	1530	95					1526–30		1540
1545			90	1560		1545		1555	
	1555	100			1570		1556–60		1570
1580			105	1591–95		1585		1596	
	1590	110			1611–15		1601–05		1603
1615			125	1621–25		1621–25		1640	
	1635	120			1645		1636–40		1654
1655			125	1660		1660		1662	
	1680	120			1690		1681–85		1688
1700			120	1710		1701–05		1700	
	1715	120			1726–30		1726–30		1730
1735			110	1741–45		1741–45		1740	
	1755	115			1761–65		1756–60		1753
1775			110	1781–85		1766–70		1772	
	1790	105			1791–95		1786–90		1785
1815			105	1811–15		1815		1812	
	1825	100			1821–25		1831–35		1835
1840			100	1841–45		1846–50		1855	
	1860				1861–65		1865		1864
1875				1876–80		1881–85		1873	
	1890								1895
1910?									

TABLE II

SUN-SPOT EPOCHS (WOLFER)

Epochs of 11-Year Cycle		Interval			Epochs of 36-Year Cycle		Relative-Numbers		Interval		Ratio	
Max.	Min.	Max. to Max.	Min. to Min.	Smooth'd Means	Max.	Min.	At Maxima	Av'ge Min. to Min.	Min. to Max. a	Max. to Min. b	$a:b$	$a:b$ Smooth'd
1	2	3	4	5	6	7	8	9	10	11	12	13
	1610.8											
1615.5			8.2	9.4	1615				4.7		1.34	
	1619.0	10.5		11.2						3.5		
1626.0			15.0	13.0					7.0		0.87	
	1634.0	13.5		13.2		1630				8.0		
1639.5			11.0	11.3					5.5		1.00	0.94
	1645.0	9.5		10.2						5.5		
1649.0			10.0	10.2	1647				4.0		0.66	1.07
	1655.0	11.0		10.7						6.0		
1660.0			11.0	12.3					5.0		0.83	1.14
	1666.0	15.0		13.2		1670				6.0		
1675.0			13.5	12.8					9.0		2.00	1.08
	1679.5	10.0		11.2						4.5		
1685.0			10.0	9.3			67		5.5		1.22	1.18
	1689.5	8.0		8.8	1690					4.5		
1693.0			8.5	9.7			68		3.5		0.70	1.25
	1698.0	12.5		11.7						5.0		
1705.5			14.0	13.1		1707	50	17	7.5		1.15	0.97
	1712.0	12.7		12.7						6.5		
1718.2			11.5	11.2			67	27	6.2		1.17	0.87
	1723.5	9.3		10.4						5.3		
1727.5			10.5	10.3	1725		92	43	4.0		0.61	0.95
	1734.0	11.2		10.9						6.5		
1738.7			11.0	11.3			88	41	4.7		0.74	0.97
	1745.0	11.6		10.9		1745				6.3		
1750.3			10.2	11.0			68	33	5.3		1.08	0.85
	1755.2	11.2		10.9						4.9		
1761.5			11.3	10.2			86	52	6.3		1.26	0.82
	1766.5	8.2		9.5						5.0		
1769.7			9.0	8.6	1770		116	63	3.2		0.55	0.74
	1775.5	8.7		9.0						5.8		
1778.4			9.2	9.2			158	69	2.9		0.46	0.78
	1784.7	9.7		10.8						6.3		
1788.1			13.6	13.5		1790	141	50	3.4		0.33	0.69
	1798.3	17.1		14.3						10.2		
1805.2			12.3	13.5	1807		40	30	6.9		1.28	0.91
	1810.6	11.2		12.1						5.4		
1816.4			12.7	12.5			49	19	5.8		0.84	0.92
	1823.3	13.5		12.3		1820				6.9		
1829.9			10.6	10.5			72	40	6.6		1.65	0.97
	1833.9	7.3		9.2	1835					4.0		
1837.2			9.6	9.3			147	65	3.3		0.52	0.83
	1843.5	10.9		11.0						6.3		
1848.1			12.5	11.8			132	52	4.6		0.58	0.75
	1856.0	12.0		11.9		1853				7.9		
1860.1			11.2	11.2			98	50	4.1		0.58	0.59
	1867.2	10.5		11.1	1865					7.1		
1870.6			11.7	11.8			140	57	3.4		0.41	0.61
	1878.9	13.3		11.9						8.3		
1883.9			10.7	11.4		1880	75	32	5.0		0.88	
	1889.6	10.2		11.0						5.7		
1894.1			12.1				88	36	4.5		0.61	
	1901.7									7.6		

TABLE III

EPOCHS OF SUN-SPOT MAXIMA (FRITZ)

EPOCHS OF MAXIMA	INTERVAL MAX. TO MAX.	EPOCHS OF 36-YEAR CYCLE		EPOCHS OF MAXIMA	INTERVAL MAX. TO MAX.	EPOCHS OF 36-YEAR CYCLE	
		Max.	Min.			Max.	Min.
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1057		1060		1324			
	12				10		
1069				1334		1335	
	12				14		
1081			1080	1348			
	15				12		1360
1096				1360			
	8	1100			12		
1104				1372			
	13				8		
1117			1120	1380		1380	
	13				8		
1130				1388			
	8				13		
1138		1140		1401			1405
	10				14		
1148				1415*			
	13				10		
1161			1160	1425*		1425	
	16				10		
1177				1435			
	8	1180			13		
1185				1448*			1450
	8				14		
1193				1462			
	10		1195		10		
1203				1472		1470	
	9	1205			11		
1212*				1483			
	13				16		1490
1225			1225	1499*			
	13				12		
1238		1240		1511		1510	
	9				7		
1247				1518			
	13		1255		11		1525
1260				1529			
	10				9		
1270		1270		1538		1540	
	8				11		
1278				1549			
	13		1285		11		1555
1291				1560			
	9				12		
1300*		1300		1572			
	8				8	1575	
1308				1580			
	16		1320		11		
1324				1591			1590
					14		
				1605			

TABLE III—*Continued*

APPROXIMATE EPOCHS OF THIRTY-SIX-YEAR SOLAR CYCLE, 295 A. D. TO 1100 A. D

Maxima	Minima	Maxima	Minima	Maxima	Minima
295			570	845	
	315	585			865
325			600	880	
	345	615			900
355			630	915	
	375	645			930
390			665	945	
	415	680			960
430			700	975	
	455	720			995
470			740	1015	
	490	760			1040
510			780	1060	
	530	800			1080
550			825	1100	

TABLE IV

EPOCHS OF MAGNETIC DECLINATION RANGE (FRITZ)

Epochs		Interval			Epochs of 36-Year Cycle		Average Range at Epochs	
Maximum	Minimum	Max. to Max.	Min. to Min.	Smoothed Means	Max.	Min.	Max.	Min.
1	2	3	4	5	6	7	8	9
1778.0							14.5	
	1784.8	9.4						9.8
1787.4			15.6	13.7		1792	15.1	
	1800.4	16.1		14.4+				7.0
1803.5			11.5	13.7	1805		9.2	
	1811.9	13.6		12.5−				6.6
1817.1			12.3	12.9+		1820	9.2	
	1824.2	12.8		11.8				6.6
1829.9			10.4	10.1			12.4	
	1834.6	7.0		9.1−	1835			9.2
1836.9			9.8	9.3			14.1	
	1844.4	11.2		11.3				8.6
1848.1			12.9	12.2+		1852	12.5	
	1857.3	12.6		11.9				6.5
1860.7			10.2	11.0			11.4	
	1867.5	10.2		10.5−	1864			7.9
1870.9			11.0	11.4			12.8	
	1878.5	13.0		11.8+				6.7
1883.9			11.3	11.4		1880		
	1889.8	9.9						
1893.8								

TABLE V

Epochs of Auroral Frequency (Fritz)

Epochs		Interval			Epochs of 36-Year Cycle		Epochs of Visibility	
Max.	Min.	Max. to Max.	Min. to Min.	Smoothed Means	Max.	Min.	Max.	Min.
1	2	3	4	5	6	7	8	9
1692								
	1700	15.4						
1707.4			12.4	13.4+		1700		
	1712.4	12.3		12.1				1710
1719.7			11.6	11.4				
	1724.0	10.4		10.3				
1730.1			8.8	9.1−	1730			
	1732.8	8.2		9.6				
1738.3			11.8	10.2			1738	
	1744.6	10.5		10.7				
1748.8			9.8	10.4				
	1754.4	10.8		10.4				
1759.6			10.7	11.5+		1760		
	1765.1	13.1		11.6				1765
1772.7			11.1	10.6				
	1776.2	7.6		8.3				
1780.3			6.2	7.1−	1780			
	1782.4	7.6		10.1				
1787.9			16.4	13.6			1788	
	1798.8	16.8		15.1+		1800		
1804.7			12.1	14.2				
	1810.9	13.7		12.4				1810
1818.4			11.3	12.1				
	1822.2	11.2		11.5				
1829.6			12.1	11.3				
	1834.3	10.7		10.8				
1840.3			9.6	10.0−	1840			
	1843.9	9.6		10.5				
1849.9			12.4	10.9			1848	
	1856.3	10.7		11.0+		1855		
1860.6			10.0	10.2				1860
	1866.3	9.9		10.7	1865			
1870.5			12.2	11.9			1870	
	1878.5	13.5						
1883.5								

TABLE VI

Epochs of the Three-Hundred-Year Cycle

Solar Spottedness		Auroral Frequency		Length of 36-Year Cycle: Solar		Length of 36-Year Cycle: Meteorological		Temperature Shown by Time of Grape Harvest		Length of 11-Year Cycle	
Max.	Min.	Max.	Min.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
1	2	3	4	5	6	7	8	9	10	11	12
350				325–30						325–10.40	
	450				475–39						440–11.55
550		575		600–30						575–10.20	
	700		750		800–42						750–11.40
850		900		925–30						975–10.37	
	1000		1050		1050–42		1050–40				1100–11.73
1150		1175		1225–31		1200–32				1225–10.50	
	1450		1450		1410–45		1415–41		1425		1450–11.00
1550		1550		1550–32		1525–32		1550		1550–10.03	
	1680		1700		1700–41		1675–41		1675		1650–11.37
1850		1860		1850–30		1850–32		1825		1750–10.55	

Washington, D. C.,
December, 1904.

AN ELEMENTARY DISCUSSION OF THE ACTION OF A PRISM ON WHITE LIGHT

By J. S. AMES

It is generally recognized at the present time that white light, so called, is not due to the superposition of regular trains of waves, but to irregular disturbances or "pulses." When such light, however, is dispersed by a grating or prism, it is resolved into components which have a certain periodicity impressed upon them. How closely this periodic phenomenon resembles homogeneous waves has been discussed by Gouy and others. From this point of view the observed periodicity is not innate in the incident white light, but is directly caused by the grating or prism. An infinitely thin pulse will be spread out by the dispersive mechanism into an extended disturbance in which there are periodicities.

Only a word need be said in regard to the action of the grating on white light, because the matter has been discussed so fully by Schuster in a paper in the *Philosophical Magazine* for June 1894, (5) **37,** 509, entitled "On Interference Phenomena." In a later paper[1] on "Talbot's Bands" Schuster again outlines the theory; and in his book, *The Theory of Optics* (London, 1904), he also discusses the same subject. The simplest case to consider is that of a single thin plane pulse falling normally on a plane reflecting grating. Let us assume that the grating consists of reflecting and non-reflecting strips, parallel and evenly spaced, and that the *only action of the grating* is to produce reversed cylindrical pulses at its reflecting strips. These pulses will be reflected back by the strips; and if the grating be viewed from any angle, a series of regularly spaced pulses will enter the eye or telescope. Thus, if the parallel lines A_1M_1, A_2M_2, etc., are drawn, making any angle, θ, with the normal to the grating, and if a line A_1A_2 be drawn perpendicular to these lines, a pulse will reach A_1 a definite interval of time before one reaches A_2, and in turn the pulse reaches A_2 by the same interval

[1] *Phil. Mag.*, (6) **7,** 1, Jan. 1904.

of time ahead of the pulse reaching A_3, etc. If a is the grating space, and V the velocity of the pulses, this definite interval of time is $(a \sin \theta)/V$. The quantity $a \sin \theta$ marks, therefore, the periodicity imparted to the pulse by the grating for an observer viewing the grating from the angle θ. These "rays" $M_1 A_1$, $M_2 A_2$, etc., may be regarded as meeting at infinity, or as being brought to a focus by a converging lens. If the latter is the case, there will be a periodic phenomenon at the focus, with the period $a \sin \theta/V$, which will persist until all the spherical pulses produced by the single incident plane pulse have passed by; that is, until N pulses have passed,

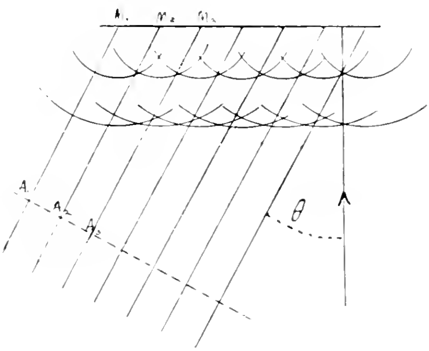


FIG. 1

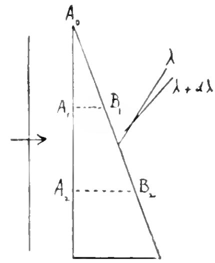


FIG. 2

if N is the number of reflecting strips of the grating. Since the "resolving power" of a grating is a measure of its efficiency in distinguishing between periodicities which are nearly equal, it is evident why this is fixed by the number of lines in the grating, as it is. On the assumption made above in regard to the action of the grating, all the light would be dispersed into two first-order spectra, one on each side of the normal. The fact that this is not true in practice, but that there are series of spectra of different orders is explained by analyzing the true action of the grating, as has been done by Rayleigh and by Schuster.

The explanation of the process by which a prism imparts periodicity to white light is by no means as self-evident as is that just given for a grating. Larmor in his *Æther and Matter*, p. 239, gives a most artificial solution, which does not appeal to one's feeling that a simple elementary explanation is possible. Schuster in the

Philosophical Magazine for January 1904, and again in his *Optics*, outlines a solution which apparently was given in full before the British Association meeting of 1903. What follows in this note may be identical with Schuster's British Association paper, but as the writer has no knowledge of this, and has not seen it in print, it may not be repetition, especially as the considerations brought forward in this discusison and all the formulæ are entirely independent of the papers mentioned.

The simplest type of pulse that one may treat mathematically is a so-called "group" consisting of two homogeneous trains of waves, of the same amplitude, but different velocity, and of slightly different wave-length. Such a group is defined by the equation:

$$y=\cos\frac{2\pi}{\lambda}(x-Vt)+\cos\frac{2\pi}{\lambda^1}(x-V^1t) \ ,$$

where

$$\lambda^1=\lambda+d\lambda$$

and

$$V^1=V+\frac{dV}{d\lambda}d\lambda \ .$$

By ordinary trigonometrical transformation this becomes

$$y=2\cos\pi\left[x\left(\frac{1}{\lambda}+\frac{1}{\lambda^1}\right)-t\left(\frac{V}{\lambda}+\frac{V^1}{\lambda^1}\right)\right].\cos\pi\left[x\left(\frac{1}{\lambda}-\frac{1}{\lambda^1}\right)-t\left(\frac{V}{\lambda}-\frac{V^1}{\lambda^1}\right)\right]$$

$$=2\cos\frac{2\pi}{\lambda}(x-Vt).\cos\pi\frac{d\lambda}{\lambda^2}\left[x-\left(V-\lambda\frac{dV}{d\lambda}\right)t\right] ,$$

which gives a physical idea of the appearance of the group. The most important property of such a group is at once apparent, viz., a group does not advance as such, keeping its identity, but, on the contrary, continuously changes its nature as one train of waves gains on the o her.[1] If, however, the appearance of the group at any point is noted at any instant, that same character will reappear at a later time in a position displaced in the direction of advance of the waves or, in certain cases, displaced backwards.

The time, T, required for the group to reappear, i. e., the "period" o the group, is $1/\frac{dV}{d\lambda}$, as follows at once from the c nsideration

[1] The only places in print in which attention has been called to this fact, so far as is known to the writer, are in the recent papers of Schuster.

that in this time one train of waves mus have gained a distance equal to the difference in wave-length; in symbols

$$T(V^1 - V) = \lambda^1 - \lambda$$

or

$$T\frac{dV}{d\lambda} = 1 \ .$$

Similarly, the distance X between the points at which any one feature of the group is restored must be such that

$$VT = X + \lambda,$$

or

$$X = \left(V - \lambda \frac{dV}{d\lambda}\right)T \ .$$

We may call the ratio $\frac{X}{T}$ the "velocity" of the group; that is, $U \equiv V - \lambda \frac{dV}{d\lambda}$. It follows, then, that $X = \lambda U/(V - U)$.

Applying the above statements to the theory of light, it is evident that a group in the pure ether advances preserving its identity, because $V^1 = V$; but when it enters a dispersive medium, its character proceeds to undergo periodic changes, reversing, reappearing, etc., as would be noted by an observer advancing with the "group velocity." Let us consider the action of a prism upon such a group, and for the sake of simplicity let the group have a plane front and fall perpendicularly upon the face of the prism. We may choose any feature of the group by which to recognize it and note its periodic recurrence, e. g., the condition marked by the sum of the two amplitudes of the component trains. As the group advances toward the prism, this "crest" moves forward with the velocity V_e, that of waves in the pure ether; when the group enters the prism, it changes its form, the "crest" recurring at intervals equal to X; consequently at certain points B_1, B_2, etc., on the second face of the prism, such that $A_1B_1 = X$, $A_2B_2 = 2X$, etc., the "crest" will emerge. Thus the vertex A_0, and the points B_1, B_2, etc., may serve as centers of secondary disturbances in a Huygens' construction, and a plane drawn tangent to these secondary spheres may be called the "group-front." It is apparent, however, that in the time T required for the "crest" to reappear at B_1 after disappearance at A_1 the component *trains of waves* have advanced a greater distance than A_1B_1, and have emerged

from the prism and passed on as two separate trains in slightly different directions, owing to their different indices of refraction.

There is thus a periodicity in the group-front, due to the fact that at certain regularly spaced intervals there is the maximum amplitude. This is caused obviously by the superposition of the two crests of the component *trains of waves* whose wave-fronts cross at a small angle. We can therefore study the direction of advance of any one "crest" in the group-front, and at the same time calculate the periodicity produced when the group is received by a telescope, by plotting the traces of the two trains of waves.

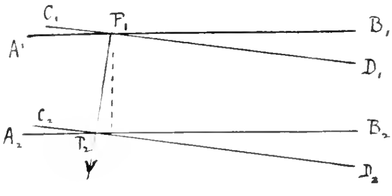


FIG. 3

Let the lines A_1B_1 and C_1D_1 be the traces of the crests of the two trains of waves at any instant, P_1, their point of intersection will then be a "crest" of the group-front; at a certain time later the two wave-crests will have advanced through equal distances to positions A_2B_2 and C_2D_2, and their point of intersection, P_2, will mark the new position of the "crest" of the group-front. In other words, P_1P_2, a line perpendicular to the bisector of the angle between A_1B_1 and C_1D_1, may be called the direction of advance of the group; that is, the receiving telescope must have this direction.

To deduce the periodicity observed by the telescope, one has but to draw the crests of the two trains of waves as they are at any instant, for a distance of several wave-lengths.

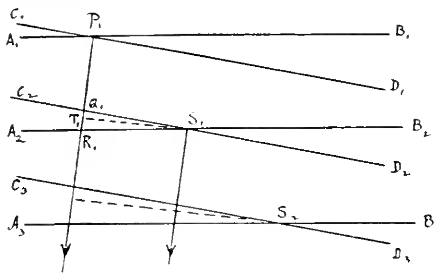


FIG. 4

Thus, let A_1B_1, A_2B_2, A_3B_3, etc., be the traces *at any one instant* of the wave-crests of the train whose wave-length in the pure ether is $\lambda_e + d\lambda_e$; let C_1D_1, C_2D_2, C_3D_3, etc., be those of the train whose wave-length is λ_e, *at the same instant;* and let P_1, S_1, S_2, etc., be their points of intersection. As the trains *advance*, the "crest" P_1 moves, as has just been shown, in the direction $P_1Q_1R_1$; the "crest" S_1 moves in a parallel

direction, etc. Consequently, the periodicity observed by the telescope is given by the distance P_1T_1, where T_1 is the foot of the perpendicular dropped from S_1 upon $P_1Q_1R_1$. If the angle between A_1B_1 and C_1D_1 is called a, this periodic distance $P_1T_1 = \frac{1}{\cos\frac{a}{2}}\left(\lambda_e + \frac{d\lambda_e}{2}\right)$; and therefore in the limit equals λ_e.

The case of a more complicated group or of a pulse is, to a certain extent, equally simple. Any group or pulse may be analyzed into a number of simple groups like those discussed above, each such

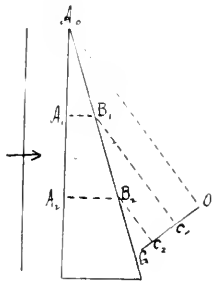


FIG. 5

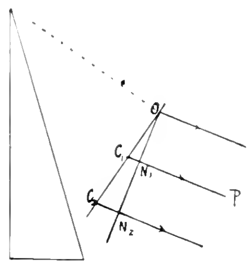


FIG. 6

group being "associated" with a certain train of waves of wave-length λ. If such a complex group enters a dispersive medium, two things must be noted: (1) Since the velocity of any simple group is $V - \lambda\frac{dV}{d\lambda}$, the different component groups will have different velocities, and so their group-fronts will be differently refracted, both on entering and on emerging; (2) since the distance required for a certain feature of a group to reappear, i. e., the length X, is different for the different groups, they will recur at different intervals, and therefore the complex group itself could not reappear. These complications might be avoided if a dispersive medium could be found for which $V - \lambda\frac{dV}{d\lambda}$ and $\left(V - \lambda\frac{dV}{d\lambda}\right)\frac{1}{\frac{dV}{d\lambda}}$ are both constant. These conditions are satisfied if the dispersion formula for the medium obeys the relation $V = A + B\lambda$, where A and B are constants; for, in this case, the group-velocity is A, and the periodic distance X is A/B: both of which are independent of λ, and therefore the same for all the component simple groups.

To avoid any refraction of the *wave-fronts* of the ultimate trains of waves on entrance into a prism, we may, as before, consider normal incidence. Then, again, we will have what may be called a "group-front" for the emerging light by drawing a plane tangent to secondary spherical disturbances having A_0, B_1, B_2, etc., as centers, where $A_1B_1 = X$, $A_2B_2 = 2X$, etc. Let the trace of this plane be OG. It will contain periodicities, for the conditions are the same at O, C_1, C_2, etc., the points of tangency. As is seen by considering the complex group made up of simple ones, the condition at these points is due to a superposition of trains of waves, and, as these advance, the different component simple groups separate out and give rise to different periodicities proceeding in different directions. We may trace these in the following manner: Let OC_1C_2 . . . . be the "group-front;" then the effects propagated in the direction C_1P—which is taken at random—have the periodicity C_1N_1 where the line ON_1N_2 . . . . . is drawn perpendicular to the direction C_1P; for $C_2N_2 = 2C_1N_1$, etc. We will prove that this periodic distance C_1N_1 is equal to λ_e, where this is the wave-length of the *train of waves* which, after normal incidence on the prism, would on emergence have the *wave-front* ON_1N_2. . . . . . The difference in time required for the group-front and the train of waves to traverse the prism along the line A_1B_1 is

$$X\left(\frac{1}{U}-\frac{1}{V}\right) \quad \text{or} \quad X\frac{V-U}{UV},$$

which, as proved above, equals $\frac{\lambda}{V}$, where λ is the wave-length of the train of waves while in the prism. Hence the distance of the wave-front in advance of the group-front, after emergence, along the line C_1P is $\frac{V_e\lambda}{V}$ or $\mu\lambda$, which equals λ_e. That is, the distance C_1N_1 equals λ_e.

It is thus seen that if a telescope is pointed in different directions toward the prism, disturbances of different periodicities will be brought to a focus; and, further, that the periodicity corresponding to any one direction is exactly that of the train of waves which would be brought to a focus if this train had been incident upon the prism instead of the group. In other words, a complex group gives rise,

through the agency of the prism, to periodic effects advancing in different directions, which are identical—with an important limitation, to be noted presently—with the effects which would have been produced if a complex train of waves had been incident upon the prism. Accordingly, the fact that a prism produces approximately homogeneous trains of waves when white light falls upon it is not a proof of the existence in the white light of periodic component trains of waves. The "resolving power" of the prism is evidently proportional to the number of periodicities which occur in the emergent "group-front," and, if Δ is the thickness of the base of the prism, this number equals $\frac{\Delta}{\lambda}$ or $\frac{\Delta}{U}\frac{dV}{d\lambda}$. This limits, then, the periodic nature of the resolved components.

In thus explaining how an arbitrary group or pulse may, by means of a prism, produce what to a certain extent may be called trains of waves, a particular kind of dispersive medium has been considered. This is, however, no limitation upon the argument.

For those interested in the discussion of the nature of white light, a list of the most important papers on the subject is added:

SCHUSTER: "On Interference Phenomena." *Phil. Mag.*, (5) **37**, 509, 1894. "A Simple Explanation of Talbot's Bands." *Phil. Mag.*, (6) **7**, 1, 1904. *Boltzmann's Festschrift*, p. 569. *The Theory of Optics*, London, 1904, Chapter. XIV. Also *Phil. Mag.* (6) **5**, 344, 1903; *Nature*, **53**, 268, 1895-6; *Cam. Phil. Trans.*, **18**, 108, 1899.

RAYLEIGH: "Wave Theory of Light." *Ency. Brit.*, **24**, 1888. *Collected Works*, III, p. 47. "On the Character of the Complete Radiation at a Given Temperature." *Collected Works*, III, p. 268. *Phil. Mag.*, (5) **27**, 460, 1889. Also *Nature*, **57**, 607, 1898; *Phil. Mag.* (6) **5**, 238, 1903.

LARMOR: *Æther and Matter.* Cambridge, 1900, pp. 239, 247.

PLANCK: "Ueber die Natur des Weissen Lichtes." *Annalen der Physik*, (4) **7**, 390, 1902.

GOUY: "Sur le Mouvement Lumineux." *Journal de Physique*, (2) **5**, 354, 1886. See also GARBASSO: *Journal de Physique*, **7**, 252 and 346, 1898.

STONEY, G. JOHNSTONE: *Phil. Mag.* **45**, 535, 1898; **46**, 253, 1898.

CARVALLO: *Journal de Physique* (3), **9**, 136, 1900.

CORBINO: *Ibid.* (4), **1**, 512, 1902, and papers in the *Comptes Rendus:* **120**, 757, 1895, by Poincare; **130**, 79, 401, 1900, by Carvallo; **130**, 238, 1900, by Fabry; **130**, 241, 560, 1900, by Gouy; **133**, 412, 1901, by Corbino.

JOHNS HOPKINS UNIVERSITY,
Baltimore, April 1905.

THE VARIABLE RADIAL VELOCITY OF γ *GEMINORUM*

By V. M. SLIPHER

The spectrum of γ *Geminorum* is the same as that of *Sirius*. As it contains many sharp metallic lines, the radial velocity of the star can be quite accurately observed. The variability of the star's velocity was announced by Campbell in *Lick Observatory Bulletin* No. 70 and in the *Astrophysical Journal* for March 1905. I had secured, with the Lowell spectrograph, two observations:

1902, October 15: Velocity = −22 km,
1903, November 24: Velocity = − 1 km,

which showed clearly variable velocity, and the binary character of the star was awaiting announcement here at the time it was published by the Lick observers.

The publication of the Lick observations encouraged me to make more spectrograms with the hope of being able to determine the star's orbital period. To this end, Professor Frost has very kindly communicated to me the observations of the star, made at the Yerkes Observatory by himself and Mr. Adams.[1] In the accompanying table are given, in chronological order all the velocity determinations available, beginning with the early ones by Vogel and Scheiner at Potsdam.[2]

Except Vogel's measures on the first two plates, the Potsdam velocities depend upon measures of the displacement of the $H\gamma$ line, which is not suited to accurate measurement, and the determinations are therefore not comparable in accuracy with the more recent ones made elsewhere, which depend upon measures of the sharp metallic lines. It might be well to note in passing that the agreement between Vogel's and Scheiner's measures of the same plate is in reality not so satisfactory as given above, for the values here have received Vogel's

[1] Three of these observations were referred to in *Astrophysical Journal*, **15**, 217, 1902, but have not been previously published.

[2] *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, **7**, Theil I.

TABLE I

Observatory	Date	Velocity	Measurer	Remarks
Potsdam......	1888, Dec. 14	−18.1 km	Vogel	
		−19.0	Scheiner	
"	1889, Jan. 6	−14.2	Vogel	
		− 9.9	Scheiner	
"	1890, Apr. 5	−17.3	Vogel	
		−18.2	Scheiner	
"	1891, Feb. 7	−12.0	Vogel	
		− 8.1	Scheiner	
Lick.........	1899, Sept. 21	−17	Campbell	Approx. measures
		−15.4	Burns	Definitive measures
"	1899, Oct. 24	−17	Campbell	Approx. measures
		−15.1	Burns	Definitive measures
Yerkes.......	1901, Nov. 15	−15.4	Adams	Photo'd by Frost
"	1901, Nov. 20	−14.9	Adams	Photo'd by Adams
"	1901, Nov. 27	−16.4	Adams	Photo'd by Frost
Lowell........	1902, Oct. 15	−22.5	Slipher	
"	1903, Nov. 24	− 0.8	Slipher	Comparison weak
Lick.........	1904, Jan. 27	− 4.7	Burns	
Yerkes.......	1904, Dec. 6	− 8.7	Frost	Photo'd by Frost
Lick.........	1905, Feb. 13	−10.4	Burns	
Lowell........	1905, Mar. 10	−11	Slipher	Underexposed
"	1905, Mar. 20	−12	Slipher	Underexposed
"	1905, Apr. 6	−11	Slipher	
"	1905, Apr. 13	−11.8	Slipher	

empirical correction, which decreases his negative velocity by 0.9 km, and increases Scheiner's by 2.8 km; i. e., for example, the actual disagreement of the second and fourth plates was about 8 km. And, contrary to my hopes, I could not derive much assistance from the Potsdam velocities in determining the period.

To take up the later observations. Those of the Lick and the Yerkes made from 1899 to the end of 1901 gave no evidence of variation, but agreed closely, giving a velocity of −15.5 km. The first to show variation were those by the writer in October 1902 and November 1903, which give the widest range so far observed. The Lick observation at the end of January 1904 also shows clearly the variation in the velocity, confirming my observation of a marked decrease in the negative velocity near the end of 1903 or the beginning of 1904. From this epoch down to the middle of April 1905 the negative velocity has been gradually increasing, and it is probable that the maximum will be reached near the middle of 1906. The observations seem to be satisfied with a period of about three and one-half years. However, more observations near the times of

maximum and minimum velocity are needed to determine the period with accuracy, and it is hoped that this approximate period will serve to indicate the times when observations should be made to be of most value in a final investigation of the system. It is evident that the present observational data are insufficient to justify an attempt to determine the orbital elements of the star.

A curve extended backward with this period passes fairly near the first, second, and fourth of the Potsdam observations, but about 12 km above the third one.

According to this period, the orbit would seem to be quite eccentric, for the change from maximum to minimum velocity takes place in much less time than the change from minimum to maximum.

This spectroscopic system is interesting from its length of period, which is comparable with that of the telescopic system δ *Equulei*, and is, excepting the secondary period of *Polaris*, the longest yet met with among spectroscopic binaries.

LOWELL OBSERVATORY,
May 3, 1905.

Minor Contributions and Notes

STARS HAVING PECULIAR SPECTRA[1]

An examination of the photographs of the Henry Draper Memorial has led to the discovery by Mrs. Fleming of a number of variable stars and other objects having peculiar spectra. A list of these, together with three additional stars having bright hydrogen lines to which attention was called by Mr. Edward S. King, is given in Table I. The constellation and number in the *Durchmusterung* are given in the first two columns. The approximate right ascension and declination for 1900 and the catalogue magnitude, except in the case of variable stars, are given in the third, fourth, and fifth columns. The class of spectrum and a brief description of the object are given in the sixth and seventh columns. The designations for

TABLE I

PECULIAR SPECTRA

Constellation	DM. No.	1900 R. A.	Dec. 1900	Mag.	Spectrum	Description
		h m	° ′			
Cassiopeia.....	....	0 4.3	+51 0	...	Md.	Variable. H 1166.
Cepheus.......	....	0 7.6	+71 32	...	Pec.	Bright lines and bands.
Cassiopeia....	+49°41	0 12.2	+49 44	...	Na.	Variable. H 1167.
Phoenix.......	−48.313	1 10.2	−48 47	9.4	Pec.	Dark bands.
Cassiopeia.....	....	1 50.2	+62 49	...	Pec.	Gaseous Nebula.
Eridanus......	−57.400	2 2.2	−57 37	...	Md.	Variable. H 1168.
Horologium....	−43.1198	3 46.7	−43 50	8.5	Pec.	Dark bands.
Camelopardalus	+58.804	4 57.4	+58 50	6.0	B Pec.	$H\epsilon$, $H\delta$, $H\gamma$, and $H\beta$, bright.
Puppis........	−22.1874	7 22.8	−22 53	6.0	B Pec.	$H\beta$ bright.
Cancer.........	....	9 2.2	+21 58	...	Pec.	Dark bands?
Hydra.........	−22.2684	9 36.7	−23 8	5.0	B Pec.	$H\beta$ bright.
Ursa Major....	....	12 34.4	+59 2	...	Md.	Variable. H 1169.
Virgo.........	....	12 57.4	+ 5 43	...	......	Variable. H 1170.
Lupus.........	....	14 46.8	−46 12	...	......	Variable. H 1171.
Ophiuchus.....	−18.4282	16 21.2	−18 14	5.2	B Pec.	$H\epsilon$, $H\delta$, $H\gamma$, $H\beta$, bright.
Draco.........	....	17 53.2	+64 9	...	Pec.	Dark bands.
Cygnus........	....	19 57.8	+32 18	...	Pec.	Bright lines. Type V.
Cygnus.......	+46.3133	20 56.4	+47 8	5.3	B Pec.	$H\zeta$, $H\epsilon$, $H\delta$, $H\gamma$, $H\beta$ bright.
Cygnus........	....	21 28.7	+44 10	...	Pec.	Bright lines. Gaseous Neb.
Pegasus........	+22.4508	21 51.7	+22 24	...	Na	Variable. H 1172.
Pegasus........	+20.5071	21 59.7	+20 34	8.7	Pec.	Dark bands.
Piscis Austrinus	−25.16142	22 46.7	−25 40	...	Mc.	Variable. H 1173.

[1] *Harvard College Observatory Circular* No. 98.

stars north of declination $-23°$ are taken from the *Bonn Durchmusterung*, for stars between declinations $-23°$ and $-52°$ the *Cordoba Durchmusterung* is used, and for stars south of declination $-52°$ the *Cape Photographic Durchmusterung* is used. Each of the new variables has been confirmed independently by a second observer. Additional information regarding these objects will be found in the remarks following the table. In the case of new variable stars, the right ascension is followed by the designation described in the *Annals*, **48**, 93, which gives the approximate position and also the designation described in the *Annals*, **53**, No. VII, which indicates the number in the series of variables found at Harvard. This number is also given in the table for convenience of future reference.

REMARKS

h m

0 4.3. 000451=H 1166. An examination of this star on five chart plates, taken between November 23, 1898, and December 6, 1902, shows a variation of about 2.5 magnitudes. Estimates from these plates give the approximate limits, 9.0 to 11.5.

0 7.6. This object is *N.G.C.* 40. Five bright lines or bands appear in its spectrum, λ3869, 4101, 4340, 4688, and 4861. The first band is broad, and apparently coincides with the band seen in certain gaseous nebulæ. The second, third, and fifth lines are $H\delta$, $H\gamma$, and $H\beta$. The fourth band, which is the strongest of all, has the position of the characteristic line in the spectra of the fifth type. The nebular lines near wave-length 5000 are not seen. This object may therefore be intermediate between a nebula and a fifth-type star.

0 12.2. 001249=H 1167. An examination of this star on sixteen chart plates, taken between January 3, 1890, and December 10, 1902, shows a variation of about 1.5 magnitudes. Estimates from these plates give the approximate limits, 7.5 to 9.0. Spectrum already known as fourth type.

1 10.2. This spectrum is of the same type as *C. DM.*$-47°6614$, described in Circular No. 76.

1 50.2. Assumed to be the following and southern of two faint and difficult objects, which also appears somewhat hazy. The spectrum consists of a bright band having wave-length of about 5000. Therefore this object has been assumed to be a gaseous nebula.

2 2.2. *020257*=H 1168. An examination of this star on six chart plates, taken between July 9, 1896, and September 1, 1903, shows a variation of about 2.5 magnitudes. Estimates from these plates give the approximate limits, 7.5 to 10.0.

3 46.7. Announced as Type IV, *Astron. Nach.*, **138**, 175. The spectrum is of the same type as *C. DM.*$-47°6614$, described in Circular No. 76. The broad, dark bands in these spectra have approximately the same wave-lengths as the bands in stars of the fourth type, Class Nd, but the intensity of the light of the spectrum between each band increases toward the violet, while in the peculiar stars the light between each band is of uniform intensity throughout.

4 57.4. Fine bright lines superposed on broad, dark bands.

7 22.8. This spectrum was marked "Peculiar" on Plate C 15655, by Mr. Edward S. King. The line $H\beta$, and three faint lines whose approximate wave-lengths are 4594, 4640, and 4727, are bright in this spectrum.

9 2.2. This object is very faint and difficult, but it appears to be of the same type as *C. DM.* $-47°6614$, described in Circular No. 76.

9 36.7. $H\beta$ narrow and bright superposed on broad, dark band.

12 34.4. 123459=H 1169. An examination of this star on ten chart plates, taken between April 11, 1890, and April 7, 1905, shows a variation of more than 2.3 magnitudes. Estimates from these plates give the approximate limits, 9.7 to < 12.

12 57.4. 125705=H 1170. Found from chart plates, while examining *RT Virginis*, which follows 11ˢ, 5'.2 north. Four plates, taken between April 7, 1890, and May 7, 1898, show a variation of more than a magnitude. Estimates from these plates give the approximate limits, 10.3 to 11.5.

14 46.8. *144646*=H 1171. This star follows S *Lupi* 0ˢ.4, south 12", and on most of the plates is a difficult object, especially when *S Lupi* is bright. It has been observed on 140 chart plates, taken between June 13, 1889, and September 4, 1901. Measures of these plates give the brightest and faintest magnitudes, 10.44 and 12.81.

16 21.2. This star is χ *Ophiuchi*. Attention was called to the peculiarity of this spectrum, "Bright line, and doubling of lines," on Plate C 15590, by Mr. Edward S. King. The presence of the bright line $H\beta$ in this spectrum was announced in the *Astron. Nach.*, **126**, 163. The photograph taken on March 14, 1905, shows $H\zeta$, $H\epsilon$, $H\delta$, $H\gamma$, and $H\beta$ as fine bright lines superposed on strong dark bands. As in other spectra of this class, the bright hydrogen lines diminish progressively in intensity, those of shortest wave-length being faintest.

17 53.2. This spectrum is of the same type as *C. DM.* $-47°6614$, described in Circular No. 76.

20 56.4. This star is P^1 *Cygni*. Attention was called to the peculiarity of this spectrum, "Bright lines" on Plate C 15177, taken on November 15, 1904, by Mr. Edward S. King. This plate shows $H\zeta$, $H\epsilon$, $H\delta$, $H\gamma$, and $H\beta$ as fine bright lines superposed on strong dark bands. These lines diminish in intensity corresponding to their relatively shorter wave-length. Other fine bright lines at approximate wave-lengths 3993, 4440, 4597, and 5009 are visible. Plate C 15203, taken on November 24, 1904, shows the bright $H\delta$, $H\gamma$, and $H\beta$. The two last named lines are on the edge of shorter wave-length of the dark lines $H\gamma$ and $H\delta$. This change seems to indicate a spectroscopic binary, one component having bright hydrogen lines.

21 28.7. This object is exceedingly faint.

21 51.7. An examination of this star on nine chart plates, taken between September 18, 1892, and October 27, 1904, shows a variation of more than a magnitude. Estimates from these plates gave the approximate limits, 8.1 to 9.4. The range from direct examination is certainly greater.

21 59.7 215122=H 1172. This spectrum is of the same type as *C. DM.* $-47°6614$, described in Circular No. 76.

22 46.7. *224625*=H 1173. An examination of this star on twenty-five chart plates, taken between July 23, 1889, and October 3, 1901, shows a variation of about 0.8 of a magnitude.

SPECTRA OF KNOWN VARIABLES

The spectra of a number of known variables have also been determined from these photographs and are given in Table II. The first column contains the designation, and the second, the name of the variable. The third column gives the class of spectrum.

TABLE II

SPECTRA OF KNOWN VARIABLES

Desig.	Name	Spectrum	Desig.	Name	Spectrum	Desig.	Name	Spectrum
004958	*W Cassiopeiae*	Mb 5 c	054974	*V Camelop.*	Md 6	093178	*Y Draconis*	Md 6
015912	*S Arietis*	Md 4	055353	*Z Aurigae*	Mb	093934	*R Leonis Min.*	Md 0
031401	*X Ceti*	Mb	060450	*X Aurigae*	Md 3	104814	*U Leonis*	Md 7
032043	*Y Persei*	Na	064030	*X Geminorum*	Md 5	122532	*T Canum Venat.*	Mc.
043065	*T Camelop.*	Pec.	071713	*V Geminorum*	Md 5	175510	*RY Herculis*	Md ?
043274	*X Camelop.*	Md 6	073508	*U Canis Min.*	Mb	195202	*RY Aquilae*	Md
045307	*R Orionis*	Md ?	083350	*— Ursae Maior.*	Md	203816	*S Delphini*	Mc.

REMARKS

004958. The spectrum of this star is given as N ? in the "Provisional Catalogue of Variable Stars," *Annals* **48,** No. III. Plate I 32618, taken on February 10, 1905, shows the spectrum of this star as Class Mb 5 c.

032043. The spectrum of this star is given as N ? in the "Provisional Catalogue of Variable Stars," *Annals*, **48,** No. III. Plate I 32279, taken on November 7, 1904, shows the spectrum as Na, or similar to *U Hydrae*.

043065. The spectrum of this star is given as N in the "Provisional Catalogue of Variable Stars," *Annals* **48,** No. III. Plate I 32380, taken on December 1, 1904, shows that this spectrum is Peculiar. The lines $H\beta$ and a wide band about 4670 are present and dark, and the faint spectrum extends to $H\epsilon$. On a poor plate this might readily be mistaken for a fourth-type spectrum. This image was carefully compared with a chart plate, I 14129, taken on December 3, 1895.

195202. The spectrum of this star is given as Md ? in the "Provisional Catalogue of Variable Stars," *Annals* **48,** No. III. Plate B 22023, taken on October 10, 1898, shows that the spectrum is Md, having the lines $H\epsilon$, $H\delta$, and $H\gamma$ bright, and resembling *R Hydrae*.

In many, if not all, of the variable stars of long period the bright hydrogen lines are not present when the star is faint. Accordingly, stars whose spectra are given as Mb, or Mc in Table II, may later be found to have spectra of Class Md.

EDWARD C. PICKERING.

MAY 5, 1905.

A PROBABLE NEW STAR, *RS OPHIUCHI*[1]

New stars can be distinguished from variables, in many cases, only by their spectra. The usual life of a new star is marked by its sudden appearance, where no star is previously known to have existed, and a gradual

[1] *Harvard College Observatory Circular* No. 99.

fading away during which it changes into a gaseous nebula. But the New Star of 1866, *T Coronae*, had already been recorded in the Bonn *Durchmusterung*, and is still visible as a star of the tenth magnitude. The New Star of 1901, *Nova Persei*, No. 2, was shown by earlier photographs to be a very faint variable, and the New Star of 1600, *P Cygni*, is still well seen as a star of the fifth magnitude, which does not now vary perceptibly. Moreover, η *Carinae* appears as a star of the seventh magnitude, after undergoing great and irregular changes in light during nearly a century.

From the Draper Memorial photographs it has been shown that the spectra of variables of long period are generally either of the third type, Class Md, in which one or more of the hydrogen lines, $H\delta$, $H\gamma$, and $H\beta$, but not $H\epsilon$, are bright, or of the fourth type, Class N. The spectra of the bright novæ are very complex, but when faint even at maximum only a few bright lines are visible. These consist of the hydrogen lines $H\epsilon$, $H\delta$, $H\gamma$, and $H\beta$, and one or more bright lines between λ4600 and 4700, which appear to coincide with the characteristic bands of spectra of the fifth type. *Nova Centauri*, however, had a wholly different spectrum.

In Circular No. 76, Mrs. Fleming pointed out that the spectrum of the star *174406, RS Ophiuchi*, on July 15, 1898, was of the third type in which the hydrogen lines $H\zeta$, $H\epsilon$, $H\delta$, $H\gamma$, and $H\beta$ were bright, and also two lines which appear to coincide with the bright bands 4656 and 4691, in γ *Velorum*. As these bands have a width of several units and are sometimes brighter on one edge than on the other, it is impossible to give their exact wave-lengths. This spectrum, therefore, closely resembles that of *Nova Sagittarii* and *Nova Geminorum*. A photograph taken on the preceding day, July 14, confirms the presence of these lines, while a photograph taken on August 28, 1894, showed that at that time the spectrum was of Class K, with no evidence of bright lines. Mrs. Fleming's record in 1899, after examining the first of these was "Nova?"

TABLE I

ANNUAL RESULTS

Year	No.	Mean	A. D.	Year	No.	Mean	A. D.
1888	1	10.86	..	1898	11	.....	.
1890	2	10.88	0.02	1899	14	10.56	0.26
1891	2	10.78	.02	1900	24	9.67	.32
1892	4	10.65	.11	1901	34	9.82	.23
1893	7	10.26	.10	1902	41	9.97	.18
1894	6	10.46	.28	1903	26	10.28	.20
1895	6	10.36	.07	1904	51	10.24	.18
1896	9	10.21	.18	1905	4	9.78	.26
1897	11	10.50	.23	....	..	.....	..

Miss Cannon, from an examination of the light-curve, called attention to the remarkable increase in the light of this star which took place in 1898. The star has been photographed at the Observatory each year since 1888, except in 1889. The year, number of photographs, mean magnitude, and average deviation of the separate results are given in Table I. The individual results in 1898 are given in Table II.

TABLE II

RESULTS FOR 1898

Date	Mag.	Date	Mag.
April 2	10.70	August 15	9.27
May 27	10.76	August 20	9.32
May 31	10.81	September 7	10.00
June 30	7.69	September 29	10.28
July 14	8.26	October 8	10.81
July 15	8.22		...

It will be seen from these tables that the star appears to have had the magnitude 10.9, before 1891, then increasing gradually about half a magnitude to 10.4, and retaining this brightness during 1893 to 1897. In 1898 it was at first faint, magnitude 10.8, until May 31. A month later, on June 30, it was more than three magnitudes brighter, or 7.7 and decreased regularly about a magnitude a month until October 8, when it was again magnitude 10.8. The following year, 1899, it remained faint, 10.6, but in 1900 it attained the magnitude 9.3 in April, diminishing to 10.0 in September. This change accounts for the large average deviation in the fourth column. Since then the variations have been slight. An examination of several good chart plates shows only one star in this position.

Nearly all of the objects mentioned above are shown on the Harvard Map of the Sky. *RS Ophiuchi* appears on Plate 31, [118, 58]; *T Coronae*, on Plate 18, [113, 59]; *Nova Persei*, No. 2, on Plate 12, [131, 185]; *P Cygni*, on Plate 20, [72, 149]; η *Carinae*, on Plate 50, [153, 64]; γ *Velorum*, on Plate 49, [155, 168]; and *Nova Geminorum*, on Plate 13, [39, 107].

Both the spectrum and the light-curves therefore indicate that this object should be regarded as a Nova, rather than a variable star, and its proper designation will be *Nova Ophiuchi*, No. 3, the new stars of 1604 and 1848 having also appeared in the same constellation.

EDWARD C. PICKERING.

MAY 15, 1905.

THE

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ON THE DENSITY OF *ALGOL* VARIABLES[1]

By J. H. JEANS

I. INTRODUCTION

It is a well-known fact that for a mass or system of masses of incompressible homogeneous fluid of density ρ, the value of ω^2/ρ depends solely on the shape and configuration of the masses, and not on the dimensions of the system. If we accept that explanation of the *Algol* variables which attributes the variation of the light to alternate eclipses of the two components of the system, then the knowledge of the value of ω, the angular velocity, which we deduce from the observed period of the light, leads to a certain inference as to the value of the density ρ, subject to the assumption that the matter may be treated as homogeneous and incompressible. It has been remarked that in almost every case which has been examined, this density is found to be surprisingly small. Generally, the calculated density is so small that the original hypothesis of an incompressible fluid is seen to be quite untenable; it is obvious that if the densities obtained are accurate, the system must be supposed to be in the gaseous state. This consideration tends to cast doubt over the whole investigation; for a gaseous mass is certainly neither incompressible nor homogeneous

[1] I have used the term "*Algol* variable" as a convenient designation for all variables showing regular variations of light, although the variables with which the paper is principally concerned are not of the strict *Algol* type.

and, as we shall see later, if the original assumption breaks down, the relation between ω and ρ does not hold, even approximately. Indeed, the calculated values of the mean density may be increased almost without limit.

Also, in some cases, it is found that the two components of the system must be supposed to be almost or quite in contact. We ought, therefore, to consider the enormous tidal forces which will be at work, so that the system of forces exerted by the masses on one another will comprise not only the direct gravitational attraction which may be supposed to act between their centers, but also forces arising from the tidal distortion, and also possibly repulsive forces produced by the pressures exerted through the neck of fluid which joins the two masses, or through the intersection of their atmospheres.

The effect of allowing for tidal forces would be to decrease the value of ρ; but if a neck is formed, the pressure transmitted by this neck would tend toward the increase of ρ. The object of the present paper is not so much to explain how to allow for these various corrections as to examine to what extent we are justified in drawing any deductions from observations on the variables.

II. THE CONFIGURATIONS OF EQUILIBRIUM OF ROTATING INCOMPRESSIBLE FLUID SYSTEMS

We may begin by recapitulating what is known as to the relation between ω and ρ for systems of incompressible fluid. With no rotation, the configuration of equilibrium is a sphere. If a spherical mass of liquid is set gently in rotation, it assumes a spheroidal shape, and if the angular momentum of the rotation is increased, the spheroidal form is maintained until the rotation is such that the angular velocity is given by

$$\omega^2/2\pi\rho=0.18712\ . \tag{1}$$

These figures are the spheroids of Maclaurin. After passing the momentum given by equation (1), the figure is no longer spheroidal, but ellipsoidal, and remains ellipsoidal until the momentum is such that

$$\omega^2/2\pi\rho=0.14200\ , \tag{2}$$

this momentum being greater than that given by (1), although the

angular velocity is less. These series of ellipsoidal figures are Jacobi's ellipsoids.[1]

When the angular velocity exceeds the critical value corresponding to equation (2), the configuration is no longer ellipsoidal, but is initially pear-shaped. This series of pear-shaped figures was first discovered by Poincaré,[2] and have been discussed by Poincaré and Darwin.[3] It is found that the angular velocity for these figures is less than that given by equation (2), although the angular momentum increases continuously. As the angular momentum further increases, beyond that corresponding to these pear-shaped curves, nothing is known as to the configurations which the mass of fluid will assume, beyond what may be inferred from an investigation of the corresponding problem in two dimensions.[4] From this and from various considerations of a general nature, it seems almost certain that the mass of fluid will ultimately split into two parts. The equilibrium of two such masses of fluid, assuming them to be nearly spherical, but allowing for their mutual tidal influence, has been discussed by Darwin[5] and Roche.[6] It appears that it is impossible for an infinitesimally small mass to rotate in contact with a primary; this is only possible if the mass of the satellite is at least equal to (about) one-thirtieth of that of the primary. Darwin shows that when the masses are rotating in contact, we have approximately

$$\omega^2/2\pi\rho=0.13 \ , \tag{3}$$

while in the case in which the masses are equal, the value is approximately,[7]

$$\omega^2/2\pi\rho=0.09 \ . \tag{4}$$

[1] G. H. Darwin, "On Jacobi's Figure of Equilibrium for a Rotating Mass of Fluid," *Proc. R. S.*, **41**, 39.

[2] "Sur l' équilibre d'une masse fluide," *Acta Math.*, **7**, 3 and 4, 1885.

[3] H. Poincaré, "Sur la stabilité de l'équilibre des figures pyriformes," *Phil. Trans.*, **198** A, 333. G. H. Darwin, "On the Pear-Shaped Figure of Equilibrium," *ibid.*, **198** A, 301, and "The Stability of the Pear-Shaped Figure," *ibid.*, **199** A, 253.

[4] J. H. Jeans, "On the Equilibrium of Rotating Liquid Cylinders," *Phil. Trans.*, **200** A, 67.

[5] "On Figures of Equilibrium of Rotating Masses of Liquid," *Phil. Trans.*, 1887, p. 379.

[6] *Montpellier Acad. Sci. Mémoires*, **1**, 243.

[7] I find this value by interpolation between Darwin's two cases in which the masses are almost touching and are just more than touching. *Phil. Trans.*, 1887, Plate 22.

All these investigations, it will be noticed, take full account of the tidal influences and of the pressure of the neck of liquid where the two bodies are merged into one. Assuming the matter of which our system is composed to be adequately represented by a homogeneous incompressible fluid, the process which we have been tracing is that of the birth of a double star. The process may be divided into two periods. The first is terminated when the point represented by equation (2) is reached. Through this period the star is spheroidal or ellipsoidal, and the emitted light would, from the symmetry of the star's figure, exhibit equal maxima and minima. After this point is passed, we come to a period in which the figure of the star possesses only two, instead of three, planes of symmetry. Here, if we suppose the surface of the star to be of uniform brightness, we should expect the star, except in very special cases, to exhibit two equal maxima and two unequal minima. Hence it will be during this second period that the star will first appear as a variable of the *Algol* type. Up to the moment of separation into two masses, the emitted light will constantly change throughout the whole period. After this separation, the amounts of light emitted will remain stationary at the maxima and minima for certain times, these times depending on the extent of the separation.

III. CALCULATION OF ρ

Let us now consider the former class of variables, representing a star throughout what we have called the second period, up to its separation into two parts. At the end of the period, ω lies between the values given by equations (3) and (4). Throughout the intermediate period it seems highly probable that ω decreases steadily. But the only definite and accurate knowledge in our possession is that contained in equations (2), (3), and (4). Fortunately, these equations give approximately the same relation between ω and ρ. As an average, we may take

$$\omega^2/2\pi\rho=0.12\ . \tag{5}$$

We have to admit that this equation may be subject to an error of about 25 per cent., but it represents the most accurate knowledge at our disposal. It enables us to calculate ρ to within about 25 per cent.

If we use any units other than astronomical, we must replace ρ by $\gamma\rho$, where γ is the gravitational constant, of which the value is 648×10^{-10} in C. G. S. units. We have also $P\omega = 2\pi$, where P is the period, so that equation (5) becomes

$$\rho = \frac{2\pi}{0.12 \times \gamma P^2} = \frac{8 \times 10^8}{P^2} \text{ (nearly)} \ . \tag{6}$$

As an example, we may take the case of the star β *Lyrae*, belonging to this class of variable, for which $P = 12.91$ days,[1] or $= 1.11 \times 10^6$ seconds. The value of ρ given by equation (6) is

$$\rho = 0.00065 \ , \tag{7}$$

agreeing closely with the value found by Professor Myers.

As a second example, let us take *V Puppis*,[2] another variable of this type, for which $P = 1^d \ 10^h \ 57^m \ 26^s = 125{,}846$ seconds. The value of ρ given by equation (6) is

$$\rho = 0.051$$

a value comparable with, although far from equal to, that given by Roberts ($\rho = 0.02$).

THE BIRTH OF A NEBULOUS DOUBLE STAR

The results just given show that the densities found by supposing the star to be liquid are much too small to belong to a liquid, so that our calculation is rendered nugatory. Further instances of densities too small to belong to liquid stars will be found in lists of calculated densities of double stars given by A. Roberts and H. N. Russell.[3] We accordingly turn to the consideration of a nebulous double star.

Here the birth of a double star is a process very different from that just considered.[4] Mathematically, the main distinction is that there is no longer any uniform relation between the density and the angular velocity of rotation, and, so far as can be seen, the process of separation may take place with an extremely small angular velocity. For the agency which effects the separation will no longer be rotation alone; gravitation also will tend toward separation. Indeed, a

[1] G. W. Myers, "The System of β *Lyrae*," *Astrophysical Journal*, **7**, 1, 1898.

[2] A. W. Roberts, "On the Orbits of the *Algol* Variables," *ibid.*, **13**, 177, 1901.

[3] *Astrophysical Journal*, **10**, pp. 314 and 317, 1899.

[4] J. H. Jeans, "The Stability of a Spherical Nebula," *Phil. Trans.*, **199** A, 1.

nebula can split into two parts under gravitation alone, the two nuclei being held apart by the pressure of the layer of gas which separates them, instead of by the so-called centrifugal force.

From numerical results obtained in the various papers of my own to which reference has already been made, I have been led to the conclusion that a gravitational instability of the kind described must be regarded as the primary agent at work in the actual evolution of the universe, Laplace's rotation playing only the secondary part of separating primary and satellite after the birth of the satellite.

Let us go back to the primitive nebula, out of which our universe may be supposed to have grown. We may, for convenience, regard this nebula as a gas of uniform density, at uniform temperature throughout, and extending to an indefinite distance in all directions.

A gas in this state is obviously in equilibrium, so long as we do not take account of the existence of its boundaries. The equilibrium may, however, be either stable or unstable, and a brief calculation shows that it is unstable.[1] The gas will tend to form into clusters about a number of nuclei.

To obtain the most probable disposition in space of these nuclei is a problem of statistical mechanics, to which I have not tried to obtain an exact solution. If, however, a solution had been obtained, we should be able to calculate, among other things, the most probable mean distance apart of adjacent nuclei. This mean distance can depend on only three quantities, namely:

1. The gravitation constant, γ.
2. The density of the original gas, ρ.
3. The pressure of the original gas, p.

The only way in which these quantities can be combined so as to form a length is through the expression

$$\sqrt{\frac{p}{\gamma\rho^2}}\;;$$

or, if we put $p=k\rho$,

$$\sqrt{\frac{k}{\gamma\rho}}\,. \tag{8}$$

Hence, by an argument from physical dimensions which I have

[1] J. H. Jeans, *Phil. Trans.*, **199** A, p. 49.

explained elsewhere,[1] it follows that the mean distance apart of the nuclei must be comparable with expression (8).

Denoting this quantity by a, the mass M of the gas which concentrates about each nucleus must be comparable with ρa^3, and therefore with

$$\sqrt{\frac{k^3}{\gamma^3\rho}} \cdot \tag{9}$$

Hence M/a must be comparable with k/γ.

To examine to what extent this result agrees with the formation of the universe as we know it, let us make use of Lord Kelvin's estimate as to star density, that there are 10^9 stars within a parallactic distance of $0''.001$ of the Sun.[2] Assuming these stars to be arranged in regular cubical piling, this gives $0''.62$ as the mean parallax of adjacent stars in terms of the earth's orbit, so that a is roughly $10^{18.7}$ cm. The value of γ is 648×10^{-10}. It is difficult to know what value to assume for k, but as a rough estimate we may take $k = 10^9$, which would be the exact value for a gas of molecular weight 30 at a temperature of 350° absolute, or for hydrogen at a temperature of 24° abs.

The value of ak/γ is found to be $10^{34.9}$, and this, on the gravitational hypothesis, ought to be comparable with the *average* mass of a solar system. The mass of our own system is $10^{33.3}$, which, considering the extreme vagueness of the data, must be regarded as in good agreement with the number $10^{34.9}$. The agreement is vastly improved on taking an average parallactic distance greater than $0''.62$; I do not know what evidence there is as to the number of dark stars between us and the nearest stars which are bright enough for us to see them. Statistics of binary stars suggest that only a small proportion of the stars in the universe are bright.

As the evolution of the universe progresses, the original nuclei doubtless separate further into distinct nuclei, so that in any case there is no ground for surprise in finding that the gravitational theory predicts masses greater than that of our present system. The confirmation of the gravitational hypothesis must not be expected to

[1] "On the Vibrations and Stability of a Gravitating Planet," *Phil. Trans.*, **201** A, 158.

[2] "On Ether and Gravitational Matter through Infinite Space," *Phil. Mag.* [6], **2**, p. 161.

lie in exactness of fulfilment of its predictions, so much as in the fact of its predicting enormous masses of quadrillions of quintillions of tons as the masses of the systems; the theory which relies on pure rotation cannot even predict the order of magnitude of masses to be expected.

The supposition that gravitational forces preponderate, not only in the formation of the original nuclei, but also in the evolution of a solar system out of each nucleus, leads to the prediction of a relation between the masses and densities of planets possessing satellites, provided that the planet was in a liquid or molten state when the satellite was thrown off.[1] By the method of dimensions, it is easily shown that the mass of the primary must be comparable with

$$\sqrt{\frac{\lambda^3}{\gamma^3\rho^4}} ,$$

where ρ is the density and λ the elasticity of the material of the primary. Assuming λ to have a constant value for all the planets of a solar system, we find that the mass ought to vary as ρ^{-2}, so that the product of density and radius ought to have the same value for all the planets of the system. This law is approximately obeyed in the solar system.[2] Moreover, taking $\rho=1$, $\lambda=10^{12}$ as rough values,[3] we find that the mass of a primary possessing a satellite ought to be comparable with $10^{28.8}$ grams. The mass of the earth is $10^{27.8}$, that of *Jupiter* is $10^{30.3}$.

Let us compare the predictions of the gravitational hypothesis with a similar prediction which can be deduced from the rotational hypothesis.

Consider a mass of gas, ultimately to form a solar system, consisting of N molecules each of mass m, moving with a veloctiy of mean square C, and occupying a cube of edge a in the primitive homogeneous nebula. The moment of momentum about an axis x through its center is

$$\mu_x=\Sigma m(vz-wy) ,$$

so that the square of this moment of momentum is

$$\mu_x^2=\Sigma m^2(v^2z^2-2vwyz+w^2y^2) ,$$

[1] "On the Vibrations and Stability of a Gravitating Planet," *Pail. Trans.*, **201** A, 157.

[2] "On the Vibrations and Stability," *loc. cit.*, p. 175.

[3] $\lambda=10^{12}$ is approximately the value for steel.

and the square of the total momentum is

$$\mu_x{}^2+\mu_y{}^2+\mu_z{}^2=\Sigma m^2[w'^2(y^2+z^2)+\ \ldots\ -2vwyz\ \ldots\ ]\ .$$

Assuming both the velocities and positions of the molecules to be distributed initially at random, we find on summing,

$$\begin{aligned}\mu_x{}^2+\mu_y{}^2+\mu_z{}^2&=\tfrac{2}{3}C^2\Sigma m^2(x^2+y^2+z^2)\\&=\tfrac{1}{6}C^2a^2Nm^2\ .\end{aligned}$$

The square of the moment of momentum remains constant throughout the contraction of the gas, being equal to $M^2\theta^4\omega^2$, where ω is the angular velocity, and θ the radius of gyration at any time. Thus

$$M^2\theta^4\omega^2=\tfrac{1}{6}C^2a^2Nm^2\ ;$$

or, since $Nm=M$,

$$M\theta^4\omega^2=\tfrac{1}{6}C^2a^2m\ .$$

Let θ and ω refer to the epoch at which a satellite is first formed, then as regards order of magnitude, we have on the rotational hypothesis, as already explained, $\omega^2=\gamma\rho=\gamma M/\theta^3$, so that

$$\gamma M^2\theta=\tfrac{1}{6}C^2a^2m\ .$$

Taking as rough values, $\gamma=65\times10^{-9}$, $a=10^{19.6}$, and noticing that $\frac{1}{3}C^2=k=10^9$, we obtain

$$\theta=10^{-34.5}\ .$$

Thus, on this hypothesis, the contraction of the nebula surrounding a nucleus would have to continue until the mass of, say, $10^{33.3}$ grams had contracted into a linear dimension of about $10^{-34.5}$ cm, and therefore to a density of about 10^{137}, before rotation alone could effect the birth of a satellite.

Two remarks must be made on the subject of this calculation. If we could assume that the original material of the universe was in the solid state before the evolutionary processes began, then the calculation would be less unfavorable to the rotational hypothesis. We should have to suppose the gas of our calculations replaced by a swarm of meteorites in the solid state, each meteorite figuring as a molecule of a quasi-gas. This conception of cosmic evolution has been put forward by Professor Norman Lockyer,[1] and has been to some extent developed by Professor G. H. Darwin,[2] but, in view of

[1] *Proc. R. S.*, **117**, 1887.

[2] "On the Mechanical Conditions of a Swarm of Meteorites," *Phil. Trans.*, 1889, p. 1

more recent knowledge and spectroscopic evidence, it can hardly be regarded as probable. The stellar systems which appear to be in the earliest stages of development mostly seem to be in a gaseous state.

Again, the amount of angular momentum predicted by the foregoing calculation is small compared with that possessed by our system. But as soon as we admit gravitational instability as a possibility, we must suppose variations of density to occur at an early stage of the evolutionary process, and the tidal influence of one nucleus on another will tend to increase the angular momentum of both. Indeed, by a well-known theorem of statistical mechanics, the tendency will be toward a continual increase of angular momentum until the energy of this momentum becomes equal to the average energy of proper motion of each nucleus, or, of course, until the condensation of the nuclei has proceeded to such an extent that their mutual tidal influence becomes negligible.

There is, therefore, I think, sufficient evidence, not only that gravitational instability may not be neglected, but that it is the principal agency to be considered in discussing questions of cosmic evolution. As regards the special question of the *Algol* variables, it becomes at once obvious that the smallness of the calculated density is merely a consequence of the neglect of the most important factors in the question. Knowing nothing but the period of rotation, we may not legitimately draw any inference as to the structure of the system.

TRINITY COLLEGE,
Cambridge, England,
May 10, 1905.

THE FIGURE OF THE SUN

By CHARLES LANE POOR

The following investigation of the figure of the Sun was suggested by the number of solar photographs taken by Lewis M. Rutherfurd in his private observatory and by him presented to the Observatory of Columbia University. In a series of investigations of the Rutherfurd star plates, Jacoby has shown that the plates have suffered no deterioration, and that they are capable of giving results comparable in accuracy with the best heliometer determinations. It was hoped, therefore, that the Rutherfurd photographs of the Sun would serve to determine with great precision the shape of that body.

After the investigation was well under way and some preliminary results were obtained, I decided to compare them with Auwers' reductions of the heliometer measures made in connection with transits of *Venus* in 1874 and 1882. This comparison led to some interesting results, which are given in the second section of this paper.

RUTHERFURD PLATES

The Observatory of Columbia University has in its possession a series of 139 solar photographs taken by Rutherfurd during the years 1860–74. This series of plates may be divided into two groups; one group of plates covering the years 1860 to 1866, and a second group taken during the years 1870–74. The plates of the first series were made with a small lens; those of the second group, with his 13-inch photographic objective, which was completed in 1868. The earlier plates were simple photographs of the Sun, without orientation marks or data of any kind. In 1870 he began to place orientation marks on the plates, but even after that date fully one-half of the plates lack this essential. In these four years Rutherfurd took one hundred plates, grouped as follows:

1870, February 16—October 14	- - -	61	plates
1871, April 17—August 19	- - -	14	"
1872, January 2—November 27	- - -	13	"
1874, April 5—December 9	- - -	12	"
Total	- - - - -	100	plates

Of the sixty-one plates taken in 1870, only four were available for measurement, the remaining plates not having sufficient data to orient them. These four plates were rejected in the preliminary investigation, but were afterwards measured and found to give satisfactory results.

Of the fourteen plates taken in 1871, eight were found to be measurable. A ninth plate was measured, but the measures were so discordant that it was rejected. The remaining five plates were either poorly developed, or did not have orientation marks upon them.

Of the thirteen plates taken in 1872, ten were found to be measurable. An eleventh plate was discarded after measurement, the separate measures being very discordant.

Of the fourteen plates taken in 1874, only one had on it the full data for orientation. This was a very poor plate, and was discarded after an attempt to measure it.

This left available for measurement a series of twenty-two plates, of which four were taken in 1870, eight in the spring and summer of 1871, and ten in the spring and summer of 1872. The measures were all made on the Repsold measuring-machine of Columbia University, and all measures were made in duplicate by Miss Harpham and Miss Davis, of the Observatory computing staff. On each plate twenty-eight points on the Sun's limb were measured—seven points at or near each pole, and seven points at or near each extremity of the equator. In each of these four groups the separate points were five degrees apart, each group thus covering an arc of 30° or 15° on each side of the pole or equator, respectively. The measurement of each point consisted in the determination of its polar co-ordinates, position angle and distance, as referred to the center of revolution of the plate in the machine. This center of revolution does not coincide with the center of the Sun's disk, but the plates can be quite accurately adjusted in the machine. In no case did the center of revolution differ from the true center of the disk by more than 0.05 mm, or $1''.2$.

The measures were corrected for differential refraction, the formulas as given by Chauvenet being used. From these corrected measures were then found the co-ordinates of the center of the Sun and the most probable value of the Sun's radius. The measured

co-ordinates were then transferred from the center of revolution to the center of the Sun's disk as origin, and thus were found, for each plate, the values of twenty-eight radii of the Sun.

The mean of the fourteen polar radii, as thus found, for each plate, was taken as the value of the polar radius, and the mean of the fourteen equatorial radii as the value of the equatorial radius for that plate. The difference between these values of the polar and equatorial radii was then formed, in the sense polar minus equatorial, and the results for the various plates are exhibited in the following tables:

1870

Date	P.-E. (arc)		Wt.
Aug. 18	+0."49	±0."28	2.4
Sept. 24........	−0.12	±0.31	2.0
Sept. 28........	+0.81	±0.25	3.0
Oct. 5........	+0.60	±0.26	2.9

1871

Date	Polar	Equatorial	P.−E. (arc)		Wt.
Apr. 21..............	39.0052	39.0314	−0."63	±0.36	1.8
June 16..............	38.6212	38.6286	−0.18	±0.22	5.6
July 16..............	38.6382	38.6665	−0.68	±0.19	6.7
July 20..............	38.5628	38.5906	−0.67	±0.35	1.6
July 21..............	38.6286	38.6586	−0.72	±0.30	2.4
July 22..............	38.5671	38.6086	−1.00	±0.30	2.4
Aug. 12..............	38.7458	38.7440	+0.04	±0.21	7.1
Aug. 19..............	38.7322	38.7172	+0.36	±0.20	5.0

1872

Date	Polar	Equatorial	P.−E. (arc)		Wt.
May 7..............	38.2086	38.1963	+0."30	±0.17	10.0
May 10..............	38.1743	38.1556	+0.45	±0.23	5.3
May 27..............	38.1213	38.1078	+0.32	±0.27	2.3
June 15..............	38.0063	37.9864	+0.48	±0.29	2.5
June 29..............	38.0341	38.0175	+0.40	±0.27	3.0
July 6..............	38.0121	37.9802	+0.77	±0.33	1.8
July 17..............	38.0575	38.0425	+0.36	±0.22	5.9
Aug. 10..............	38.1160	38.1249	−0.21	±0.14	14.3
Aug. 12..............	38.2410	38.2275	+0.32	±0.33	2.0
Sept. 21..............	38.5375	38.5132	+0.40	±0.30	2.5

In the above tables the first column gives the date on which the plate was taken; the second and third columns give the respective values of the polar and equatorial radii in scale divisions; the fourth column, the difference between the polar and equatorial radii in arc, together with the mean error of this result as determined from the separate measures; the fifth and last column gives the relative weights of the different plates as determined from their mean errors. The scale value differed for each plate, and was determined by assuming that the value of the mean radius of the Sun at distance unity is equal to 961″. Approximately one division of the scale is equal to 24.″6 of arc.

The different plates in each year give quite consistent results, but the mean results for the different years differ radically. The plates in 1871 show the equatorial radius to exceed the polar by some 0.″3; while the plates of 1870 and 1872, on the other hand, show the polar radius to be the greater by some 0.″2. Forming the mean by weights of the results obtained from the plates in the different years, we see that the yearly means are as follows:

1870, Sept. 22	$+0.''50 \pm 0.''10$
1871, July 19	-0.32 ± 0.16
1872, July 2	$+0.22 \pm 0.09$

These measures thus seem to indicate a change in the relative sizes of the polar and equatorial radii of the Sun. During the interval 1871–72 the polar radius was increasing relatively to the equatorial, and by 1872 was decidedly the greater. These changes in the shape of the Sun are apparently real changes, and can hardly be accounted for in any other way. The plates were all taken with the same instrument and under the same conditions, and in corresponding seasons of the year. They were nearly all taken in the morning hours, and at approximately the same distance from the meridian. So far as can be determined from the data at hand, there is no instrumental explanation for the difference between the results in the different years.

The conclusion from this investigation is that during this period, 1870–72, there was a real change in the shape of the Sun; the equatorial diameter first increasing and then shrinking relatively to the polar diameter.

HELIOMETER MEASURES

While adjusting and determining the constants of the heliometers which were used in observing the transits of *Venus* in 1874 and 1882, the German observers made a great number of determinations of the Sun's diameter. In all some 2,692 separate measures of the diameter were made by twenty-three observers. Five heliometers were used, measures with the same instrument being made in various stations by the same observer, and in the same station by various observers. Thus Heliometer A was used by Adolph, Wittstein, Valentiner, Ambronne, Peters, Kobold, Deichmuller, Hartwig, Küstner, and Weinek in Strassburg; by Adolph and Valentiner in Tschifu; and by Franz and Kobold in Aiken.

This immense mass of data was most thoroughly discussed by Dr. Auwers in *Die Venus-Durchgänge, 1874 und 1882.* He reached the conclusion that the diameter of the Sun at distance unity is

$$1919''.26$$

and that the polar diameter exceeds the equatorial diameter by the amount

$$\text{P.} - \text{E.} = +0''.038 \pm 0''.023.$$

This apparent anomaly in the shape of the Sun was explained by Dr. Auwers as being due to the tendency on the part of an observer to measure vertical diameters greater than horizontal diameters. And this is quoted by Newcomb as conclusive evidence that the Sun is sensibly a sphere, and that there can be no non-symmetrical distribution of matter in the Sun sufficient to explain the anomalies in the motion of Mercury.

In forming his means, from which the above result was obtained, Auwers kept together all observations made with a single instrument, and thus observations of different years were grouped together. As arranged by Auwers, these observations do not afford any indication of a change of the relative diameters with the time. In order to investigate this point, I rearranged the series of observations, as given by Auwers, arranging them in order of the time without regard to the observer or the instrument. When thus arranged, the observations fall into two series: one extending from September 1873 to January 1875; the other, from May 1880 to June 1883. There is an isolated

observation in July 1877, and another isolated one in March 1884. These do not fall into either series.

There is an uncertainty of some days in assigning a date to each determination of the ratio of the solar diameters; for the value of the difference between the polar and equatorial diameters (P.—E.), as given by Auwers for each observer, is found by him as the mean result of a number of observations, extending in many instances over a period of a month or more. In very few cases did an observer measure the polar and equatorial diameters on the same day, nor is the number of polar and equatorial diameters the same in any series. In reducing the observations of any one observer, Auwers took the mean of all diameters measured within 15° of the poles, and called such mean the polar diameter. He similarly took the mean of all diameters measured within 15° of the equator, and called such mean the equatorial diameter. The mean dates to which these mean diameters belong are not given by Auwers. For example, the observations made by Adolph in Strassburg were all made on fifteen days between September 2 and September 25, 1873. In this series Adolph made in all some fifty-seven determinations of the Sun's diameter; of which fifty-seven measures ten fall within the 15° limit of the pole, and nine within the corresponding limit of the equator. The polar measures were made on September 8, 14, 18, and 21; the equatorial measures, on September 18, 20, 21, 23, and 25. The remaining thirty-eight observations of this series were not utilized by Auwers in this investigation.

As a result of these nineteen measures by Adolph, Auwers finds the value for the ratio between the polar and equatorial diameters, $-0''.16$, in the sense Polar minus Equatorial; and this value, I have assumed, is the value for September 18, the mean date of the observations. Such an assumption is, of course, more or less approximate, but it gives a date sufficiently close for the purpose in hand.

SERIES OF 1873–75

In the first series of observations, extending from 1873 to 1875, there are in all thirteen such sets of observations. These are tabulated below, being arranged according to the mean dates of the observations; the weights being those assigned by Auwers.

TABLE I

Date	Observer	P.−E.	Weight
1873, September 18............	Adolph	−0″.16	0.5
September 20............	Borgen	+0.03	0.5
October 20............	Valentiner	−0.21	0.5
December 15............	Wittstein	+0.05	1.0
1874, February 4............	Weinek	+0.15	0.8
March 18............	Schur	+0.09	0.2
April 3............	Adolph	−0.32	0.2
May 15............	Schur	+0.15	1.0
December 26............	Adolph	+0.16	1.7
December 28............	Borgen	+0.21	0.1
December 29............	Valentiner	+0.23	0.2
1875, January 5............	Schur	+0.44	0.4
January 5............	Seeliger	+0.13	0.2

While the separate determinations vary, a simple inspection of the above table shows that there was a progressive change in the difference between the polar and equatorial diameters. In the earlier measures the equatorial diameter was slightly the greater; in the later measures the polar diameter was decidedly the larger. This is shown not only by the average result, but by the measures of each observer. Adolph, Borgen, Valentiner, and Schur made observations in the fall of 1873 and the spring of 1874; again, these same observers made other series of observations in the latter part of 1874 and in January 1875. In the case of each of these four observers, the difference, P.—E., is greater in the latter series. These results are shown in the following table:

Observer	1873–74	1874–75
Adolph................	−0″.20	+0″.16
Borgen................	+0.03	+0.21
Valentiner.............	−0.21	+0.23
Schur.................	+0.14	+0.44

Again, divide the entire series of observations into three groups, placing in the first group all the observations made in 1873, in the second group those made in the spring of 1874, and in the third group those made in December 1874 and in January 1875. Give to each observer the weight assigned by Auwers, and form the weighted mean of each of the three groups. The observations then fall into the following order:

Mean Date	P.−E.	Weight
1873, October.	−0.″01	2.5
1874, March.	+0.10	2.2
1874, December.	+0.21	2.6

And these means show the same progressive change as do the observations of the separate observers.

Thus these heliometer measures point toward a real change in the relative sizes of the polar and equatorial diameters of the Sun. It can hardly be doubted that this change is real, for it is shown by the observations of the individual observers, as well as by the observations of the different observers when grouped according to the time. Furthermore, this change is in the same direction as that indicated by the Rutherfurd plates made during the years 1871–72. The two series of measures, heliometer and photographic, thus supplement and confirm one another.

TABLE II

Date	Observer	P.−E.	Weight
1880, May 9.	Ambronn	+0.″19	1.1
July 15.	Ambronn	+0.08	4.0
1881, September 30.	Franz	−0.51	0.3
November 10.	Schur	+0.22	2.8
1882, March 14.	Kobold	+0.28	0.5
March 25.	Peter	+0.06	5.0
March 28.	Müller	+0.37	0.5
April 4.	Kobold	−0.05	2.8
April 6.	Marcuse	+0.07	0.7
April 15.	Küstner	.00	0.4
April 15.	Kempf	+0.09	1.7
May 3.	Deichmüller	−0.04	4.0
May 15.	Hartwig	+0.09	6.2
May 15.	Schur	+0.13	4.0
May 22.	Franz	+0.06	0.5
June 4.	Wislicenus	+0.02	4.0
July 2.	Bauschinger	−0.05	1.7
November 20.	Franz	−0.06	1.0
November 25.	Küstner	+0.13	1.3
November 25.	Kobold	−0.01	1.7
November 30.	Deichmüller	−0.30	0.1
November 30.	Müller	−0.34	0.2
November 30.	Auwers	−0.01	1.7
December 2.	Wislicenus	−0.28	0.3
December 4.	Peter	+0.15	0.5
December 5.	Kempf	+0.32	0.6
December 5.	Hartwig	+0.30	0.7
1883 May 15.	Wislicenus	−0.21	6.2
June 4.	Hartwig	−0.02	2.8

SERIES OF 1880–83

In this series there are in all twenty-nine sets of observations, of which number, however, twenty-three were made in 1882. These are tabulated in Table II, being arranged according to the date of observation in a manner entirely similar to the former table for 1873–74.

An inspection of these results will again show a change in the difference between the polar and equatorial diameters. This change is not at once apparent, for the relative weights of the separate determinations in this series differ greatly, the largest being 6.2, the smallest 0.1. Some of the determinations of small weight differ considerably from adjacent and better observations, and these poor values tend to conceal the progressive change in the ratio between the diameters. This change, however, is clearly brought out when the observations are divided into groups and the weighted means of each group formed. When this is done, we find that the observations arrange themselves as in the following table:

Date	P.–E.	Weight
1880, June	+0.″10	5.1
1881, October	+0.15	3.1
1882, March	+0.06	11.6
June	+0.05	20.4
November	+0.05	8.1
1883, May	–0.15	9.0

We thus see that during the interval from 1881 to 1883 there is a progressive change; the equatorial diameter apparently growing longer in relation to the polar diameter. While the division of the observations of the year 1882 into three groups is more or less arbitrary, yet, no matter how these observations had been grouped, the progressive character of the change would have been apparent. The mean of all the determinations for the year 1882 is +0.″05 with a weight of 40.1.

The change thus found for the period 1880–83 is the reverse of that found for the former period 1874–75, at which time the equatorial diameter was found to be growing shorter relatively to the polar diameter.

NORTHFIELD PLATES

Under the direction of Professor W. W. Payne, Dr. H. C. Wilson has taken a long series of solar photographs at Northfield, Minn. Only a few of these photographs are available for measurement; most of the plates lack satisfactory orientation, and in many the edge of the image is blurred, owing to a defective shutter. Dr. Wilson selected and sent to Columbia University for measurement five plates, which were taken during the years 1893 and 1894, all of which were well oriented and had on them the necessary data for measurement and reduction.

These five plates were measured in the same manner as were the Rutherfurd plates, with the following results:

1893

Date	P.-E. (arc)		Wt.
Sept. 8..........	−1″.10	±0″.24	2.8
Sept. 9..........	−0.94	±0.21	3.7
Sept. 11.... ...	−0.72	±0.18	5.9

1894

Date	P.-E. (arc)		Wt.
July 10..........	−0″.72	±0″.24	2.8
July 17..........	+0.36	±0.23	3.1

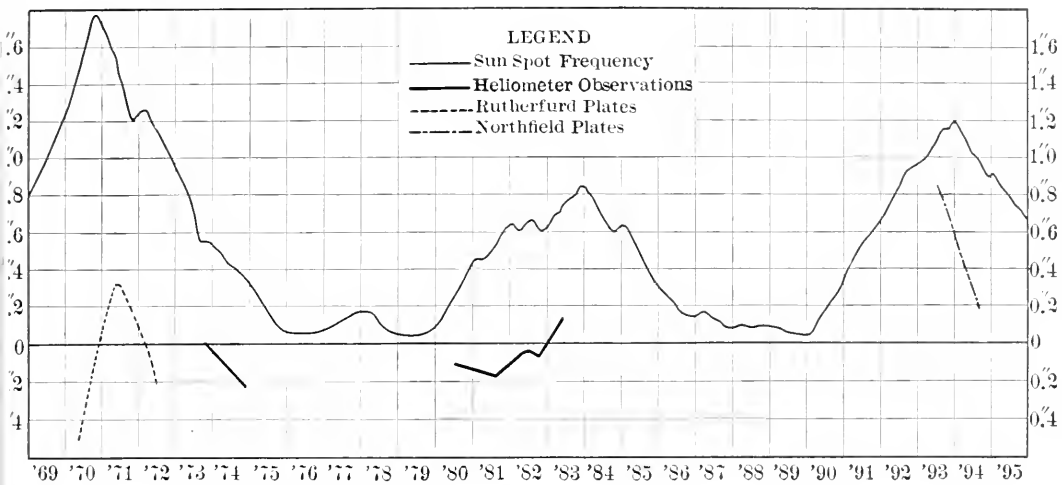
Forming the means by weight we find for the yearly means:

1893, September 10	- - - - -	−0″.87±0″.10
1894, July 13	- - - - -	−0.20±0.23

These measures again indicate a change in the ratio between the polar and equatorial radii. The plates are too few in number to give any conclusive result, but they seem to point toward a shrinkage of the equatorial with respect to the polar diameter. By themselves these plates would have but little weight in forming any conclusion, but taken in conjunction with the Rutherfurd plates and the heliometer measures, they confirm the general result, that the shape of the sun is variable.

RELATION OF THE SUN'S FIGURE TO THE NUMBER OF SUN-SPOTS

A sun-spot maximum occurred in the latter part of 1870, and from this time until 1876 there was a steady diminution in the number of spots. In 1870 and 1871, just previous to the maximum, the Rutherfurd plates show that the equatorial diameter was increasing, but during the period 1871 to 1876 the Rutherfurd photographs and the heliometer measures both show that the equatorial diameter was shrinking relatively to the polar diameter. The period from 1880 to 1883 was a period when the number of sun-spots was increasing, the



RELATION OF THE SUN'S FIGURE TO THE NUMBER OF SUN-SPOTS

sun-spot maximum occurring in the latter part of 1883. During this period the heliometer measures indicate that the equatorial diameter was increasing relatively to the polar diameter. But the 1883 sun-spot maximum was hardly half as high as the maximum in 1870, and we should expect, therefore, to find the changes in the Sun's diameter less marked in later period than in that of 1870. This is exactly what the determinations show. A third sun-spot maximum occurred in the latter part of 1893, and during 1894 the number of spots rapidly decreased. The Northfield plates show that during this period the equatorial radius was decreasing relatively to the polar radius.

This relation between the frequency of sun-spots and the figure of the Sun is shown in the accompanying diagram. The curve of

sun-spot frequency is taken from Miss Clerke's *Problems in Astrophysics*. The dotted curve represents the relation between the equatorial and polar radii of the Sun, as deduced from the Rutherfurd plates; the full heavy curves, the changes as exhibited by the heliometer measures, and the broken line, the changes as indicated by the Northfield plates. These curves are plotted from the weighted means of the observations, as given in the above tables, but with the signs reversed, so that the portions of the curves above the zero line represent those observations which show the equatorial diameter to exceed the polar. The slopes of the observational curves are nearly parallel to the corresponding portions of the sun-spot curve, and the general character of these curves shows a striking resemblance to the curve of sun-spot frequency.

The present investigation would seem to show, therefore, that the ratio between the polar and equatorial radii of the Sun is variable, and that the period of this variability is the same as the sun-spot period. The Sun appears to be a vibrating body whose equatorial diameter, on the average, slightly exceeds the polar diameter. At times, however, the polar diameter becomes equal to and even greater than the equatorial—the Sun thus passing from an oblate to a prolate spheroid.

In this variable figure of the Sun may lie the explanation of the anomalies in the motions of *Mercury*, *Venus*, and *Mars*.

Columbia University Observatory,
New York City,
June, 1905.

THE ORBIT OF THE SPECTROSCOPIC BINARY ζ *TAURI*

By WALTER S. ADAMS

The star ζ *Tauri* was included in a list of stars with variable radial velocities published by Professor Frost and the writer in 1903.[1] Attention was called at that time to the peculiar character of its spectrum, and because of the interest attaching to it for this reason, as well as on account of its comparatively long period, it was observed by us with considerable regularity. Previous to my leaving the Yerkes Observatory, twenty-two plates had been obtained, and I am indebted to Professor Frost for three others, secured since that time, which he has kindly placed at my disposal. These have proved of great value in the determination of the star's period.

The general features of the spectrum in the region covered by the plates may be described briefly as follows: A strong, well-defined line of considerable breadth at $H\gamma$; very weak and diffuse helium lines at λ 4388 and λ 4472; a similar, though slightly stronger line due to magnesium at λ 4481; and, finally, a number of faint broad lines identical for the most part with the enhanced lines of iron and titanium.

The question at once arose, in connection with the measurement of the plates, whether increased accuracy would be attained by the use of any lines in addition to $H\gamma$, and a few measures were made on the basis of assigning weights to the various lines at the time of measurement. On account of the extremely high relative value of $H\gamma$, however, this plan did not prove satisfactory, and it seemed best finally to base the determinations upon that line alone. As the width of $H\gamma$ is considerable, amounting in the average to about 0.75 tenth-meters, which would correspond to 52 kilometers in velocity, duplicate measures have been made throughout with a view to reducing the accidental errors of setting upon the line. With one exception, these have agreed reasonably well for all of the plates. This was in the case of the first plate of the series, B 219; the first measurement gave a value of +17.6 km; the second, a value of +5.9 km. It was evident that a totally different estimate of the position of the

[1] *Astrophysical Journal*, **17**, 150–153, 1903.

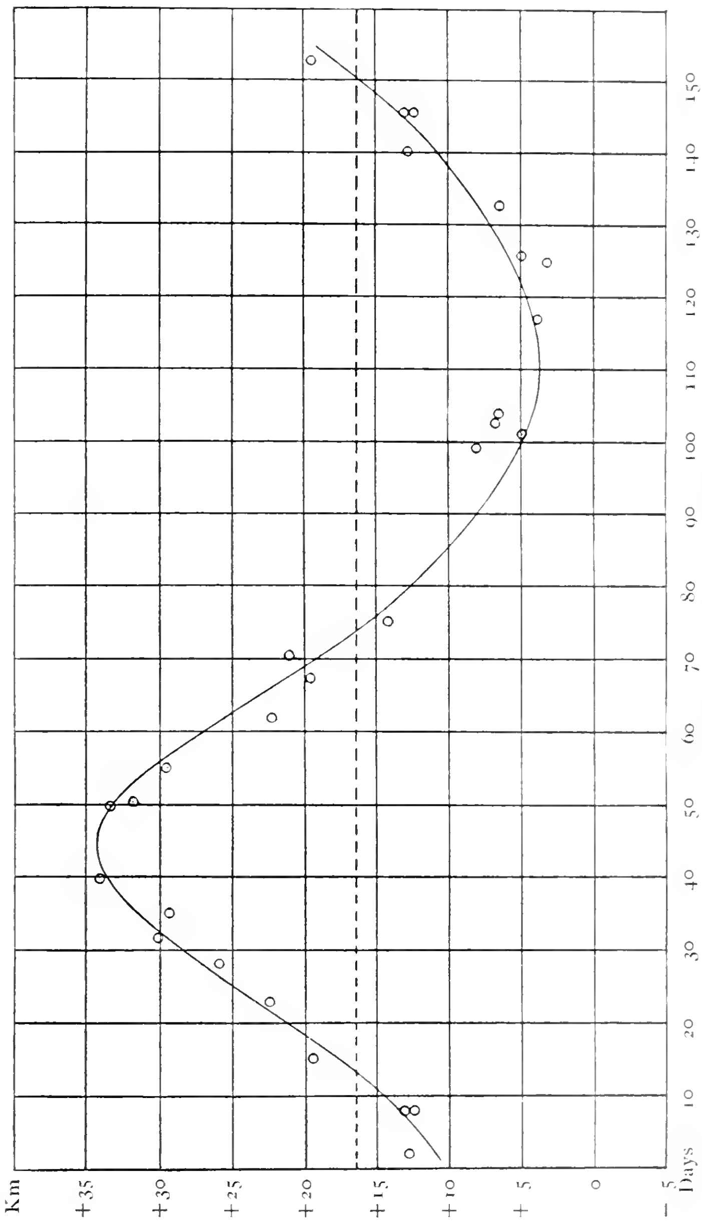
line had been made in the two cases, and, as neither the comparison nor the stellar spectrum upon this plate is satisfactory, it has been omitted in the discussion.

The table which follows gives a summary of the material used in the determination of the orbit, including the values given by the separate measurements as well as their mean. As in previous communications, the series letters A and B refer to the cameras employed.

Plate Number	Date, G. M. T.	First Measure	Second Measure	Mean	O.-C.
		km	km	km	km
A 317	1902, February 12.7	+23.0	+19.2	+21.1	+2.2
B 332	April 23.6	14.7	11.1	12.8	+1.8
B 410	September 13.8	13.2	13.3	13.2	−0.4
B 425	October 15.8	33.8	34.1	34.0	+0.7
B 440	October 30.8	31.2	28.0	29.6	−1.0
B 452	November 6.9	19.4	25.2	22.3	−3.1
B 462	November 19.9	13.5	14.8	14.2	−1.4
B 470	December 17.8	6.5	7.1	6.8	+2.2
B 473	December 18.8	6.7	5.9	6.3	+1.8
B 476	December 31.8	4.0	3.8	3.9	−0.5
B 482	1903, January 8.8	2.3	4.1	3.2	−2.4
B 485	January 9.7	4.5	5.4	4.9	−0.9
A 386	January 16.8	4.4	8.7	6.5	−1.3
B 489	January 29.8	14.3	10.4	12.4	−1.2
A 397	February 5.8	18.7	20.3	19.5	+1.7
A 403	February 13.7	22.5	22.2	22.4	−0.8
A 407	February 18.7	26.3	25.5	25.9	−0.9
A 412	February 26.5	28.9	29.7	29.3	−2.3
A 420	March 13.7	31.4	31.9	31.7	−1.0
A 447	April 30.6	7.9	8.5	8.2	+2.9
B 533	December 13.6	34.1	32.2	33.2	+0.1
A 494	1905, January 13.7	30.6	29.8	30.2	+0.8
B 571	February 18.6	19.7	19.5	19.6	−1.4
B 582	March 24.6	5.0	4.9	4.9	+0.1

Plate B 452 is underexposed, and the considerable difference between the values given by the two measurements is probably due to this fact.

A brief inspection of these results showed the period to lie in the vicinity of four and one-half months, and a few trials gave a value of 138 days as most consistent with the observations. The latter were then plotted with this period, and a smooth curve drawn through them. The departure of this from a sine curve made it evident that the eccentricity must be considerable, and accordingly the graphical method of Lehmann-Filhés was adopted for the computation of the orbit.



VELOCITY-CURVE OF *ζ TAURI*

With the use of a planimeter to adjust the areas, the following quantities were obtained as a basis from which to derive the elements:

$$\text{Velocity of system } V = +16.4 \text{ km}$$
$$A = 17.6 \text{ km}; \; B = 12.3 \text{ km}; \; z_1 = +4.23; \; z_2 = -4.50 \; .$$

The notation is that of Lehmann-Filhés.

These quantities gave the following elements:

$$u_1 = 100^\circ \; 13'$$
$$\omega = 9^\circ \; 45'$$
$$e = 0.180$$
$$T = 1902, \text{ January } 19.9$$
$$a \sin i = 27{,}900{,}000 \text{ km}$$
$$\text{Period } U = 138 \text{ days}$$
$$\mu = 2^\circ\!.609$$

The time of periastron passage T nearly coincides with the time of maximum radial velocity, which shows that the major axis of the orbit must be nearly perpendicular to the line of sight.

The velocities for the dates of observation were computed from these elements, and the differences between the observed and the computed values are given in the column O.-C. of the table above. The largest residual is -3.1 km, a result which is quite satisfactory in view of the fact that the determinations are based upon only one line. The quantity of material at present available for discussion does not seem to the writer sufficient to make a least squares solution of value.

The accompanying plate shows the velocity-curve derived from the set of elements, and the positions of the observed velocities in reference to it. The broken line drawn parallel to the axis of abscissas is at a distance of 16.4 above it, which is the velocity in kilometers of the center of gravity of the system.

No trace of the spectrum of the second component has been found upon any of the plates, the evidence afforded by the remarkably well-defined character of $H\gamma$ being especially conclusive.

SOLAR OBSERVATORY,
Mount Wilson,
June 1, 1905

ON THE MAGNESIUM SPARK

By W. W. STRONG

In 1902[1] Dr. Mohler used the shift of the spark lines to measure the velocity of the particles driven off from the electrodes. He found a probable velocity of about 400 meters per second for certain iron, aluminum, magnesium, and cadmium lines. While getting this Doppler effect, it was found that the character of the magnesium lines when photographed "end-on" was different from that of the same lines when the spark-gap was revolved 180°. Using magnesium and iron electrodes, Dr. Mohler found that when the spark-gap was "end-on" and the magnesium electrode was next the slit, the lines λλ 2795, 2802, and 2852 would be strongly reversed, like λ 2852 in the arc. If, however, the iron electrode was nearest the slit, these same lines would be without reversal. The purpose of this investigation was to find the cause of this reversal.

The conditions under which the work was done were the same as those under which Dr. Mohler worked.[1] A concave 4-inch Rowland grating, ruled 14,400 lines per inch, was used for photographing the spectra. A 110-volt current with a Wehnelt break was used in the primary circuit. The Rühmkorff coil used was capable of giving an 18 cm spark. The capacity used in the secondary circuit was usually a small Leyden jar of 0.0025 microfarads capacity, and the maximum length of spark was about 1.5 cm, this length varying with the size of the platinum electrode exposed in the solution of the Wehnelt break. The usual length of spark-gap was from 3 to 8 mm. A resistance box was used in the primary circuit, and the resistance was changed to suit the break. The work was done in the second and third order of spectra, and all photographing was done in a basement where the temperature was very constant and the spectroscope was free from jars.

It was found that when magnesium electrodes were used, the "principal series" lines λλ 2802 and 2795, with the line λ 2852, would be widely reversed for the two "end-on" positions, for the vertical

[1] *Astrophysical Journal*, **15**, 125, 1902.

and transverse positions. The "first subordinate series" lines λλ 2798 and 2791, were usually reversed also, but the width of their reversals was much smaller. The line λ 2779, the middle line of the quintuple group, was sometimes reversed. The "second subordinate series" lines, λλ 2936 and 2928, and the line λ 4481, were never reversed. When there was a heavy current in the primary circuit, reversal did not always occur. Spectrograms of the edges of a vertical spark-gap showed the lines λλ 4481, 2936, and 2928 somewhat weakened.

If either iron, copper, or zinc were substituted for one of the magnesium electrodes, and the magnesium electrode was away from the slit of the spectroscope, the lines λλ 2852, 2802, 2798, 2795, and 2779 would be without reversal. When the spark-gap was revolved 180° and the magnesium electrode was next the slit, these lines were reversed just as they were when both electrodes were of magnesium. As a rule, iron was substituted for one of the magnesium electrodes on account of the abundance of iron lines in this region. When such a spark-gap was vertical, the width of the reversed parts of the above magnesium lines was less than for the "end-on" position. For the "across" position ("end-on" position revolved 90°), sections made near the magnesium electrode showed reversal; whereas sections made near the iron electrode gave less or no reversal. The following table gives the results as found for the "end-on" positions:

Wave-length of magnesium lines	4481	2936	2928	2852	2802	2798	2705	2791	2779
Width of reversal, *Mg* next the slit	0	0	0	0.2	0.24	0.15	0.27	0.15	0.1
Width of lines, *Mg* next the slit	1.5	0.6	0.5	0.6	0.00	0.60	1.00	0.60	0.3
Width of reversal, iron next the slit	0	0	0	0	0	0	0	0	0
Width of lines, iron next the slit	1.3	0.6	0.5	0.3	0.45	0.40	0.45	0.43	0.3

It will be seen from the above table that the lines λλ 2852, 2802, and 2795, the ones that are most reversed when the magnesium electrode is next the slit, decrease very considerably in width when the iron electrode is next the slit, while the other lines remain of the same width, especially the lines λλ 2936 and 2928. It should be remembered that the above figures are only rough approximations.

The above results would seem to indicate that there is a kind of an enveloping "reversing" layer of magnesium particles around the spark, and that this layer or sheath does not extend across the spark-

gap as far as the inner portion of magnesium particles, or at least that it is much thinner near the iron electrode. That the spark should be differentiated into different layers does not seem remarkable, since the arc itself, as shown by Lockyer,[1] Baldwin,[2] and Foley,[3] is composed of different layers which emit different spectra. The lines that showed greatest reversal, $\lambda\lambda$ 2852, 2802, and 2795, are also reversed in the arc. Henry Crew[4] finds that the width of the reversal of these lines and λ 2779 is "greatly increased" in an atmosphere of hydrogen. A. S. King[5] finds that self-induction very much reduces the lines $\lambda\lambda$ 2791, 2795, 2798, and 2802 in the spark, while λ 2852 is made blacker and the reversal is narrowed. All this would seem to indicate that the phenomenon is due to absorption by the relatively cool vapors of magnesium, and that the lines $\lambda\lambda$ 4481, 2936, and 2928, are not thus subject to absorption. For if one accepts the view that hydrogen reduces the temperature of the arc, one would expect a greater amount of absorption. Also, in a spark with capacity in the circuit, the discharge is oscillatory, and the series of sparks passing back and forth in quick succession tend to change the spark into an arc. Now, self-induction[6] "increases the time the oscillations persist, and so enables the vapor of the metal to get well diffused through the spark-gap." One would then expect the lines to be much less broadened, and hence not reversed. To prove this an absorption effect, a vertical spark-gap (*Mg* 1, *Mg* 2, Fig. 1) of magnesium electrodes, was placed between the slit and an "end-on" spark-gap (*Fe*, *Mg* 3) of iron and magnesium, with the iron electrode nearer the slit. Now the unreversed lines from the iron-magnesium spark-gap should be reversed by the double magnesium spark-gap. Reversal was found to occur.

During the work, efforts were made to get a Doppler effect. The quintuple group showed a slight shift which was hardly within the limits of measurement. Two "end-on" spark-gaps of magnesium electrodes, the sparks being made to go in opposite directions, were photographed together. Such a spectrogram should show twice

[1] *Proc. R. S.*, **28**, 425, 1879.

[2] *Physical Review*, **3**, 370, 448, 1896.

[3] *Ibid.*, **5**, 129, 1897.

[4] *Astrophysical Journal*, **12**, 167, 1900.

[5] *Ibid.*, **19**, 225, 1904.

[6] J. J. Thomson, *Conduction of Electricity through Gases*, p. 396.

the Doppler effect on one exposure, but the lines were so diffuse that measurement was impossible. It would be very interesting to find whether the reversed lines would show the same shift as the other unreversed lines. It may be that the different values found for different lines of the same element is due to the fact that these lines are produced by different parts of the spark.

To try further the reversal effect, the spark was made to pass through a fine hole. For holes less than 0.5 mm in diameter the

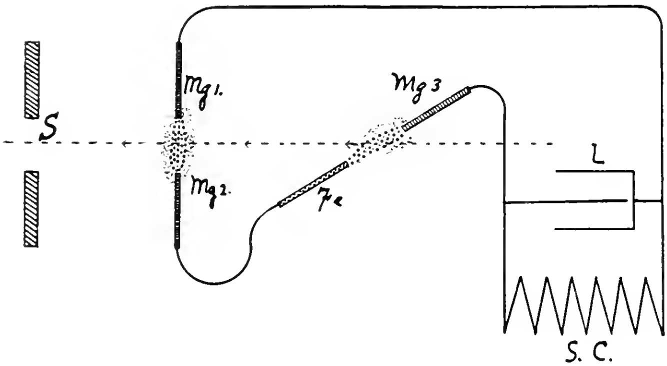


FIG. 1

"reversing" layer was entirely cut off. Spectrograms taken of the spark between the hole and the iron electrode never gave any reversal. For these small holes very long exposures were necessary, as very little magnesium came through the hole. All lines are of course faint, but the lines λλ 2928 and 2936 do not appear at all through a hole 0.16 mm in diameter; while λ 4481 is very faint, it usually occurring as the strongest line of all. Only the arc lines remain, and their relative intensity is the same as in the arc. Further work on this point is intended.

In conclusion, the writer wishes to thank Dr. Mohler for the suggestions and aid which he has given, and also for his constant kindness and courtesy.

DICKINSON COLLEGE,
Carlisle, Pa.,
May 1, 1905.

DIFFRACTION GRATING REPLICAS

By ROBERT JAMES WALLACE

In the early part of 1901 the writer entered upon a series of experiments having for their object the duplication of the plane grating, with the idea of producing a method which would yield definitely reliable results under conditions which might be easily satisfied. As these experiments have continued at intervals since then up to the present time, and have resulted in the manufacture of replica gratings of high grade, which are being widely used, it seems advantageous that a description of the method employed in their manufacture should be placed on record, for the guidance of those interested.

The publication of these details has been purposely delayed in order that sufficient time should elapse to preclude the possibility of deterioration; and also that opportunity might be afforded for the collection of data relative to their behavior. As both of those points have been answered in a satisfactory manner, there is therefore no further reason for delay.

The superior suitability of a transparent grating for a very considerable amount of work is of course evident, while the ease and certainty of production renders it possible that gratings need not be (as at present) excluded from high schools and kindred institutions on account of their cost. Apparatus may be constructed, or experiments undertaken, which would not be deemed advisable if one had to risk an original grating, while the duplication of gratings giving abnormal spectra is rendered not only possible, but easy.

It seems unnecessary that we should here enter upon a résumé of the various endeavors which have been made by earlier workers in this direction, beginning with Strutt in 1872 and continued by him as Lord Rayleigh in 1896 and down to the present time. However, there is one name which should not be lightly passed by in this consideration, for Thorp, of Manchester, England, was the first worker to produce a really presentable grating-duplicate of considerable efficiency, and there is certainly owing to this worker from the scientific world a decided debt of gratitude.

Mention may also be made of still another effort which is subsequent to that of the author. Mr. F. E. Ives, of New York, after receiving a few replicas from the writer became interested in the subject and himself undertook the problem of making a successful replica. After a series of experiments, he succeeded in producing a cast which differed only in respect to its method of mounting. Owing to the fact that application has been made for patent, complete details of the process are not available, but it is sufficient to state that claim is made for a replica in "a harder and less elastic material than celluloid," and with a smaller contraction coefficient. This is pressed face down in contact with a piece of selected plate glass, and then covered by and cemented to another similar plate with a cement whose refractive index is the same as that of the cast. The writer was honored by the receipt of one of these "new process replicas" upon their introduction about the beginning of the present year. When tested upon the spectrometer, it gave very good results—comparable with those manufactured by the method about to be described. Unfortunately, these casts do not seem to be permanent, as the cementing medium appears to be a solvent of the film, so that now diffraction colors are only to be seen in isolated patches.

Thorp's method[1] consisted in flowing the original grating with a thin film of high-grade oil, upon which was poured celluloid in solution. When dry, this was peeled from the previously oiled surface and mounted face up on a plate of glass by means of a solution of gelatine and glycerine; the film being lowered gently and gradually into contact.

In all of the Thorp grating casts a very large number of air-bubbles are evident between the grating film and the glass support, the presence of which serves to scatter the light and impair the brilliancy and definition of the spectrum. In the method of mounting employed by the writer these air-bubbles are entirely eliminated, the replicas presenting a clean and brilliant appearance.

When in 1901 the matter of casting from the Rowland's grating was begun, the method employed was that indicated by Thorp, viz., celluloid. A solution was made of gun cotton in amyl acetate, and then camphor was added in sufficient quantity both with and without

[1] Patented in England.

the addition of alcohol. Innumerable difficulties were encountered which, when suppressed or surmounted, simply gave place to others; and although considerable experimental work was performed, it but served to show the unreliability of this solution. These difficulties lay mostly in the direction of uneven shrinkage and opalescence of the film; while gratings of quality sufficiently good to define well in the spectroscope constituted only about 20 per cent. of the entire number.

In 1902 the results of further experiment led to the discontinuance of the preliminary coating with oil, and the exclusion of camphor in the solution. This change (together with an alteration in the method of stripping and mounting) resulted in much greater success in the production of replicas of a high grade, giving also a decidedly more brilliant film. This solution (which has since been in use without change) is composed of

Amyl acetate, pure (Mallinckrodt) - -	64 cu. cm ($2\frac{1}{4}$ oz.)
Anthony's snowy cotton - - -	2.5 grams (38 grains)

The cotton should be added to the amyl acetate in small quantities at a time, and well shaken until dissolved, after which it is allowed to stand during twenty-four hours. At the end of that time the resultant collodion is precipitated by being poured in a very thin stream into a large tray filled with water. The collodion should be poured from a height of at least three or four feet, and the water meanwhile should be constantly stirred with a glass rod. The precipitation does not immediately occur, the collodion collecting in an oily scum upon the surface of the water, which must be stirred from time to time during the course of the ensuing twenty-four hours.

When precipitation is complete, it will present the appearance of white or very light gray flocculent masses, which float upon the surface of the water, and are collected upon a clean filter paper and set aside to dry.

When thoroughly dry, it is again dissolved in the following proportions:

Amyl acetate, pure (Mallinckrodt) - -	64 cu. cm ($2\frac{1}{4}$ oz.)
Precipitated cotton - - - -	2.5 grams (38 grains)

and the collodion carefully filtered through paper—a process which may be advantageously hastened by the use of an aspirator or other form of air-pump.

The writer has prepared and used this collodion both with and without precipitation, but preference is given to the former as producing a film which is not only more brilliant, but has a much more even and regular shrinkage in the stripping.

The grating to be duplicated is first carefully leveled in a roomy drying-cabinet, and, after dusting with a soft camel-hair brush, the necessary amount of solution is flowed over the face. The exact quantity lies within wide limits; too small an amount produces a film so thin that one has difficulty in handling it, while too much gives a film which dries with a more or less matt surface. By using always the same container one may drop the necessary quantity, and then by inclining the grating cause it to flow over the surface. From the container used by the writer twenty-five drops is the average amount for a two-inch grating.

It seems hardly needful to indicate that this flowing of the grating should be performed in an atmosphere as free from air-currents as possible, thus minimizing the danger of dust particles settling upon the surface during the operation. The grating is then placed upon the leveled support in the drying-cabinet, and the door carefully closed.

The drying is rather slow, a two-inch grating taking about eight to twelve hours, but it cannot be advantageously hurried. In the opinion of the writer, the slower the drying, the better the result, as the solution gets time to fill perfectly the minute grooves made by the cutting diamond. It is a notable fact that casts made from collodions of different composition, and drying quicker, did in no case give results which were at all comparable with those which had been obtained by the slower method. It has also been noted that this film may be advantageously left in contact with the original for a considerable length of time, up to about three or four days, as it is much more easily handled in the process of stripping and mounting.

After an extended trial of various mounting mediums, which need not be enumerated, preference was given to a very thin layer of plain hard gelatine (with which the glass is previously coated and dried on a leveling-slab). The mounting is performed in the following manner:

The gelatine-coated glass and the thoroughly dry grating are placed (face up) in a tray containing filtered distilled water at normal temperature, and the tray covered over with a clean glass. After a

few minutes the extreme edges of the ruling will begin to show shadow bands caused by the contraction of the film, and thus pulling the lines "out of step." When this is observed, the grating should be removed from the water, and any adhering globules shaken from its face; then by a gentle pressure of the thumb nail at the edge of the clear portion of the polished circle, the film will be caused to spring apart from the original. This loosened portion is then grasped by the blades of a pair of wide "cover-glass" forceps, and with an even, slow motion raised from the original in a direction parallel to the length of the ruled line. *Immediately* the film is free it is laid face up upon the gelatine-coated plate (which is removed from the water for that purpose) in the same manner as in lowering a cover-glass upon a microscope slide; the plate is tilted to drain it of superfluous water, and the edge of the replica is clamped in contact, by means of a wide spring "letter clip" with matched edges. A piece of the softest velvet rubber, with a carefully cut edge, is now drawn *very lightly* and evenly over the replica in the same direction as in stripping, viz., parallel to the line length, and the plate set aside to dry.

This entire operation of stripping and mounting is very rapidly performed, and although the description may appear lengthy in the recital, it can only be laid to the fault of the author.

It has also been found advantageous to "ring" the replica with the casting solution after it has dried, and thus prevent the separation of the replica under extreme hygrometric conditions.

The contraction of the film during the process of mounting alters the number of lines to the inch, but such shrinkage is very small and is easily controlled within limits, viz., length of drying time. In those manufactured by the writer, which have a drying time of twenty-four hours, this contraction has been determined by careful measurement of over thirty replicas, with the following result:

$$\left.\begin{array}{l}\text{Width of original ruling, 28.867 mm}\\ \text{Width of replica ruling, 28.691 mm}\end{array}\right\}\text{Mean of 10 settings each,}$$

which gives a difference of 0.176 mm on the entire width of ruled surface.

The total number of lines on the original is 16,397; hence 568 = 1.0 mm. On the replica the total number of lines divided by the width gives the new constant, viz., 572 nearly, or an increase of about six lines to each one thousand.

This contraction of the replica, and the consequent increase in the number of lines to the mm, result in a greater dispersion of the spectrum. In a photograph of the region between $\lambda 3933$ and $\lambda 4308$, with the original ruling, the separation of the lines K and G was found to be 27.68 mm, as against 28.12 mm on the negative taken through the replica.

An enlargement of these negatives is shown in Plate III, which illustrates the capability of the cast under similar conditions with the original, taken with a spectrometer having an aperture of 25.0 mm and a camera focal length of 300 mm. Examination of the original negatives under a glass shows that everything which is resolved with the original grating is equally well shown in the negative from the replica (which was not especially selected for this purpose, but was taken indiscriminately from the stock of "First quality" replicas).

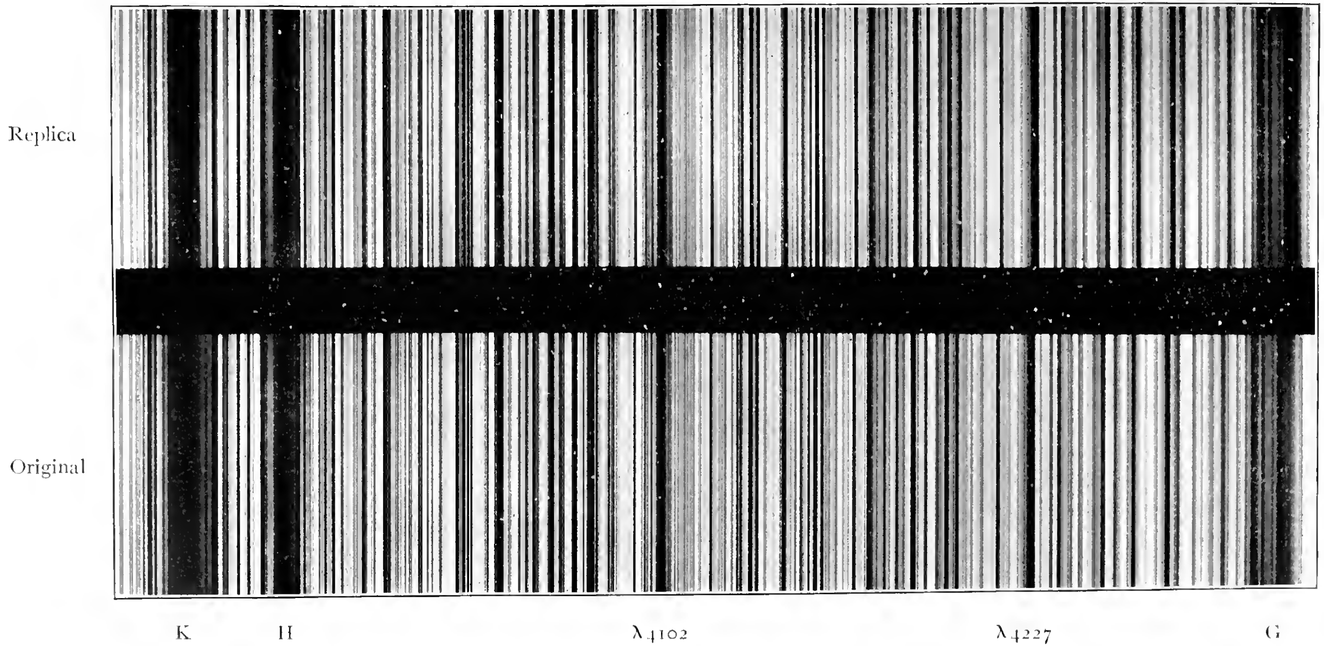
The quality of glass upon which the replica is mounted has much to do with its efficient performance in the spectroscope. It is not essential that one use worked flats, but it is necessary that the surfaces be of fairly good quality; the glass in use by the writer is "white optical crown" which has been reground and polished, and which may be graded by preliminary observation in a spectroscope.

Not all casts are of first quality or give equally good definition, for, while under apparently identical conditions of manufacture, the results vary. This is undoubtedly due to the "personal equation" in the process of mounting, and for this reason they are tested and graded. Those of "second quality" are useful for projection purposes, and also in the chemical laboratory for the flame test of *K*, *Na*, etc.

Not every sample of gun cotton will give an equally good film, and from many varieties tested by the writer the brand before specified was with much care selected as the best—not only on account of the smoothness of the resultant film, but principally because it was found to be entirely free from any trace of acid. It will, of course, be evident that if this were not the case, it would only be a question of the number of casts made, which would determine the life of the original grating.

If a replica grating be superposed face down upon the original ruling, and observed at an angle which equals that formed by the

PLATE III



SOLAR SPECTRUM AS PHOTOGRAPHED WITH A GRATING AND ITS REPLICA
APERTURE 25 mm; 15,000 LINES TO THE INCH

incident light, a series of more or less symmetrical shadow bands will be noted, which run approximately parallel with the ruling, and are caused by interference on account of the slight difference in the line spacing. If the replica were *absolutely* perfect, the bands would be straight, but in general they are slightly curved, such curvature forming (at the grating edge) an arc of a circle of radius approximately 1.5 meters. In but few cases have they come closer to a straight line than this, which may be considered as a fair average. The counting of these shadow bands was the method used by Thorp

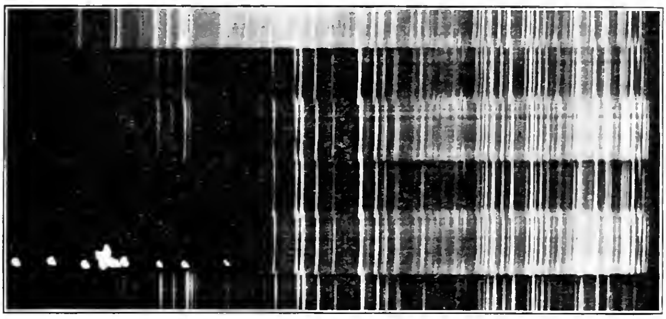


FIG. 1.—Spectra of cadmium-lead-zinc alloy.

No. 1—No screen.
No. 2—Thin mica screen.
No. 3—Same (with longer exposure).
No. 4—Grating film very thick.
No. 5—Same (with longer exposure).
No. 6—No screen, shorter exposure than No. 1.

in determining the actual number of lines per inch in his casts—their number corresponding to the increase by contraction.

On examination of these films in a quartz spectrograph, to determine their power of transmission for ultra-violet light, it was found that the absorption was practically *nil* up to $\lambda 2613$, and with slightly longer exposure they would transmit up to $\lambda 2314$. As glass practically stops everything beyond $\lambda 3400$, it has been suggested by Professor R. W. Wood (to whose kindness the writer is indebted for this examination) that they be mounted upon selected thin sheets of mica, which are even more transparent to light of shorter wave-length. Fig. 1 shows the photographic record[1] of this examination.

[1] From negative by Professor R. W. Wood with quartz spectrograph of the Johns Hopkins University.

For special apparatus these casts may be readily mounted in a variety of ways suitable to the end in view, while for direct vision they may be mounted upon 30° prisms of light crown. This method is very suitable in the construction of small spectroscopes, micro-spectroscopes, etc., and largely eliminates the possibility of poor definition due to multiple reflection from the faces of the glass plate when not accurately parallel.

In conclusion, it may be remarked that the process appears to be absolutely devoid of any evil effect upon the original grating; in fact, the opposite is strictly the case, for concerning a one-inch grating owned by the writer, from which upward of one thousand replicas have been taken, the surface appears to be as brilliant as when newly ruled. In cases where gratings have been in use in class instruction for a number of years, and consequently present a very bad appearance, being dull, surface-scratched, and greasy, the making of a number of successive casts restores in a great degree the original brilliancy. The explanation is obvious: the "dirt" on the grating is imbedded in the film while fluid, and, "setting" therein, is removed with the cast. A series of such casts from a dirty grating presents a good object-lesson as to the efficiency of this method of cleaning, being much superior to the means usually employed, viz., alcohol, ether, ammonia, etc. The replicas themselves are sufficiently tough to bear careful washing, their elasticity allowing them to be rubbed with cotton without injury.

Methods are now under consideration for giving a suitable reflecting surface to these casts, so that they may be produced either in the form of transmission or reflection gratings. Numerous experiments have also been tried in an attempt to duplicate the concave grating, but these, up to the present writing, have not given a sufficient measure of success to warrant their being recorded here.

Yerkes Observatory,
May 15, 1905.

THE SPECTRA OF ALLOYS

By P. G. NUTTING

In connection with the general problem of the relative intensity of two spectra coming from the same source at the same time, a study of the spectra of alloys is of importance, and in practice, if working with arc or spark spectra with impure electrodes, it is desirable to know the probable influence of this impurity on the spectrum worked with, and by what means this influence may be miminized. Or, if the electrodes are alloys of nearly equal parts of two metals, is it not possible, by varying the current, capacity, inductance, or atmosphere, to cause the spectrum of either component metal to preponderate at will? While spectroscopic phenomena connected with the spark are extremely complex in detail, effects involving spectra as a whole are not difficult to observe.

The only previous work on the spectra of alloys appears to be that of Lockyer and Roberts.[1] They showed that the so-called "long" lines were the first lines to appear in the spectrum of an impurity as the proportion of that impurity was gradually increased. They even sought to build up a spectroscopic method for quantitative analysis. Had they taken account of the widely differing intensities with which the spectra of different metals appear under the same conditions, and had they chosen for comparison lines those homologous according to the more modern developments of spectroscopy, more might have been accomplished.

After preliminary work had shown that the gas used as ambient atmosphere was of little consequence, hydrogen was chosen for the work on account of the simplicity of its spectra. The greater part of the work was done in hydrogen at atmospheric pressure, the remainder in the open air. The spark tube was of the special design described in a previous paper.[2] The spark was excited by a small 10,000-volt transformer fed at 100 to 400 watts by means of a control rheostat in the primary. Spectra were recorded photographically by

[1] *Proc. R. S.*, **21**, 507–508, 1873.

[2] P. G. Nutting, "Metal-Gas Spectra," *Bureau of Standards Bulletin* No. 3.

means of a large model Fuess spectrograph. With each alloy, five spectra were taken side by side on the same plate. These five spectra were taken (1) with large capacity; (2) with large capacity and inductance; (3) with small capacity and series spark; (4) with series spark and large inductance added; (5) with neither capacity nor inductance.

The alloys were usually fused in graphite crucibles and cast in a mold in cylindrical rods 2 mm in diameter. Among the sixty combinations tried, fully a third proved to be eutechtic; on solidifying the fused mixture separated into parts having different composition. Cadmium and lead fuse up nicely with aluminium, but on cooling a button of pure, or nearly pure, aluminium freezes out on top. Tin and zinc, however, appear to form true alloys in any proportion with aluminium. Some of the magnesium alloys could not be cast and were glass-hard, but chips were ground into shape for use as electrodes on an emery wheel. Except in the case of lead, the tellurium alloys appeared to go over chemically into nearly infusible masses. The antimony-zinc combination is eutechtic, but as the eutechtic alloy mixes with either of the two metals in all proportions, it may, for spectroscopic purposes, be treated as homogeneous.

The problem of preponderance in the spectra of alloys clearly resolves itself into one of the vaporization of the electrodes, and of the excitation of this vapor to luminosity. Are the metals composing an alloy used as electrode vaporized by the passage of the spark in the proportion in which they are present? Once vaporized, does the mixed vapor obey the spectroscopic laws of mixed gases?

EXPERIMENTAL RESULTS

1. *Effect of presence of second metal.*—Considering the spectra as a whole, the spectra of an alloy are found to be entirely independent of each other. There are a few cases where the presence of a second metal appears to affect slightly the relative intensity of some of the lines in the spectra of the first, but the effects are insignificant. Four series of tests were made. In the first series the metals were mixed in equal parts; in the second, lead and zinc were mixed in varying proportion ranging from pure lead to pure zinc. In the third series, carried out really to test the identity of the so-called "long" and

"short" lines with those variant or invariant with inductance, two different pure metals were used as electrodes. Finally the spectra of five of the more easily vaporizable metals were re-examined[1] in Plücker tubes.

When alloys were made up of equal parts by volume of two metals, in every case the metals appeared to be vaporized in the proportion in which they were present; that is, there was no selective vaporization (fractional distillation) such as might occur on heating to a high temperature. In other words, the vaporization produced by a spark is a *skin* effect, taking place just at the bounding surface between electrode and gas. This might be expected from the fact that the electrode fall of potential is chiefly confined to the region immediately adjacent to the electrodes, and the product of fall of potential and current is a measure of the energy loss. The results obtained are tabulated below. A plus sign indicates that spectra of the alloy were photographed; a minus sign indicates lack of success in preparing the alloy, or that the alloy proved to be eutechtic, while blank spaces are left where no trials were made.

	Zn	*Sn*	*Sb*	*Pb*	*Mg*	*Hg*	*Cd*	*Bi*
Al............	+	+	—	—	+	—	—	+
Bi............	+	+	+	+			+	
Cd............	+	—	—	—		+		
Hg............	+	—	—	+				
Mg............	+	—						
Pb............	—	—	+					
Sb............	—	+						
Sn............	+							

Tests were also made of the alloys copper-zinc, copper-cadmium, arsenic-lead, tellurium-lead, thallium-lead and thallium-zinc.

There was also considered the possibility that, if one of the metals of an alloy electrode oxidized more readily than the other, a fleck of the oxide of one metal might form on the surface and the current pass out chiefly through this fleck on account of the low electrode fall from oxides. Such an effect would alter the relative intensities of the two metallic spectra. Some metals, notably lead and tellurium, form black hydrides in great quantities when the spark passes in hydrogen, but in no case was the effect mentioned observed. Some

[1] "The Spectra of Mixed Gases," *Astrophysical Journal*, **19**, 105–110, 1904.

of the lead and zinc alloys were used in oxygen and air as well as hydrogen, but without observable effect on spectral preponderance.

The nine lead-zinc alloys, mixed in percentages of 5, 10, 20, and 50 per cent. of each metal, showed the same independence of the two spectra. From these spectrograms the proportion in which the metals were present might be estimated with fair accuracy by comparing homologous lines, but there would always be a tendency to overestimate a small proportion; 5 per cent. lead in zinc (or zinc in lead) appears more like 10 per cent. As an extreme case, some impure tellurium, estimated to contain 15 per cent. lead, was found, on analysis, to contain but 3 per cent. In agreement with Lockyer, the variant ("short line") group of lead lines $\lambda\lambda$ 3828, 3833, 3842, 3854, do not appear until the proportion of lead reaches about 30 per cent., while all the six prominent invariant ("long") lines appear prominently when but 5 per cent. lead is present. The relative natural intensities of spectra to be taken into account in making estimates of relative intensity is given in the accompanying table of data taken from an earlier paper.[1] The results all refer to the spark in hydrogen. The fourth column refers to the spark with large capacity; the fifth, to the spark with large capacity and large inductance added; the sixth, designated "arc," refers to data taken with neither capacity nor inductance added. The following column, marked "per cent. invariant," indicates roughly the proportion of the lines of the metallic spectra which are not cut out by adding inductance. The last column, headed "hardness," refers to the brightness with which the metallic spectrum comes out in hydrogen. Calling silver and platinum 1 and lead 10, intermediate estimates for the other metals were made from the photograph by two observers at different times. Estimates of "hardness" rarely differed by more than unity. In air the spectra of aluminium, antimony, tin, and tellurium are much "softer" than in hydrogen, while cobalt, chromium, iron, and nickel are much "harder," the spectra of the remaining metals being but little affected in hardness by change of atmosphere.

The use of two different pure metals as electrodes affords a convenient and effective means of classifying lines according to variability. The "short" lines of Lockyer are the lines cut out by

[1] "Metal-Gas Spectra," *Bureau of Standards Bulletin* No. 3.

	Atomic Weight	Temp. of Fusion	No. Lines E. and H.	Intensity in *H* Cap.	Ind.	Arc	Per cent. Invariant	Hardness
Al	27	657°	105	30	30	10	40	5
Bi	208	268	98	90	90	80	60	8
Cd	112	321	113	40	40	30	90	7
Cu	63	1084	260	10	10	10	95	2
Hg	200	−39	94	70	90	70	95	8
Mg	24	750	46	60	60	40	95	6
Pb	205	326	74	80	90	30	90	10
Sb	119	632	160	50	40	20	80	7
Sn	118	232	87	60	50	30	70	8
Te	127	525	79	80	10 ?	10 ?	20	5
Tl	204	290	16	40	60	30	80	8
Zn	65	419	108	30	30	10	95	5

inserting inductance or removing capacity, and these same lines show more or less stubby in the spectrograms, according to variability. On the other hand, the "long" or "arc" lines which are invariant, extend entirely across the spark image. Adding capacity lengthens all lines, while adding inductance shortens them. There exist all intermediate degrees of variability, so that, instead of classifying lines as either variant or invariant, it would be preferable to grade them on a scale of ten or one hundred. And by this method of throwing a real image on the spark longitudinally upon the spectrograph slit, even the faint lines of a "soft" spectrum like that of silver may be easily graded. It is of interest here to note that, while the long invariant lines show in the spectrum of an impurity only 2 per cent. of which is present in an alloy, the variant lines may not show until the proportion reaches 30 per cent.

Finally the results for the spark spectra of alloys were connected with the work on mixed gases in Plücker tubes by vaporizing zinc, cadmium, mercury, tellurium, thallium, and arsenic, with hydrogen and with each other, in Plücker tubes. Even in this case the invariant "arc" lines are the first to appear, whether in the presence of hydrogen or the vapor of another metal. And inductance appears to cut out the same lines from a Plücker tube spectrum that it cuts out from the spark spectrum of the same metal. However, the only lines studied were half a dozen belonging to cadmium, thallium, and tellurium, there being no other variant lines available in the Plücker tube spectra of metals.

2. *Effect of varying electrical conditions.*—A 10,000-volt spark, with and without capacity, inductance and series spark, and a 120-volt arc were used in these tests. The first series of photographs were taken with the schedule given on page 132; a later series was taken with the schedule: (1) spark with large capacity; (2) spark with capacity and inductance; (3) spark with neither capacity nor inductance, i. e., a 10,000-volt, 10-milliampere alternating arc; (4) 120-volt, 4-ampere direct current arc; (5) same with but one ampere current.

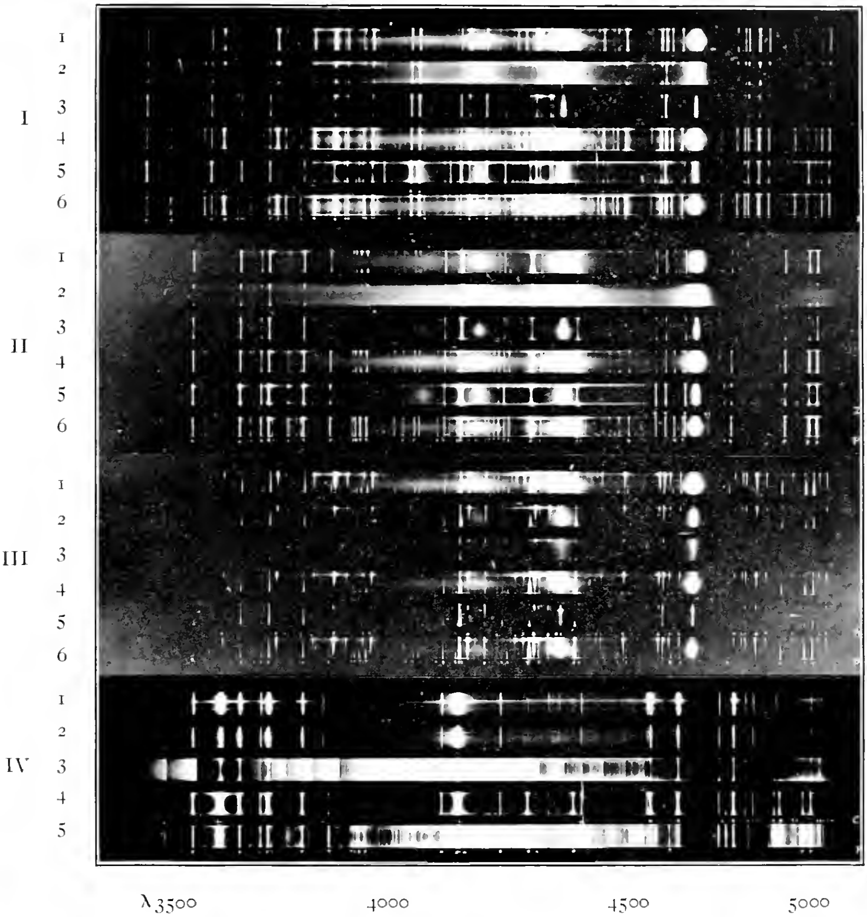
While these changes in the excitation produce enormous changes in the relative intensities of individual lines, the spectra of the different metals in an alloy are affected independently. If the manner of excitation does affect preponderance, it does so but slightly, and the effect is masked by the changes in individual lines.

3. *Effect of atmosphere.*—Tests made in atmospheres of hydrogen, oxygen, air, and in arsenic and mercury vapor, showed no effect of change of atmosphere, so far as the "long" invariant lines were concerned, and only a very few cases where even the variant lines were affected. For example, the relative intensity of the variant lead group near λ 3800 is different in air and arsenic from what it is in hydrogen and oxygen, but the smilar variant groups in the aluminium spectrum near $\lambda\lambda$ 3600 and 4500 remain unaffected even in an atmosphere of mercury vapor.

4. *Effect of atomic weight.*—From results obtained with other gases in Plücker tubes, we should expect, other things being equal, and the metals in the electrodes being vaporized in the proportion in which they are present, that the spectrum of the metal of greater atomic weight would predominate in the spectra of the low potential arc and the spark with inductance, while in the spark with capacity, the spectra of the two component metals (allowance being made for "natural intensity" and "hardness," see page 134 above) would be of equal prominence. This appears to be the case. The heavier metals, lead, mercury, thallium, and bismuth, strongly predominate in arc and inductance spectra over the lighter, magnesium, aluminium, copper, and zinc; while cadmium, tin, antimony, and tellurium predominate or are subordinate according to the weight of the metal with which they are associated.

But in the spectra of the spark with capacity and the high poten-

PLATE IV



SPECTRA OF ALLOYS

I. Aluminium-bismuth alloy; 1, spark with small capacity; 2, spark with neither capacity nor inductance; 3, series spark with inductance; 4, series spark without inductance; 5, large capacity with inductance; 6, large capacity without inductance. II. Zinc-lead alloy, 5 per cent. zinc, 95 per cent. lead, same schedule as I. III. Spark with one electrode lead and the other bismuth, showing relation of "short" lines to inductance effect. Same schedule as I and II. IV. Arc and spark spectra, lead-cadmium alloy; 1, 120-volt, 4 ampere arc; 2, 120-volt, half ampere arc; 3, 10,000-volt, 0.04 ampere spark with neither capacity nor inductance; 4, spark with large capacity and inductance; 5, spark with large capacity and without inductance.

tial, low-current arc, the effects are more complex. Zinc spectra, for example, are relatively brighter, while antimony spectra are fainter, than we should expect. I have been unable to formulate any general conclusions to cover this case. The lack of co-ordination in the results may be due to the presence of the third gas in large proportion, the steep wave front of the electrical discharge, an alteration in the corrections for "natural intensity," and spectral "hardness," or perhaps to a slight tendency toward fractional vaporization. Like the three-body problem, the three-gas problem appears to admit of no simple solution.

Summary of conclusions.—From the results of this work it would appear that:

1. In the arc and spark spectra of alloys the spectra of the component metals are independent of one another. Intensities present in any proportion do not affect the intensity of a spectrum as a whole, except to decrease its intensity.

2. Varying electrical conditions of excitation, or varying the ambient atmosphere, does not affect the relative intensity of the component spectra.

3. In the arc and spark with inductance, other things being equal, the spectrum of the component of greater atomic weight will be brighter.

4. Spectroscopic quantitative analysis, to within an error of perhaps 5 per cent., appears to be practicable, provided: (*a*) selected lines of similar character are used for comparison; (*b*) spectra are taken with an arc or spark with capacity and inductance, with sufficient current to produce plenty of metallic vapor in proportion to the ambient gas; (*c*) allowance is made for differences in the natural intensity and hardness of the spectra of various pure metals taken under the same conditions; (*d*) allowance is made for differences in the atomic weights of components, provided these differ by a considerable amount.

In practice, (*a*) the presence of impurities in electrodes is of little consequence; (*b*) when alloys are used as electrodes, it is useless to attempt to intensify the spectrum of either component by varying the conditions under which the arc or spark is produced.

BUREAU OF STANDARDS,
Washington, D. C.,
July 1905.

CALIBRATION OF A WEDGE PHOTOMETER

By JAMES D. MADDRILL

In the spring of 1900 the Lick Observatory agreed to take part in a determination of standards for faint stellar magnitudes, in accordance with a co-operative plan suggested by Professor E. C. Pickering, director of the Harvard College Observatory. Wedge photometers of a special type (see description by Mr. J. A. Parkhurst, *Astrophysical Journal*, **13**, 249), devised by Professor Pickering and provided for from the Rumford Fund of the American Academy, were distributed from the Harvard College Observatory. Photometer No. 3, containing photographic Wedge III, was received at the Lick Observatory in May 1900. The part assumed by the Lick Observatory involved the comparison of stars near the limit of the great refractor with stars of about the twelfth magnitude. With the apparatus as originally arranged, it was found that the artificial star could not be made bright enough for comparison with stars brighter than about the fourteenth magnitude in the great telescope. The small incandescent lamp originally inserted at *S* through the side of the tube (see Fig. 1) was therefore removed, and a brighter lamp *A* was placed lengthwise at the end of the tube. At the same time, the piece of blue glass supplied, was replaced by a piece *B* of darker blue. The resulting artificial star is well defined and is perhaps a trifle more reddish than the average real star. In addition, the cell carrying the two shade glasses *L* was attached to the tube *T* so that either or both could be interposed in the rays of the real star. After the drawing of Fig. 1, a slight alteration was made by which the lamp *A* can be moved an inch closer to the diaphragm if desired. The brightness is thus increased by about a magnitude and a half. The image in any possible position of the lamp can be made nearly as small and sharp as the image of the real star under the best conditions.

Owing to the pressure of work in other directions, the program was not entered upon here until June 1902. The observational part was then taken up by Dr. Aitken, assisted by several others, and completed in January 1904. The photometer has been used to some

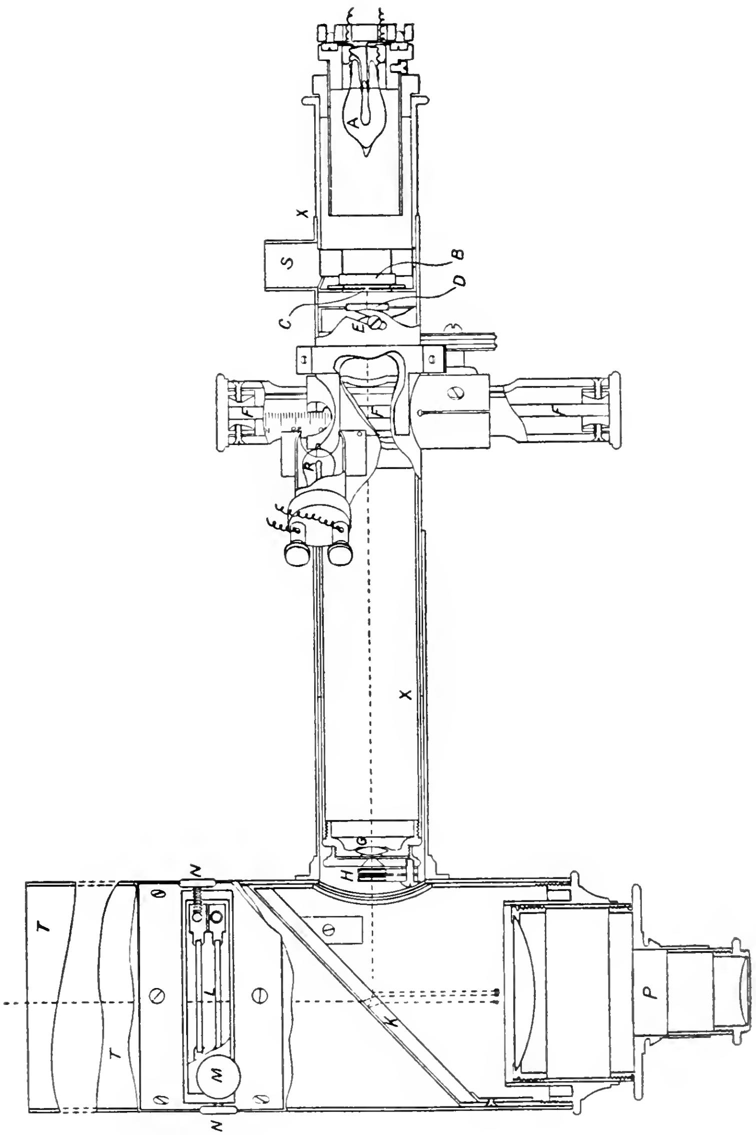


FIG. 1

extent, besides, in the observation of variable stars. The unsatisfactory condition of our knowledge of the constants of the instrument has delayed the final reduction of the measures.

Preliminary star measures of the "absorption" (strictly absorption and reflection) of the wedge, made by Dr. Aitken, early showed departures from the theoretical linear relation between scale reading and absorption in magnitudes. Rather smaller values of the relative absorption per division were found at large than at small scale readings. It became evident that for accurate determinations an "absorption" *curve* of the wedge would be required. Other observers found similar difficulties with their wedges. Mathematical and empirical absorption curves were derived. Two, as yet unpublished, have come to my notice. These were for wedges cut from the same plate as the Lick wedge; similar results might therefore be expected for the Lick wedge. In a preliminary investigation of the Virginia wedge, Professor Stone found the mathematical relation

$$\sigma=0.132-0.00080\,(m-31.8)\ .$$

This was obtained by varying the aperture of the object-glass in known ratios. The integral curve, passed through the origin, is, using our own notation,

$$m=(0.1574-0.00040d)\,d\ .$$

The empirical curve "A 12," determined by Mr. J. A. Parkhurst for Wedge V at the Yerkes Observatory, is shown in Fig. 2 (*A*). I believe this was obtained by means of a wheel or polarizing photometer, point images being compared. A few preliminary measures of *Pleiades* stars by Dr. Aitken were used here in the attempt to derive a curve, assuming the values of the absorption per division to be constant over stretches of five divisions. A curve following the general direction of Parkhurst's resulted. These preliminary measures showed that a very great number of star measures would be required to determine the curve with sufficient accuracy. On the other hand, it seemed that laboratory comparisons, made as nearly as possible under the same conditions as those of practice, should yield the curve directly, and with greater facility. Accordingly, at the suggestion of Director Campbell, the photometer was taken to Berkeley in June 1904 and adapted for comparison with the Lummer-

Brodhun photometer, kindly placed at our disposal by Professor Slate.

For convenience in making settings of the laboratory photometer, the tube X (see Fig. 1), carrying the wedge tube FF and terminating at X and X, was mounted in the optical axis on a carriage composed of the prism-box and one of the lamp-holders. This carriage was movable as a single rigid system; the lamp at the left end of the bench, together with its blue glass, diffusing surface, and diaphragm, being kept fixed. One of the moving pointers was read at a given setting, and the distance, d_2, of the screen from the fixed source, s_2, was obtained by subtracting a readily determined constant. Settings were made with the wedge at 0, 5, 10, ... , 65. The measures were then repeated in the reverse order, from 65 to 0. The following are illustrative settings, made 1904, June 13:

	Wedge				
	0	5	10	15	
	cm	cm	cm	cm	
Settings.......	143.1	145.5	158.3	187.1	Diaph. I (2.92 cm) over s_2.
	142.4	144.3	160.9	186.9	Diaph. 0.29 cm over wedge.
	143.0	143.8	159.6	184.8	Reading of pointer, with meter-stick touching s_2 and screen 163.66 cm.
	142.2	144.9	159.3	186.2	
		144.2			
	142.68	144.54	159.52	186.25	

A small correction, due to decreasing effective illuminating surface with increasing distance of the screen from the source, was taken into account by measuring distances, d_2, from *diaphragm* to screen. The distance from s_2 to the diaphragm was 1.32 cm. d_2 was therefore 64.98 cm less than the reading of the pointer. The reduction to magnitudes was carried on as follows $\left(\Delta m = \Delta \dfrac{\log(d_2^2)}{.4} = 5\Delta \log d_2\right)$:

Wedge	d_2	$\log d_2$	m
	cm		
0............	77.7	1.8904	0.000
5............	79.6	1.9009	0.052
10............	94.5	1.9754	0.425
15............	121.3	2.0839	0.968

When the length of the bench forbade further settings with a given diaphragm over s_2, a smaller diaphragm was used and settings were continued, a connected series being obtained by beginning at the last position of the wedge. The two following series resulted:

Wedge	I	II	Diff.	Mean
d	m	m	m	m
0	0.000	0.000	0.000	0.000
5	0.052	0.038	−0.014	0.045
10	0.425	0.402	−0.023	0.414
15	0.968	0.968	0.000	0.968
20	1.458	1.384	−0.074	1.421
25	1.809	1.736	−0.073	1.772
30	2.157	2.058	−0.099	2.108
35	2.601	2.512	−0.089	2.556
40	3.059	2.944	−0.115	3.002
45	3.397	3.342	−0.055	3.370
50	3.602	3.552	−0.050	3.577
55	3.655	3.566	−0.089	3.610
60	2.562	.....	.....	2.5
65	0.108	.....	.....	0.10

The difference at 15 is probably due to a particle of dust or an over-setting of the wedge in the second series. At this point the mean curve is somewhat higher, relatively, than the other curves for this wedge or the other wedges. The differences could be smoothed and perhaps ought to be, as irregularities of the sort suspected at 15 might easily occur. It has not been done here, but can be done at any time if desirable. The mean curve is plotted in Fig. 2 (*D*), and designated as the Laboratory Curve.

The marked similarity of the laboratory curve to Parkhurst's Curve A 12 for Wedge V was noticed as soon as it was plotted. Their points of inflection occur at the same scale readings; they might be brought nearly into coincidence by a simple change of slope—except near the dense end, where the wedges are certainly different. The star observations by Dr. Aitken required an average slope ("wedge-constant") about equal to that of Parkhurst's Curve. It was fully expected that the laboratory investigation would yield such a slope, special care having been taken to place the light-diffusing surface at a distance from the wedge greater than 5 cm—in accordance with the conclusions reached by Mr. E. S. King on the effect of distance of

the source of light from the photographic wedge.[1] It was necessary, however, because the Lummer-Brodhun photometer compares areas, and because the cone of rays forming the artificial star is less than a millimeter in diameter at the wedge, to place a diaphragm

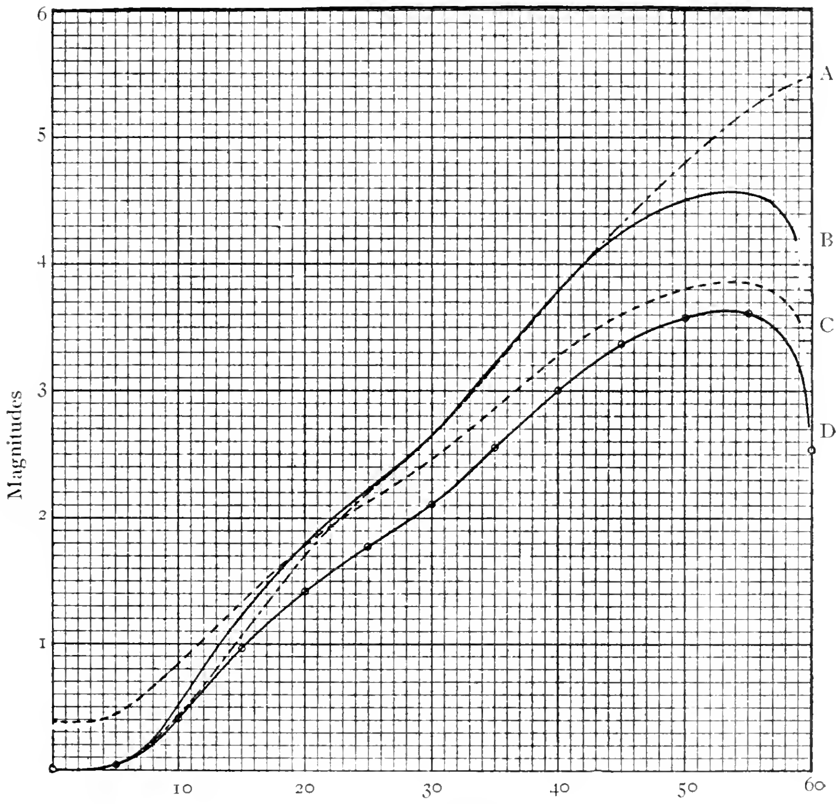


FIG. 2.—*A*=Curve A 12— V (Yerkes).
B=Wedge Curve— III (Lick).
C=Laboratory Curve— III (Harvard).
D=Laboratory Curve— III (Lick).

near the wedge. Lenses might have been used to render the divergent pencils nearly parallel at the wedge; but, aside from the departure from the conditions of practice, loss of light would have resulted, and new internal reflections would have been introduced. Light

[1] *Annals of Harvard College Observatory*, **41**, 244.

conditions required the use of a diaphragm considerably larger than would otherwise have been employed. Its aperture was 0.29 cm. The construction of the wedge tube made it convenient to place the diaphragm 1.2 cm from the wedge (film) and inconvenient to place it nearer. Just beyond the diaphragm, and separated from it by a cardboard washer, was the piece of blue glass ordinarily used with the artificial star. The distance from wedge to diffusing surface was

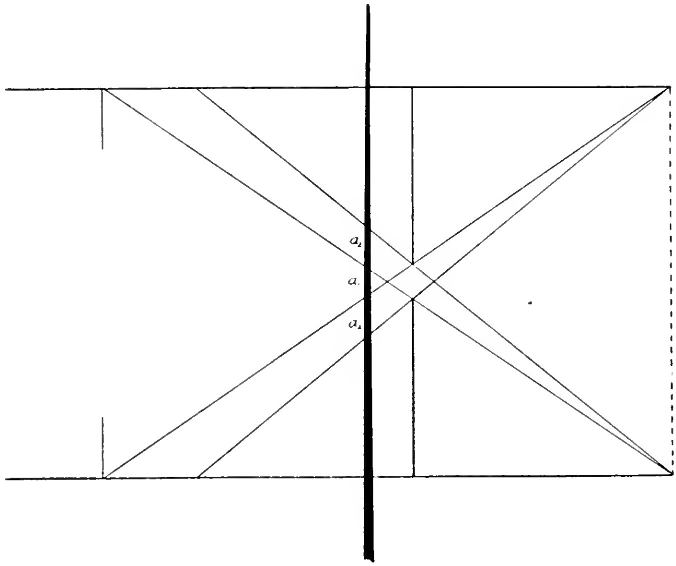


Fig. 3

8 cm. The area of wedge uniformly illuminated (a_1, Fig. 3) was 0.26 cm in diameter, and the boundary of the area (a_2) surrounding, illuminated by smaller parts of the diffusing surface, was 0.94 cm in diameter. a_1 and a_2 correspond somewhat to umbra and penumbra, with light substituted for shadow.

It has occurred to me that the following might be an explanation of the phenomena observed by King and myself. The diffusion of light by the silver grains is greater at the dense end of the wedge than at the thin end. Little of the light transmitted at a_2 without diffusion reaches the screen or field of the reference photometer, because of the greater angle of these rays with the optical axis. Much of this light is saved by diffusion at the dense end of the wedge. There will be

some loss of the light of a_1 by diffusion at the same time, but this will probably be less than the gain by a_2, because of the smaller angle of the rays of a_1 with the optical axis. Thus arrangements in which a_2 bears a large ratio to a_1 will yield smaller values of the slope of the curve than arrangements in which the ratio is smaller. Larger and practically identical values result from star determinations or comparisons with photometers employing point images, a_2 being very small.

After the measures of the wedge, determinations of the absorption of the shade-glasses were made with the Lummer-Brodhun photometer. The small shade (No. 2) next the projecting lens of the artificial star was mounted (with the projecting lens) in the optical axis, 5 cm from the blue glass that was placed against the rice-paper diffusing surface. Twelve settings, in the order: 3 shade out, 6 shade in, 3 shade out, gave 1.72 ± 0.001 magnitudes absorption.

The shade-glasses *L* were then adapted for determination. A diaphragm of aperture, 0.6 cm, approximately equal to the diameter at the shades of the cone of rays from the real star in the 36-inch or 12-inch refractors, was placed between the shades at the position of the cone in practice. Shade I was 8 cm, and Shade II 9 cm, from the source, which was left as in the measures of the small shade No. 2. Twenty settings, in symmetrical order, gave 0.89, 0.86, and 1.75 magnitudes, respectively, for the absorption of Shade I, Shade II, and both shades together.

A second laboratory investigation of the wedge was made in January 1905, with a view to determining the effect of position of diffusing surfaces, diaphragms, color, etc. Special attention was paid to securing greater intensity of illumination. The 32 candle-power incandescent lamps of June were replaced by 100 candle-power incandescent lamps, the lamps being connected always in parallel on the same circuit. A slit aperture, 0.165 cm by 1.2 cm, placed close to the glass of the wedge, was turned at right angles to the direction of motion of the wedge. The area a_1 was about 0.06 by 1.2 cm, and the area a_2 about 0.3 by 1.3 cm. Thin rice-paper, 4 cm from the lamp, and 8 cm from the wedge, diffused the light transmitted. Ten determinations of the relative "absorption" between scale readings 10 and 50, with a piece of rice-paper touching the slit, gave 3.19

magnitudes, with a mean range of ± 0.03. These observations were made with a variety of arrangements of diaphragms, with the candle-power reduced (by resistance in the circuit) to 70 and to 1, with blue glass, with red glass, etc. The sources were made similar in each comparison. No effect of color or candle-power was detected. The numerical result is practically that obtained in June (1904), 3.16 magnitudes, and seems to indicate that the blue glass placed 1.4 cm from the wedge in June behaved much as a diffusing surface. Eight determinations with rice-paper 3.0 cm from the wedge gave 3.34 magnitudes. Three, with rice-paper at both positions, gave 3.10 magnitudes. With no diffusing surface between the wedge and the rice-paper 8 cm away, three determinations gave 3.73 magnitudes. No arrangement tried would yield a result nearer 4.0 magnitudes, given by the star measures. In nineteen of the observations, settings were made at scale reading 30 as well as at 10 and 50. The mean of the nineteen ratios of the relative absorption between 10 and 30 to that between 10 and 50, was 0.548, mean residual ± 0.006, probable error ± 0.0013. The uniform value of this ratio shows that the different curves resulting from the various arrangements of the laboratory apparatus, could all be obtained by stretching one of them, say the laboratory curve of June 1904. It therefore seemed reasonable that a curve accurate enough for all practical purposes would be obtained by stretching the laboratory curve in a ratio to be determined from star measures.

A series of measures of *Pleiades* stars, selected from the list determined by Müller and Kempf,[1] was then made for this purpose by Dr. Aitken. Fifty-one pairs of stars were observed, the average range of a pair being 1.9 magnitude. The 12-inch refractor was employed, part of the time with full aperture and part of the time with 6-inch and 3-inch apertures. The shade glasses were used in some of the measures; the values of their absorption were taken from the laboratory determination. The stretching factor was derived from 51 equations of the form $f = \frac{\Delta M - \Delta m}{\Delta m}$, M being the magnitude from Müller and Kempf, and m the magnitude from the laboratory curve. The weight assigned to each determination was Δm. A

[1] *Astronomische Nachrichten*, **150**, 203, 1899.

fact already noted by Dr. Aitken, that the smaller apertures or use of shades give greater absorption per scale division, appears in the following tabulated results:

Aperture	Shades	Pairs	[Δm]	[$\Delta M-\Delta m$]	j	p. e.
12 in..........	none	15	16.23	+3.99	+0.246	±0.015
6	none	16	24.18	+6.35	+0.263	±0.020
6	both	6	12.30	−3.58	+0.291	±0.031
3	none	14	22.45	+7.37	+0.328	±0.049

It was thought best to combine these determinations in one result to be used for all apertures and conditions. In practice, a large range of wedge will rarely be used in comparisons, and single determinations will be liable to actual errors much larger than inaccuracies thus introduced. The tabulated determinations were therefore weighted in inverse ratio to the squares of their probable errors and combined, with the final results, $j=+0.261\pm0.011$. The ordinates of the laboratory curve were then increased by 0.261 of their own length. The resulting curve is plotted in Fig. 2 (*B*), and designated as the Wedge Curve. For practical use, the thousandths have been dropped, and the values tabulated as m with the argument scale division, *d:*

TABLE OF m WITH ARGUMENT d

d	00	10	20	30	40	50
0	0.00	0.52	1.79	2.66	3.79	4.51
1	0.00	0.67	1.89	2.76	3.90	4.54
2	0.00	0.82	1.98	2.87	4.00	4.56
3	0.01	0.96	2.07	2.99	4.09	4.57
4	0.02	1.10	2.15	3.10	4.17	4.57
5	0.05	1.22	2.23	3.22	4.25	4.56
6	0.10	1.34	2.31	3.33	4.32	*
7	0.17	1.46	2.39	3.45	4.38	*
8	0.26	1.57	2.47	3.57	4.43	*
9	0.38	1.68	2.56	3.68	4.47	*
10	0.52	1.79	2.66	3.79	4.51	*

PP	11	12	13	14	15
1	1.1	1.2	1.3	1.4	1.5
2	2.2	2.4	2.6	2.8	3.0
3	3.3	3.6	3.9	4.2	4.5
4	4.4	4.8	5.2	5.6	6.0
5	5.5	6.0	6.5	7.0	7.5
6	6.6	7.2	7.8	8.4	9.0
7	7.7	8.4	9.1	9.8	10.5
8	8.8	9.6	10.4	11.2	12.0
9	9.9	10.8	11.7	12.6	13.5

The following will illustrate the method of reduction that I have been using. The stars here measured are among the faintest on the Rumford program.

FIELD OF *R DRACONIS*—STANDARDS FOR FAINT STELLAR MAGNITUDES

Wed., 1903, Aug. 19, 9:50–10:56, P. S. T. Obsr., R. G. Aitken.

*	*d*	*m*	Shades	Obs.	*M*	*k*	Obs. Mag.	Mean Range	
								d	*m*
a I II.....	10.60	0.61	1.75	*k*— 1.14 =	11.12	12.26	11.51	±1.2	±0.18
b I II.....	12.78	0.93	1.75	— 0.82	11.82	12.64	11.83	0.9	.13
c I II.....	14.51	1.16	1.75	— 0.59	12.20	12.79	12.06	1.2	.14
d I II.....	18.60	1.64	1.75	— 0.11	12.38	12.49	12.54	0.7	.08
e I II.....	16.16	1.36	1.75	— 0.39	12.67	13.06	12.26	0.6	.07
								Mean	.12
q.........	38.87	3.67	....	+3.67	.....	.....	16.32	1.6	.18
r.........	40.37	3.83	.....	+3.83	.....	.....	16.48	1.4	.15
s.........	43.13	4.10	.....	+4.10	.....	.....	16.75	1.7	.14
t.........	44.14	4.18	....	+4.18	.....	.....	16.83	0.8	.06
u.........	47.99	4.43	....	+4.43	.....	.....	17.08	2.2	.10
					Mean	12.65		Mean	.13

The first three columns are taken from the record book and table. Column 4 gives the value of the shades employed (indicated in column 1 by Roman numerals). The measures being purely differential, a constant *k* is introduced in column 5 to express the absolute magnitude. The reference magnitudes *M*, in column 6, are preliminary values supplied by the Harvard College Observatory. The individual determinations of *k* from the observation equations of columns 5 and 6 are entered in column 7. The mean value, at the foot of the column, is applied successively in column 5, and the result of the observation is given in column 8. It will be noticed that the scheme at the same time conveniently exhibits data for an inter-adjustment of the reference magnitudes *M*. The corresponding data from all the observations can finally be combined and the best relative system of magnitudes *M* obtained. For example, observations of this field by the same observer on two other nights—a different combination of shades being employed each night—give:

*	II	III	Mean I. II. III
a	11.52	11.45	11.49
b	11.76	11.87	11.82
c	12.29	12.17	12.17
d	12.29	12.31	12.38
e	12.31	12.40	12.32
q	16.53	16.41	16.42
r	16.71	16.45	16.55
s	16.69	16.67	16.70
t	16.77	16.68	16.76
u	17.00	17.01	17.03

If the reference magnitudes given by Harvard College Observatory were determined from three nights' work with a photographic wedge photometer, the magnitudes to be adopted finally should probably be the means of the values found at Harvard, Yerkes, and Lick observatories. The individual and final results for *q*, *r*, *s*, *t*, *u*, are unaffected by such adjustment made from any number of *complete* observations. Columns 9 and 10 may be added to give an idea of the accuracy of the results.

Since the determination of the curve adopted, I have plotted the curve of the wedge from the means of the measures of its absolute absorption (and reflection) made at Harvard by Messrs. King, Bard and Cram (*Annals of Harvard College Observatory*, **41**, 239, Wedge III). A plot of these measures shows that the maximum lies at about 3.2 of the scale there used. The maximum of the Harvard curve was then placed opposite the maximum of the Laboratory curve in Fig. 2. If the ordinates of the Harvard curve be diminished by 0.39 magnitude so that it will pass through the origin, the differences L.O. − H.C.O. become:

d	*m*	*d*	*m*	*d*	*m*
0	0.00	20	+0.03	45	+0.17
5	0.00	25	+0.03	50	+0.18
10	−0.04	30	+0.04	55	+0.14
15	+0.03	35	+0.09	59	+0.02
		40	+0.12		

These would be reduced materially by applying a stretching factor of about +0.05.

LICK OBSERVATORY,
June 1905.

MINOR CONTRIBUTIONS AND NOTES

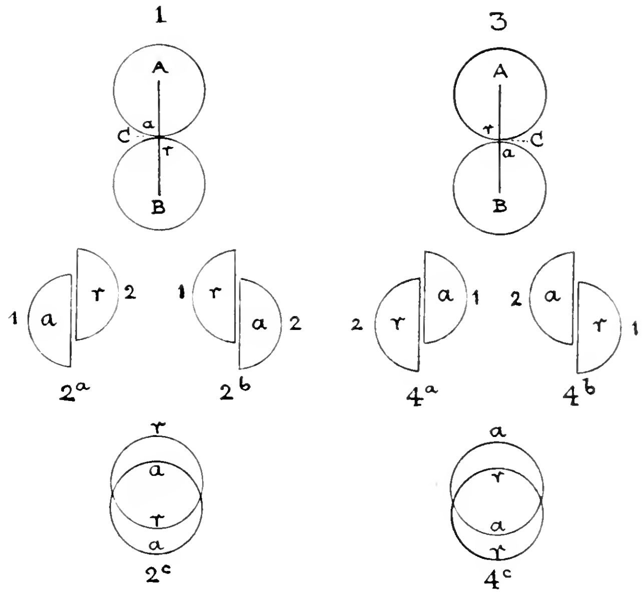
ON SPECTROSCOPIC OBSERVATIONS OF THE ROTATION OF THE SUN

The author of the review[1] of my paper has criticised the method by which the slit in my apparatus is illuminated, and he has expressed the doubt that my results on the rotation of the Sun may be seriously affected by what he considers an obvious deficiency introduced by the employment of the heliometer as the means for throwing the images of the opposite solar limbs upon the slit. He remarks that this defect in the apparatus will at once be recognized by anyone who has worked in stellar spectroscopy. I can assure him that it did not escape my attention before I began my observations, and that I did not enter upon a laborious and important research of this kind, which is indeed "exacting in its requirements," without having satisfied myself that the alleged deficiency does not exist in my measurements. Indeed, if the observations were vitiated by such errors as are pointed out by my critic, their value would, in my own opinion, be extremely doubtful. But his adverse conclusion has obviously been reached on an incomplete consideration of the apparatus and the method adopted in my observations. As the question is admittedly of general importance for solar research, and especially since the remarks against my investigation in the article referred to may be apt to misrepresent the character and quality of the work done at this observatory with regard to solar rotation, I may be allowed to correct the views of the reviewer by describing as clearly as possible the methods which I have employed in these delicate spectroscopic observations.

At the outset I remark that, apart from the design of the apparatus, the method is mainly the one originally adopted by Professor Dunér in his celebrated research. The spectra of two diametrically opposite points of the solar limb are thrown, side by side, into the focus of the viewing telescope, and in each of these two juxtaposed spectra the distances between two solar (iron) lines and two closely adjacent telluric lines are measured by means of a micrometer. If we denote the spectrum of the receding limb by r, that of the approaching limb by a, and if we call the measured distance between

[1] *Astrophysical Journal*, **21**, 385, 1905.

the solar and telluric line in the first case D_r, and in the second case D_a, then $\frac{D_r - D_a}{2}$ represents the displacement due to rotation of a point in the observed heliographic latitude. Now, in order to bring the opposite points of the Sun upon neighboring parts of the slit, I employ a heliometer mounted in a horizontal meridianal position, which receives the light from a siderostat. The full aperture of the heliometer object-glass differs not much from that



of the collimating lens, so that if the centers of the two halves coincide, the cylinder of light leaving the collimator has nearly, although not quite, the diameter of the collimating lens. Obviously, by separating the optical centers by means of the heliometer screw, and by setting the heliometer at the proper position angle, we can bring any two opposite solar points upon neighboring points of the slit. Before the two images reach the slit, however, they pass through a large rectangular prism of excellent optical quality. By this contrivance we are enabled to rotate the images so that the line joining their centers falls exactly in the direction of the slit, which is constantly kept in the vertical direction. The arrangement of the images in the focal plane of the collimator is shown in Nos. 1 and 3 of the accompanying diagram, where AB denotes the position of the slit. Let us now suppose that in No. 1 the upper image is produced by the half object-glass

1 of the heliometer, and that this image has the approaching limb at *C*, while the lower image, with the receding limb at *C*, is produced by the half object-glass 2. With a sufficiently wide slit we see the pictures of these two halves upon the surface of the grating; and obviously in the position indicated by No. 2*a*, the spectrum of the approaching limb is furnished by the semi-cylinder whose projection on the grating is denoted by the semi-circular surface on the left,[1] while the spectrum of the receding limb is furnished by the other half cylinder on the right. This is the picture which the author of the review had in his mind, and to which his criticism is fully applicable. Certainly the light of the two sections passes through different parts of the collimating lens, and also falls upon different parts of the grating, and hence the positions of the lines may be vitiated by the effects of bad focusing and of optical anomalies.

I admit that if I had confined my measurements to this one particular arrangement, my results would be open to grave doubt. But after this one observation had been made, I have invariably screwed the two half object-glasses of the heliometer into the other position. In this second observation, therefore, the former upper image has become the lower one, and *vice versa*. This does not apparently alter the arrangement in No. 1. We have still at *C* the approaching limb in the upper image, and the receding limb in the lower one. But the arrangement of the segments on the grating is now different, the light of the spectrum *r* being furnished by the left half, and that of the spectrum *a* by the right half. By combining the two observations we obtain evidently the same result as if the light of the spectrum *r* had been furnished by the full circle *rr* in No. 2*c*, and that of the spectrum *a* by the circle *aa*.

But even now the set of measurements has not been considered complete. Another series of observations has been made in which the same solar points are thrown into the position shown in No. 3, where the upper image at *C* shows the receding point, and the lower image the approaching point of the Sun. This position can be attained in two ways, both of which have been employed, viz., by turning either the heliometer or the rectangular prism through an angle of 180°. The two corresponding arrangements of the segments on the grating are exhibited in Nos. 4*a* and 4*b*. Hence by again combining the two observations we arrive at the same result as if we had observed the receding limb by means of light coming from the full cylinder *rr*, and the approaching limb through light passing along the cylinder *aa*, as shown in No. 4*c*. Consequently, in the combined

[1] The optical axis of the collimator is very nearly perpendicular to the surface of the grating.

four observations the light of both limbs has passed through identical parts of the whole apparatus, and therefore the objection raised in the review of my paper is untenable.

Far from being open to the criticism referred to, the method may even be said to possess certain optical advantages over the ordinary devices for projecting the limbs upon the slit. In as delicate an investigation as that before us it is essential that we should be able to ascertain the optical deficiencies of our instrument, and thus be in a position to eliminate, or at least to control, these during the progress of the work. It is obvious that the heliometric method, by supplying us with observations which, considered singly, are affected by errors of this kind, whereas these errors completely disappear from the general mean of a set of four judiciously arranged measurements, affords valuable tests of the optical quality of the spectroscope by which the results cannot but be benefited. The mechanical advantages of the heliometer are, I think, too obvious to require special mention. I trust the author of the review may now admit that his remarks that "grave doubt must necessarily be thrown upon some of the numerical results," and also that "it is difficult to see how the heliometer can be employed in a spectroscopic investigation so exacting in its requirements as that of the solar rotation," require essential modification.

On the other hand, I am greatly indebted to him for having pointed out the necessity of these spectroscopic observations of solar rotation, and I hope that his remarks on this point may impress upon solar physicists the importance of co-operative work in this direction.

J. Halm.

Royal Observatory,
Edinburgh, May 30, 1905.

PRELIMINARY NOTE ON "ORTHOCHROMATIC" PLATES

The usual spectrographic tests made for the purpose of determining the sensitiveness of "orthochromatic" plates are in general very much in error, owing to the means adopted for obtaining the negatives. The prismatic spectrum is altogether unsuited as a "standard": first, because of the selective absorption in the ultra-violet (and even in the visible violet) when prisms of dense flint, or direct vision spectroscopes, are used; while, second, the irrationality of the particular prism employed must be determined before any correct comparative estimate of the spectrograms can be made.

In the use of reflection gratings there is likewise an element of uncertainty on account of the selective absorption—no two gratings giving

spectra directly comparable with one another in so far as luminosity is concerned—and it is this point which chiefly concerns the photographic plate.

The conditions governing a "standard" dispersion-piece are easily met by the adoption of a *transparent* diffraction grating manufactured as described by the writer,[1] which is obviously free from abnormality due to differences in the material of which it is formed, or to the process employed in its preparation.

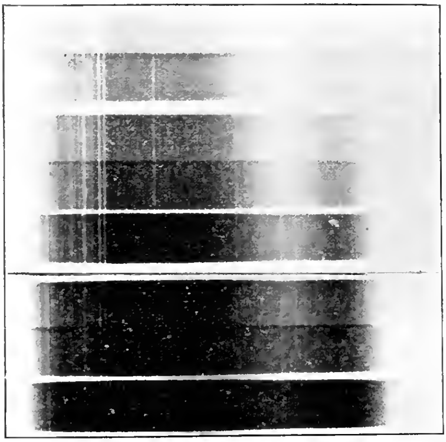


FIG. 1.—Sky Spectrum with increasing exposure on Seed "L. Ortho" plate.

Estimates based upon the negatives of the prismatic spectrum are very misleading on account of the crowding together of the colors in the red end and the elongation in the violet, thereby giving an apparently greater proportionate action in the least refrangible hues, for which the plate has been specially sensitized. Consequently there exists a considerable discrepancy in the curve of sensitiveness (even of the same plate) by different observers, no due allowance having been made for the increase in density towards the red, and decrease in the violet, consequent upon the prismatic irrationality already mentioned. It is, therefore, obvious that if the opacity of the red or yellow region be increased, while that in the violet be weakened, then due consideration must be given to this effect.

In the "sensitiveness-curves" appended to this paper, use was made of a transparent replica grating and the spectrum of the first order utilized. By means of a plate-holder furnished with rack and pinion consecutive exposures were made upon various plates with duration as follows: 2, 5, 15, 30 seconds, 1, 2, 4, 8, and 16 minutes.

In some cases a supplementary exposure of 25 to 30 minutes was given in order that the action of prolonged exposure might also be noted. A reproduction of one such plate is shown herewith.

In plotting the sensitiveness-curve a print was first made on Solio paper

[1] See page 123.

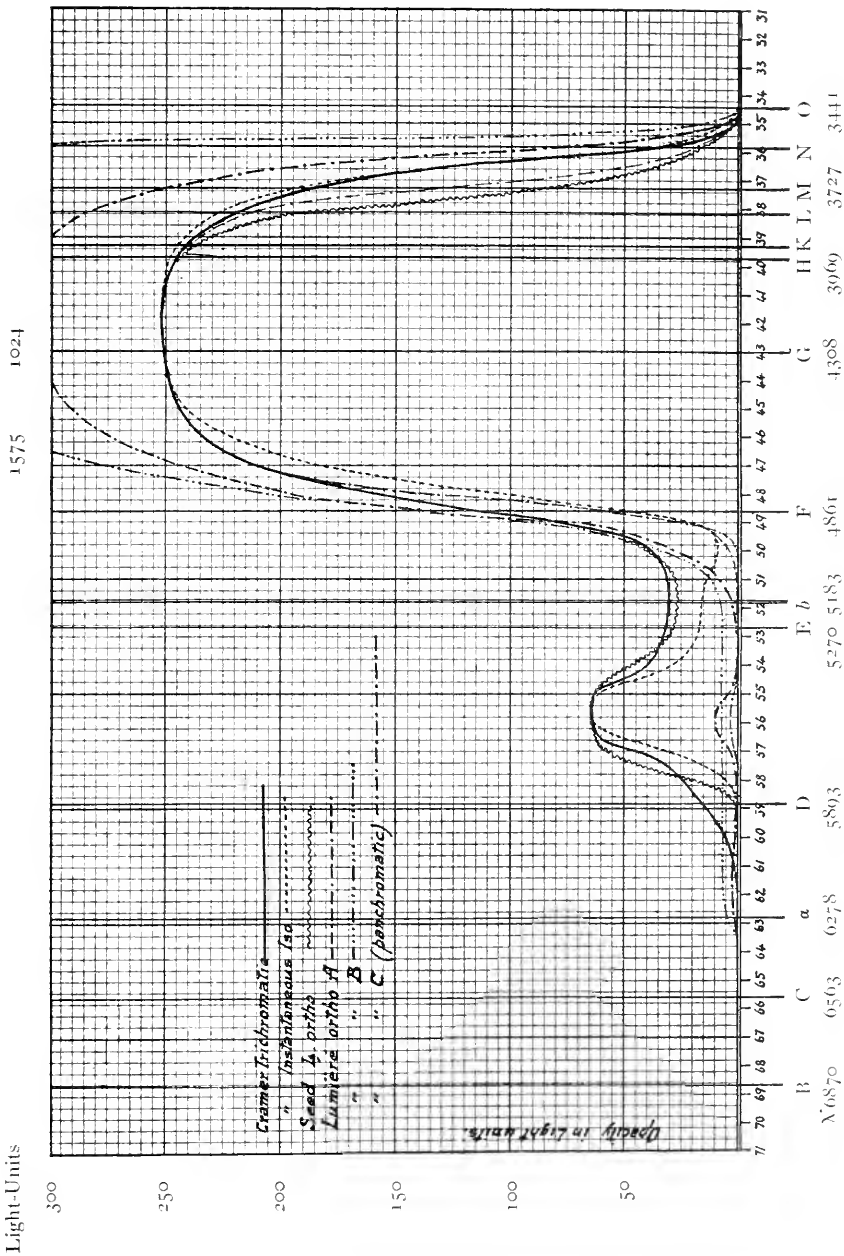


FIG. 2.—Sensitiveness-Curves of Isochromatic Plates.

from one of the plates, and that exposure selected which represented the highest allowable printing opacity—that is to say, that spectrum which was so opaque in the region of greatest sensitiveness that it was only with difficulty that the Fraunhofer lines would show on the print. This opacity was found to be represented by a light action of 256 units on a negative obtained from exposure behind a revolving Hurter and Driffield sector-disk, on the same plate, developed for the same length of time. On every other plate, therefore, that spectrum was selected for estimation whose greatest region of opacity corresponded to 256 light units on a similar plate exposed by disk. The curve was then plotted on squared paper, where the ordinates represent amount of light action (in units), and the abscissæ, wave-lengths; the variations in opacity throughout the length of the spectrum being estimated by small patches isolated by an opaque screen and in juxtaposition with the various opacities of the sector negative, the aperture ratio of which is accurately known.

It will be noted that two of the curves rise above the limit of the squared field, viz., Lumière "Ortho B" and "Panchromatic C." This was rendered necessary owing to the very slight action in the red end of the spectrum as compared with normal exposure in the blue-violet. The heights of the blue-violet curves in these cases were estimated simply by the ratio between that spectrum whose exposure time equaled normal exposure (=256 light units) and the time of the one selected for plotting, the opacity being much too intense for measurement.

It should be understood that these curves are representative only of "selective sensitiveness," no consideration whatever being given to the relative speed. Tests were made from the standpoint of the "Schwellenwerth" as being best suited to astronomical needs, where the principal point is to obtain developable action with the smallest light value. These test for the plates plotted may be tabulated in the following results:

Cramer "Instantaneous Isochromatic"	=	1
Seed "Landscape Orthochromatic"	=	3 per cent. less
Lumière "Ortho A" / Lumière "Ortho C"	=	7 per cent. less
Lumière "Ortho B"	=	14 per cent. less
Cramer "Trichromatic"	=	40 per cent. less

Acknowledgments are due to Mr. H. B. Lemon for able assistance in this work, and also to Messrs. Lumière, Cramer, and Seed, who kindly furnished the necessary plates.

ROBERT JAMES WALLACE.

YERKES OBSERVATORY,
July 15, 1905.

A FEW ADDITIONAL FLUOR-SPARS CONTAINING YTTRIUM AND YTTERBIUM

I had hoped sometime greatly to increase my list[1] of fluor-spars containing yttrium and ytterbium, but it is not probable now that I shall soon have the opportunity of doing so. However, the few additional ones that have been kindly sent to me, together with those already reported on, extend the examination practically to all parts of the Earth.

For the sake of this completeness I submit the following supplementary table:

SUPPLEMENTARY LIST OF FLUOR-SPARS EXAMINED

Where from	Furnished by	Amount Yttrium	Amount Ytterbium
AMERICA, *Canada*—			
Cameron, Ontario	R. B.	Large	Appreciable
Lot 4, Concession A, Cobden, Ontario	R. B.	Appreciable	Small
North ½ Lot 13, Concession 3, Cobden, Ontario	R. B.	Small	Small
South ½ Lot 13, Concession 3, Cobden, Ontario	R. B.	Small	Trace
Derry, Quebec	R. B.	Small	
Huell, Quebec	R. B.	Small	
JAPAN—			
Province of Bungo	H. N.		
Province of Ise	H. N.	Trace	
Province of Noto	H. N.	Appreciable	Trace
Province of Tajima	H. N.	Small	

R. B. means Dr. Robert Bell, Geological Survey, Ottawa, Canada. H. N. means Professor H. Nagaoka, Imperial University, Tokyo, Japan.

The fluor-spar from Cameron, Ontario, like the samples from other places, previously reported on, containing large amounts of yttrium, is especially sensitive to thermal effects.

W. J. HUMPHREYS.

UNIVERSITY OF VIRGINIA,
June, 1905.

WAVE-LENGTHS OF CERTAIN SILICON LINES

The three lines at $\lambda\lambda$ 4553, 4568, and 4575 are of especial utility in determinations of the radial velocity of numerous stars having spectra of the *Orion* type. They are particularly prominent and sharp in the sub-group represented by β *Crucis* and ϵ *Canis Majoris*. The identification

[1] *Astrophysical Journal*, **20**, 267–270, 1904.

of these stellar lines with silicon was independently and simultaneously accomplished by Sir Norman Lockyer[1] and Mr. Joseph Lunt.[2] They are called "Group III" of the silicon spectrum by the first-named observer.

At the time of publication of the paper on "The Radial Velocities of Twenty Stars" by Frost and Adams, the best available laboratory determination of the wave-lengths of these lines seemed to be those of Exner and Haschek (on the spark spectrum—the lines are not present in the arc spectrum), although the uncertainty of their measures, chiefly due to the diffuse character of the lines, was recognized.[3]

It therefore seemed worth while to attempt a more accurate measurement of the wave-lengths in the spectroscopic laboratory of the Observatory. The plates were all made by Brown[4] with the first order of the 10.5-foot concave grating, using a strong spark between electrodes containing metallic silicon and titanium, probably prepared in an electric furnace. Numerous experiments were tried with a view to overcoming the diffuseness of the lines, but they were hardly successful. The symmetrical character of the lines, however, justified the hope of obtaining more precise measures than those previously published.

The presence of the titanium impurity had the advantage of supplying sharp comparison lines near the silicon lines; but on some of the plates the *Ti* line at λ4552.632 (Rowland's value) evidently was confused with the adjacent silicon line. That line was therefore only measured on plates where the intensity of the *Ti* line was too weak to perceptibly affect the strong silicon line, as could be safely inferred when the *Ti* line at 4555.66, of equal intensity with 4552.63, was too faint to be at all effective.

The current was supplied by a transformer wound 110 to 30,000, with large capacity in parallel with the secondary. The intensity of the lines was increased by enlarging the capacity, while they were made to vanish entirely by putting in self-induction, as has frequently been observed.

Of the plates taken, three were well suited for measurement, and two of these plates were measured by Brown on a Zeiss comparator, with accordant results. The three were later measured by Frost, in each case with violet toward the left and violet toward the right, and with both a single thread and a double thread (actually rulings on glass reticle) on a Gaertner comparator. The silicon wave-lengths are referred to Rowland's values of the titanium lines used, in most cases to λλ 4544.864, 4548.938, 4555.662, 4563.939, 4572.156, and 4590.126 or 4617.452.

1 *Proc. R. S.*, **65**, 449, 1899. 2 *Ibid.*, **66**, 44, 1899.

3 *Publications of the Yerkes Observatory*, **2**, 157.

4 Volunteer Research Assistant at Yerkes Observatory, Summer Quarter of 1904.

As Eberhard had found in his valuable "Untersuchungen über das Spektrum des Siliciums"[1] that the sharpness of the silicon lines was increased in an atmosphere (of hydrogen), Professor Crew, in response to our request, kindly offered to have a trial made in his laboratory, with an atmosphere of coal-gas. The desired result was not attained, but an excellent concave-grating plate of the silicon spark in air was made for us by Mr. G. S. Fulcher, to whom, as to Professor Crew, our thanks are due. This plate was measured by Frost, in the same manner as the other plates, twice independently, with an interval of some weeks. The *Ti* line at λ4552 was present on this plate, so that measures were not made of the nearby *Si* line. The two measures of this plate were closely accordant for the two *Si* lines, but in the case of λ 4567 the result was quite a little larger than on the plates taken with the Observatory grating.

Inasmuch as the measures by Brown included only two of the plates, and were not made with the violet in both positions under the microscope, it has seemed best to employ only the measures of Frost in deriving the result. The means for the two observers do not differ, however, by 0.01 tenth-meter in case of any of the three lines.

The reductions were made by least squares, by Miss F. A. Graves, in the usual manner for concave-grating plates where several reference lines are used. An idea of the internal agreement of the measures can best be given by printing the results for the different plates. The measures with violet to left and violet to right were combined before they were reduced, so that they cannot be given separately.

WAVE-LENGTHS OF SILICON LINES

Plate	Thread	4553	4568	4575
No. 9	Single	4552.65	4567.91	4574.78
	Double	.66	.90	.79
No. 19	Single	.64	.87	.80
	Double	.61	.88	.79
No. 24	Single	.62	.85	not measured
	Double	.64	.89	
Crew's No. 670 (Means of two measurements)	Single	not measured	.95	.79
	Double		.94	.80
	Means	**4552.64**	**4567.90**	**4574.79**

[1] *Zeitschrift für wissenschaftliche Photographie,* 1, 346, 1903.

The average values of the residuals from the measures of the titanium lines on the plates are as follows: No. 9 (6 *Ti* lines), 0.012 t.-m.; No. 19 (8 *Ti* lines), 0.009 t.-m.; No. 24 (3 *Ti* lines), 0.005 t.-m.; No. 670 (4 *Ti* lines) 0.023 t.-m.

More accurate results can doubtless be obtained when the silicon lines can be made sharper on the spectrograms, and it is planned to experiment with vacuum tubes containing $Si\ Fl_4$, with which Eberhard got the *Si* lines in question sharply defined.

It is proper to add for comparison the results obtained by other observers, viz.:

Gill (from stars)...............	4552.79	4567.90	4574.68
McClean (from stars)..........	4552.6	4567.5	4574.5
Lockyer (spark).................	4552.8	4568.0	4574.9
Exner and Haschek (spark).....	4552.75	4567.95	4574.9

The significance of the change to the new wave-lengths from those of Exner and Haschek, previously used in work on *Orion* stars at this Observatory, is best seen when converted into kilometers:

λ 4553: Correction (F. and B. – E. and H.) – 0.114 tenth-meters = – 7.51 km
4568: " " " – 0.053 " = – 3.48 km
4575: " " " – 0.109 " = – 7.14 km

The effect on the values of the radial velocities of stars, for which considerable weight was given to the displacements of the silicon lines, is by no means inappreciable. In the case of γ *Pegasi*, for instance, the use of the new values of the *Si* wave-lengths changes the final value of the radial velocity as determined here by fully + 1.5 km.

In due time the precise values of the corrections to be applied to the published radial velocities of other stars of the *Orion* type will be communicated. In many instances these silicon lines were not used.

We incidentally measured the wave-length of the air line (nitrogen) at λ4507, which occurred on most of the plates, with the result: Mean of F. and B., λ=4507.81. The difference between the results for the two observers was 0.01 t.-m.

EDWIN B. FROST AND JULIUS A. BROWN.

YERKES OBSERVATORY,
August 10, 1905.

Reviews

Guide du Calculateur. Par J. Boccardi. Paris: A. Hermann, 1902. Première Partie, pp. x + 78. Deuxième Partie, pp. viii + 147. Price, Partie I, 4 francs; Partie II, 12 francs.

It is not easy to see why writers of textbooks have afforded so little help to the student who wishes to become proficient in computing; or why M. Boccardi, in the preface to the work that lies before us, should think it necessary to apologize for presenting to us so useful a book. Facility in computing is, to be sure, only to be acquired through practice; but this applies equally well to photography and to many similar arts, none of which can be said to suffer from a lack of works of reference for the beginner. The reviewer has often thought that there is a wide field for missionary work among all classes of professional computers. How many business accountants are there, for example, who know of the existence of such tables as Crelle's? And how few must there be whose labors would not be materially lightened with the aid of that inexpensive work! Ignorance of the tools they might command is even more surprising in the case of engineers, as the reviewer is able to attest from a somewhat extended acquaintance among them.

M. Boccardi divides his work into two parts, published under separate covers. Part I deals with general rules for computing; Part II, with special examples. Part I is again divided into three sections, entitled *before*, *during*, and *after* the calculations. The first of these sections is designed to enable the computer to make an intelligent choice of methods, tables, etc., after having decided upon the degree of accuracy that he wishes to retain. We are glad to see that the author has adequately emphasized the economy of using tables with no more than the necessary number of decimal places—something which is still not as universally recognized as it ought to be. In the reviewer's opinion, the author underestimates the utility of computing machines; his comments upon them are meager and somewhat unfavorable.

Section II, although short, contains so many hints and short-cuts of a practical character that it merits at least the perusal of every computer. Section III points out the best methods for verifying calculations, and for rapid location of errors in them.

Part II is chiefly devoted to perturbations and to the determinations of orbits; it will be found useful by the student who purposes to specialize in this subject, but it has not the general interest that attaches to Part I.

In a work of this character we must expect to find some precepts and details of methods that would not meet with the unanimous approval of computers. For the most part, in M. Boccardi's book, these are matters of opinion at best, and are not of great consequence. Possibly an exception to the last is to be found in the author's dictum (repeated throughout the work in various forms) that "there can never be too many controls." We think it would be better to advise the computer to select one "necessary and sufficient" control, and not to spend too much time on intermediate ones, at least after he has acquired some accuracy in his work.

The size of the page (35×24 centimeters) makes the two volumes somewhat clumsy; otherwise the printer has done his work well. The great merit of the essential features of the book should give it a wide circulation among expert computers as well as beginners.

F. S.

Allegheny Observatory,
August 1, 1905.

Index to the Literature of the Spectroscope (1887–1900) inclusive. By Alfred Tuckerman. *Smithsonian Miscellaneous Collections*, Vol. XLI, 1902. Pp. 373.

This is a continuation of the previous index by the same author similarly published in 1888, but it represents a decided improvement both in arrangement and accuracy. The rapid development of the subject is indicated by the fact that the present volume, covering fourteen years, nearly equals in size its predecessor, which was intended to include all articles to 1889. This comparison is not entirely fair, however, as the later volume gives more space to each article in the author-index.

The author-index and subject-index divide the book equally. The former gives full titles of articles, and will be found the more generally useful. The latter is subdivided alphabetically into a large number of sections, and this method is followed in the separate sections. This necessarily makes some anomalies: under the section "Astronomical in General" the constellations occur where the alphabet brings them, but without reference to the particular star in the constellation. Thus we have in succession "Cephei," "Cladni" [!], "Comets," "Corona" [only a single reference], "Cygni," "D. M. (Star)," etc. As the subject-index gives

only the references to the articles, without their title, it will be of less service than the author-index.

In general, the work will be found helpful, though it has a sufficiency of errors clerical and errors of discrimination. If the author had had a practical acquaintance with more of the articles, the result would have been better, but in that case he would probably have had no time for this bibliographic labor of love; and the workers in the subject certainly should be grateful for this assistance in making available the rather scattered literature. F.

Popular Star Maps: A Rapid and Easy Method of Finding the Principal Stars. By COMTE DE MIREMONT. Folio, 13×15 inches. London: George Philip & Son, 1904. 10*s.* 6*d.*

These are doubtless the clearest of all star maps for one who wishes to recognize the constellations and locate the principal stars. Ten double maps, 10×10 inches, give on the left-hand page, in white on a blue ground, the stars down to the third or fourth magnitude, with nothing else to distract. On the opposite page are shown the same stars in black on a white ground as usual, with the common name or Greek letter of the star and the constellation names, a few lines joining some of the stars and forming simple figures. The introduction contains the Greek alphabet, a list of the principal constellations, and two indexes, one arranged alphabetically by constellations, the other in order of Right Ascension, giving the name, magnitude, place for 1904, and precession, for each star mapped. A few minor points open to criticism do not materially detract from the excellent adaptation of these maps to the use of the beginner. The declination co-ordinates are not given on the maps and are not needed, hence it is superfluous to carry the declinations in the list to hundredths of a second of arc. The distortion at the corners of the polar maps, for instance in the constellation *Perseus*, masks the resemblance to the sky, and in this case there is no overlapping map to rectify the figure. With the limitations noted, this work admirably fulfils the purpose for which it is intended. P.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript*, one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE
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AND ASTRONOMICAL PHYSICS

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ON THE EVOLUTION OF THE SOLAR SYSTEM

By F. R. MOULTON

1. *Introduction.*—For about a century it has been quite generally believed that, in an earlier state, our system was a widely extended nebula; and that, contracting under the mutual gravitation of its parts, rings were left off which later developed into the planets. There have been numerous variations from the original theory as formulated by Laplace, but they have all been essentially the same dynamically, for in all of them it was assumed that the original mass was in a state of temporary hydrodynamical equilibrium, maintaining its volume largely by gaseous expansion, and that the planets have developed out of rings left off from the parent mass. In 1900 Professor T. C. Chamberlin and the writer undertook, so far as was possible, to test,[1] by an appeal to the laws of dynamics, the consistency of this ring theory with known phenomena. Contradictions were uniformly found, and in some cases the results were so conclusive as to compel us frankly to abandon it as an untenable hypothesis.

Having given up the ring theory, the problem has been to find, if possible, something more satisfactory. The result has been the

[1] Chamberlin, "An Attempt to Test the Nebular Hypothesis by the Relations of Masses and Momenta," *Journal of Geology*, February-March, 1900. Moulton, "An Attempt to Test the Nebular Hypothesis by an Appeal to the Laws of Dynamics," *Astrophysical Journal*, **11**, 103–130, 1900.

formulation of a fairly definite theory, which Professor Chamberlin calls the "Planetesimal Hypothesis," and which he has expounded in his paper, "Fundamental Problems in Geology," in *Year Book* No. 3 of the Carnegie Institution of Washington, pp. 195–258. The present paper is devoted to a brief account of some of the main dynamical features of the theory, and to some comments on the retrograde revolution of *Saturn's* ninth satellite. For the sake of brevity in the exposition, the theory will be given in categorical terms, without implying in the least that it is not yet open to question at every point.

2. *Outline of the theory.*—It is supposed that our system has developed from a spiral nebula, perhaps something like those spiral nebulæ which Keeler showed are many times more numerous than all other kinds together.[1] The spiral nebula is supposed to have originated at a time when another sun passed very near our Sun. The dimensions of the nebula were maintained almost entirely by the orbital motions of the great number of small masses of which it was composed, and only a very little by gaseous expansion. It was never in a state of hydrodynamical equilibrium, and the loss of heat was not necessary for its development into planetary masses. The planets have been formed around primitive nuclei of considerable dimensions by the accretion of the vast amount of scattered material which was spread throughout the system.

Such a spiral nebula as that described, having originated in such a way, will develop into a system having the following properties: The planets will all revolve in the same direction, and approximately (though perhaps not exactly) in the same plane; the sun will rotate in the same direction, and nearly in the same plane, and will have an equatorial acceleration; the more the planets grow by the accretion of scattered matter, the more nearly circular will their orbits become; the planets will rotate in the forward direction, and approximately (though perhaps not exactly) in the planes of their orbits; the more a planet grows by the accretion of scattered matter, the more rapidly will it rotate; the planetary nuclei may be attended originally by many satellite nuclei revolving in any direction, but the scattered material will tend to drive all those satellite nuclei down

[1] *Astrophysical Journal*, 11, 347–348, 1900.

on to the primary nucleus which do not move forward in the general plane of the system; the scattered material develops and preserves circularity in the satellite orbits, if they revolve in the forward direction, but considerable eccentricity, if in the retrograde direction; a satellite may revolve more rapidly than its primary rotates; the system may contain many planetoids whose orbits are interlocked; the small planets will be cool and dense, and the large ones hot and rare; and the greater part of the moment of momentum of the system will belong to the planets. It will now be shown that these statements are true.

3. *A possible origin of spiral nebulæ.*—In view of the relative motions of the stars, it is to be expected that two will sometimes pass near each other, and very much less frequently actually collide. At the time of near approach of two large masses the mutual tidal strains are very great. It is supposed that once a sun, which we shall call S', passed near our Sun, S, and raised on it a huge tide on the side toward S', and an almost equal one on the opposite side. It follows from the well-known theory of tidal forces that the effect on S was equivalent to a diminution of its attraction in the line passing through S', and an increase of its attractions in other directions. Roche has shown that when the bodies are nearer each other than 2.44 times their radii, the self-gravitation of one of them is more than balanced by the differential tidal forces due to the other. Our Sun was then agitated by great energies such as now produce the eruptive prominences. The enormous tidal strains attending the near approach of S' increased the eruptive tendencies of S toward and from S', and large quantities of material were ejected with great velocities in both of these directions. If it had not been for the subsequent disturbing effects of S', these ejected masses would have returned to the Sun, but S' drew them from their rectilinear paths and left them describing ellipses around the Sun.

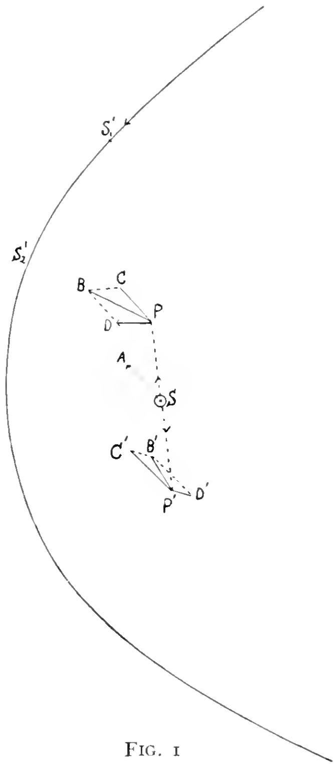


FIG. 1

To show how this was done consider Fig. 1. Suppose the masses

P and P' were ejected from S in opposite directions when S' was at S'_1, and consider the character of the disturbing forces when S' has arrived at S'_2. Let $\overline{SA}$ represent the acceleration of S' upon S in direction and amount. In the same units let $\overline{PB}$ and $\overline{P'B'}$ represent the acceleration of S' upon P and P', respectively. Resolve $\overline{PB}$ and $\overline{P'B'}$ each into two components, so that one of them in each case (PC and $P'C'$) shall be equal and parallel to $\overline{SA}$. Since $\overline{SA}$, $\overline{PC}$, and $\overline{P'C'}$ are equal and parallel, they do not disturb the relative positions of S, P, and P'. The disturbing accelerations are therefore $\overline{PD}$ and $\overline{P'D'}$ in direction and amount. It is observed that P and P' are both disturbed so as to start to revolve around S in the direction of the motion of S'.

While the character of the disturbing forces can be shown in this way, the precise results of their continued action can be determined only by computation. The labor of this computation is great, for the disturbing forces are very large, the curve described is complicated, and S' is for several years near enough sensibly to modify the motion of P and P'. The orbits described by the ejected material have depended upon the mass of S', the nearness of its approach to S, the relative position of S' at the time the material was ejected, and the velocity of ejection. At present it seems necessary in order to get a thorough understanding of the dynamics of the subject to treat by numerical processes a large number of special cases. This work is at present under way, and in all the cases so far considered P and P' have been left moving in elliptical orbits.

It remains to show that the nebula will have a spiral form immediately after the departure of S'. Figure 2 shows the positions of the masses ejected at successive intervals. The dotted lines are the actual curves which have been described, and the full lines show the apparent form of the spiral. There will be, of course, a vast quantity of fine material scattered throughout the system. The striking feature of the nebula is that there are two arms of the spiral starting from opposite sides of S. This is precisely the condition revealed by photography in the spiral nebulæ, often most unmistakably. It is to be observed that the motion is not along the lines of the spiral, and that, therefore, as the spiral grows older it will become more and more coiled.

Whether or not computation shall verify this conjecture respecting the origin and nature of spiral nebulæ, it seems probable, both from their appearance and from their spectra,[1] that the theory has led to a correct picture of their physical and dynamical condition. Doubtless those spirals which have been photographed are immensely larger than the one from which our system may have developed, and as a rule have relatively less massive centers.

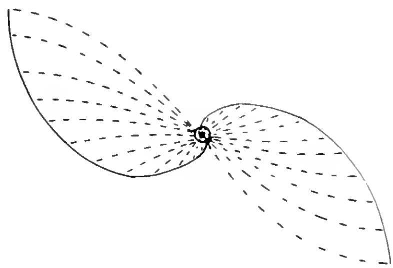


FIG. 2

4. *The revolutions of the planets.*—The matter was originally ejected more or less irregularly with occasional large nuclei which have grown into the planets by the accretion of the finer scattered material. The direction of ejection depended upon the direction of S', the lag of the tides on S, and the original direction and rate of rotation of S. It is very improbable that the plane of the original equator of S coincided with plane of the orbit of S'. Consequently the original ejections were not all exactly in the plane of motion of S'. A little consideration shows that, taking into account the ejections both toward and from S', and both before and after perihelion passage of S', the matter was distributed nearly symmetrically with respect to the plane of the orbit of S'.

It follows from the mode of generation of the elliptic orbits of the ejected material that the planets must all revolve in the same direction, and from the statements just made that their planes will nearly, though not exactly, coincide. It follows from the symmetrical distribution of the ejected material that the more a planet grows by the accretion of the scattered material, the more nearly will the plane of its orbit coincide with that of the orbit traversed by S'. Consequently we should expect but slight divergences in the planes of the orbits of the large planets, which have grown most, and on the average much greater differences in the orbits of such small bodies as *Mercury* and the planetoids. Interpreted according to this

[1] E. g., Scheiner found that the *Andromeda* nebula seems to have a dark line spectrum, *Astronomische Nachrichten*, **148**, 325, 1899.

theory, the high inclination (10°) of the orbit of *Eros*, which lies so close to the orbits of the Earth and *Mars*, is nothing to occasion surprise.

5. *The rotation of the Sun.*—The present rotation of the Sun is the resultant of its original rotation and of the disturbance due to S'. The original direction and rate of rotation of S are quite unknown. Its rotation was affected in two ways by the passage of S'. (*a*) S' raised large tides on the Sun and dragged them around in the direction of its motion. This contributed a certain rotation to the Sun in the direction of motion of S'. (*b*) Some of the ejected particles left the Sun with small velocities, and fell back upon it before their orbits were greatly perturbed by S'. But all of these particles had acquired some moment of momentum in the direction of the motion of S', and, falling obliquely into the Sun, they gave up their moment of momentum to this body. In these two ways a rotation was developed in the Sun agreeing approximately with that of the general motions of the planets.

Both of the influences which have been mentioned were most important in the equatorial zone, and extended to relatively shallow depths. Consequently the Sun was given an equatorial acceleration which still persists. The spots occur where the layers having different rates of rotation flow most rapidly past each other.

6. *The eccentricities of the orbits of the planets.*—The nuclei around which the planets formed were left by S' revolving in ellipses, presumably of considerable eccentricity. The probable amount will be revealed in the course of time by our computations. The orbits of the scattered particles had every possible orientation and crossed the orbits of the nuclei. The nuclei swept up these particles, and in the process had the eccentricities of their orbits changed. The question of interest is whether the eccentricities were increased or decreased. It is observed first that the more nearly two orbits have the same major axis and eccentricity, the more likely are bodies moving in them to collide; for under these circumstances the orbits may intersect at the most acute angle. The case of collision of two bodies moving in such orbits will be treated.

Let a_0 and e_0 be the common major semi-axis and eccentricity before collision, and a and e the corresponding elements of the orbit

of the combined mass after collision. Let N represent the mass of the nucleus, and m that of the particle. Let us neglect the slight perturbations that N and m produce in each other's motions, and consider only the effects of a collision at a point where their orbits cross. At the instant before the impact of the two bodies their kinetic energy was

$$\frac{1}{2}(Nv_N^2+mv_m^2) \ ,$$

where v_N and v_m represent their respective velocities. After impact the kinetic energy of their combined mass was

$$\frac{1}{2}(N+m)v^2 \ ,$$

where v is the velocity of the combined mass. Since part of the kinetic energy will have been transformed into heat by the impact, we have

$$Nv_N^2+mv_m^2>(N+m)v^2 \ . \tag{1}$$

But it follows from the theory of elliptic motion in the problem of two bodies that

$$v^2=k^2\left(\frac{2}{r}-\frac{1}{a}\right) \ . \tag{2}$$

Hence, since r was the same for both N and m the instant before impact, and for the combined mass the instant after impact, we have, substituting in equation (1),

$$Nk^2\left(\frac{2}{r}-\frac{1}{a_0}\right)+mk^2\left(\frac{2}{r}-\frac{1}{a_0}\right)>k^2(N+m)\left(\frac{2}{r}-\frac{1}{a}\right) \ .$$

It follows from this equation that

$$a<a_0 \ . \tag{3}$$

The moments of momentum of N and m before impact were respectively

$$k^2N\sqrt{a_0(1-e_0^2)} \text{ and } k^2m\sqrt{a_0(1-e_0^2)} \ .$$

After impact the moment of momentum of the combined mass was

$$k^2(N+m)\sqrt{a(1-e^2)} \ .$$

Since the moment of momentum of a system is not changed by collisions, we have

$$k^2N\sqrt{a_0(1-e_0^2)}+k^2m\sqrt{a_0(1-e_0^2)}=k^2(N+m)\sqrt{a(1-e^2)} \ .$$

Making use of the inequality (3), it follows that

$$e < e_0 \ . \tag{4}$$

That is, in this most important case the eccentricity of the nucleus was always decreased, in whatever manner the collision may have taken place. When the postulated conditions were anywhere nearly fulfilled, an overwhelming majority of collisions operated in the same way, though there were cases where the eccentricity was increased. The conclusion is that the more a planet grew by accretion, the more nearly circular, in general, its orbit became.

Let us compare this with the solar system. The orbits of the terrestrial planets average more than twice as eccentric as those of the great planets. Being nearer the Sun, where friction would have destroyed most irregularities, the ring theory would demand that their orbits should be more circular. The smallest planet, *Mercury*, has an orbit more than twice as eccentric as that of any other. The orbits of the planetoids are, on the average, three times as eccentric as those of the planets, and about one planetoid in four has an orbit more eccentric than that of *Mercury*.

7. *The rotation of the planets.*[1]—There is no reason to assume that the nuclei were originally rotating in any particular direction, and very probably they rotated in various directions. The present rotation of a planet depends upon the original rotation of its nucleus, upon the effects of S' on this nucleus, and upon the effects of the impacts of the scattered material. So far as S' influenced the rotation by the tides it generated, it tended to make it forward. The effects of the impacts will be considered, neglecting for simplicity the eccentricity of the orbit of the nucleus.

Suppose the nucleus N traveled around the Sun so as to fill the space between the curves a and b, and so that its center traveled along the circle c. The orbits of the small masses, m, which entered the path of N will be divided into three classes: (1) those orbits whose perihelia were inside of b and whose aphelia were between b and c; (2) those orbits whose perihelia were inside of c and whose aphelia were outside of c; (3) and those orbits whose perihelia were

[1] This idea was first developed briefly in approximately the present connection by Chamberlin, "A Group of Hypotheses Bearing on Climatic Changes," *Journal of Geology*, **5**, 653, 1897, footnote, pp. 668–669.

PLATE V

SPIRAL NEBULA, *M* 51 *CANUM VENATICORUM*

Photographed with Two-foot Reflector of Yerkes Observatory by G. W. Ritchey, 1902

between c and a and whose aphelia were outside of a. They are represented by the ellipses e_1, e_2, e_3, respectively in Fig. 3.

Case of e_1.—Since the major axis of the orbit of N was greater than that of m, it follows from equation (2) that at the time of their collision N overtook m. It is easily seen from Fig. 3 that this collision tended to give N a forward rotation.

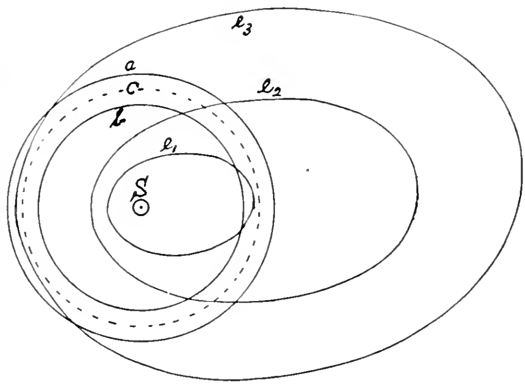


FIG. 3

Case of e_2.—In this case N and m were moving with nearly equal velocities, and it follows that the blow on N was largely radial. Therefore the effect of any single impact was small, and the combined effects of many, which tended to give rotations in various directions, cannot have been important.

Case of e_3.—In this case m overtook N outside of its center and tended to give it a forward rotation.

Thus, in the two cases where the impacts were most effective in changing the rotation, N was given a forward rotation. Therefore we should expect to find the planets rotating forward approximately in the planes of their orbits. The inclinations of the planes of their equators to the planes of their orbits might differ, because of differences in the original rotations of the nuclei, from planet to planet as in the case of *Saturn* and *Jupiter*. If there were any marked exception to the rule, we should expect to find it in the outermost planets where the particles moving in the orbits of the third type would be few. Moreover, the larger a nucleus was, the greater were the relative velocities of impact, the greater was the momentum contributed for a given mass and velocity, and therefore the larger was the amount of the moment of momentum contributed to N. For this reason, as well as for the fact that the larger nuclei probably acquired relatively larger amounts of scattered material than the smaller nuclei, we should expect to find the larger planets rotating more rapidly than the smaller.

8. *The satellites.*—When the planetary nuclei left the Sun, they were attended by many smaller secondary nuclei. When the velocity of the secondary nucleus was very small relatively to its primary nucleus, it was attracted into the larger mass and lost its separate existence. When the velocity of the secondary nucleus was large compared to its primary nucleus, it passed away from its gravitative control and began a career as an independent body. In all other cases the secondary nuclei revolved around the primary nuclei, and there is no reason for supposing they had originally any general direction of revolution.

For the purposes of the discussion, these secondary nuclei will be divided into three classes depending upon the positions of the planes of their orbits and their directions of revolution. The first class will consist of those whose orbits were highly inclined to the planes of motion of their respective primary nuclei; the second class, of those which revolved nearly in the planes of motion of the primary nuclei, but in the forward direction; and the third, of those whose revolution was nearly in the planes of motion of the primary nuclei and retrograde.

Secondary nuclei of the first class.—The problem for consideration is the effect of the impact of the scattered material with the secondary nuclei.

There is not room to go into details here, but it is easy to see that whenever the motion of the secondary nucleus was perpendicular to the plane of the general system, the scattered material acted upon it like a resisting medium. The result was that the size of the orbit was diminished. When its mass had become doubled by the accretion of the scattered material, its orbit had shrunk, by this cause alone, to one-fourth of its former dimension. In addition to this, the increase in mass of the planetary nucleus still further reduced its orbit. Consequently the secondary nuclei of the first class were in general precipitated on their respective primary nuclei. Only those have survived which were originally very remote, but not beyond the gravitative control of the primary, and which developed under the exceptional conditions prevailing near the borders of the system.

Secondary nuclei of the second class.—It can be shown, by a dis-

cussion similar to that given in section 7, that in this case the collisions tended to increase the dimensions of the orbits of secondary nuclei, and that, therefore, some of these bodies have maintained their separate existence. The increase of velocity was greatest when the secondary nuclei were remotest from their respective planetary nuclei. The principles of celestial mechanics show that this has made their orbits continually more nearly circular.

Secondary nuclei of the third class.—It can be shown, by a discussion similar to that given in section 7, that in this case the scattered material acted as a resisting medium. Moreover, up to a certain point the eccentricities were increased. Consequently most of those secondary nuclei which moved in the retrograde direction were precipitated upon the primary nuclei around which they revolved.

In a system having both direct and retrograde satellites it is to be expected, on the basis of this theory, that those moving in a retrograde direction will be very remote and have considerable eccentricities. This is the case with the ninth satellite of *Saturn*, whose orbit is about twice as eccentric as that of any other satellite.[1] The peculiar positions of the orbits of the satellites of *Uranus* seem to indicate that in this case the original secondary nuclei were moving in a special way. While the conditions prevailing in the satellite system of *Uranus* would not have been predicted on the basis of this theory, yet they do not definitely contradict it as they do the ring theory. The large planetary nuclei had gravitative control over secondary nuclei with large relative velocities, and for this reason the chances for their having several satellites are more favorable than in the case of small planets.

There is no reason why a satellite may not revolve more rapidly than its primary rotates, especially as its period continually diminishes as the mass of the planet increases.

9. *The planetoids.*—The planetoids have formed from material whose orbits did not cross a region swept by a large nucleus. Their ability to grow has been small because of their small dimensions and feeble gravitating power. The variations of the eccentricities

[1] According to the latest data, received since this statement was in print, the orbit of the seventh satellite of *Jupiter* seems to be retrograde and very eccentric.

and inclinations of their orbits are a measure of the heterogeneity prevailing after the passage of S' and before the equalizing effects of collisions had been realized. *Eros* is the one of considerable dimensions between the orbit of the Earth and *Mars*, whose orbit was so highly inclined that it did not lose its separate existence by collision with one of these larger bodies.

10. *The physical condition of the planets.*—The original nuclei which have grown into the terrestrial planets were of such small mass that they did not possess sufficient gravitative power to control true atmospheres. Consequently they cooled rapidly and became solid. The scattered material likewise lost its heat rapidly. Hence these planets have grown up from solid material, and have been solid throughout nearly their whole history since the visit of S', and their atmospheres have been acquired in a later stage of their evolution from the occluded gases which escaped as they contracted. The present internal heat and the past igneous action, of which there is abundant evidence in the Earth, are due partly to the residue of original heat which was not lost by radiation, and much more to the heat generated by the contraction of the planets from the density of the material when it fell on them to their present density. The writer has shown the quantitative effectiveness of such moderate contractions in bodies having the dimensions of the planets.[1]

The original nuclei which have grown into the great planets were large enough to retain true atmospheres. Consequently their original heat has been much more largely retained. For the same reason it is probable that they contain a larger proportion of the light volatile substances than the smaller planets do. The larger nuclei attracted the scattered material upon themselves with considerable velocities, and the heat generated by the impacts was very great. In the case of the small planets this surface heat was rapidly radiated away, but the atmospheres of the great planets retained it largely, and for this reason their original fluid state has been immensely prolonged.

11. *The moment of momentum of the system.*—It follows from the origin of the system that the remote planets should possess most of the moment of momentum of the system. This is precisely what is

[1] Chamberlin, "Hypotheses Bearing on Climatic Changes," *Journal of Geology*, **5**, 674, 1897, and Moulton, *Introduction to Celestial Mechanics*, p. 61, problem 5.

found. As Professor Chamberlin has pointed out, *Jupiter* contains about one-tenth of 1 per cent. of the mass within the orbit of *Saturn*, and more than 95 per cent. of the moment of momentum. This is an inevitable consequence of the spiral theory, but, on the contrary, the whole question of moment of momentum is a rock on which the ring theory breaks.[1]

12. *The ninth satellite of Saturn.*—The retrograde revolution of the ninth satellite of *Saturn*, taken in connection with the forward revolution of the other satellites and the direction of rotation of the planet, contradicts what has been considered up to the present time to be an inevitable consequence of the ring theory.[2] Nevertheless, the discoverer, Professor W. H. Pickering, has attempted to explain how it may have originated without contradicting the ring theory.[3] This section is devoted to an examination of these explanations.

Pickering's first suggestion is that *Phoebe* may have been originally a comet which has been captured by *Saturn*. It can scarcely be possible that the eccentricity of *Saturn's* orbit, or that the perturbations of *Saturn* by *Phoebe*, would assist in the supposed capture. Therefore we shall neglect in this discussion the eccentricity of the orbit of *Saturn* and the mass of *Phoebe*. This reduces the problem to that treated in the writer's *Introduction to Celestial Mechanics*, Chapter VII. It is shown there that if the constant of the Jacobian integral is sufficiently large, the surfaces of zero relative velocity are closed around the finite bodies, and consequently that a satellite or inferior planet in this case must remain permanently a satellite or inferior planet.[4] By the methods explained there it is found that if the constant is greater than 3.0180, the surface around *Saturn* is closed. The constant that belongs to the ninth satellite is found to

[1] Chamberlin, "An Attempt to Test the Nebular Hypothesis by the Relations of Masses and Momenta," *Journal of Geology*, **8**, 58–73, 1900. Moulton, "An Attempt to Test the Nebular Hypothesis by an Appeal to the Laws of Dynamics," *Astrophysical Journal*, **11**, 128, 1900.

[2] The observations at present seem to indicate that the sixth satellite of *Jupiter* revolves in the forward direction, and the seventh retrogrades at just about the same distance.

[3] *Annals of the Harvard College Observatory*, **53**, No. III, pp. 60–61.

[4] Hill used this method in connection with his special differential equations to prove the existence of a superior limit to the Moon's radius vector. *American Journal of Mathematics*, Vol. I.

have the value 3.0626. Consequently it can never escape from *Saturn's* control, and conversely it has never come under *Saturn's* control from a remote region. The radius of *Saturn's* largest closed surface of zero relative velocity is only about 40,000,000 miles, and it follows that *Jupiter* cannot have assisted in making the capture.

The other suggestion, which evidently was considered much more probable, is that *Saturn* once extended out to the orbit of *Phoebe*, and rotated in the retrograde direction with the angular velocity of the orbital motion of this satellite. The Sun is supposed to have raised large tides in this widely extended mass. Now, tides are generated on the side toward and the side opposite the tide-raising body, and it follows that the viscosity of the mass would in time reduce the rotation so that one side would be constantly toward the Sun. That is, tidal friction would in time give it a forward rotation with the angular velocity of the planet's motion around the Sun. As the planetary mass contracted, it would rotate more rapidly in the forward direction, and the interior nine satellites and the rings are supposed to have been left behind in this stage.

Let us examine the question quantitatively. Since the moment of momentum is a signed quantity, we may suppose that it is negative in the case of a retrograde rotation, and positive in the case of a direct rotation. Then, according to the explanation suggested by Pickering, the Saturnian nebula originally had a negative moment of momentum. The tides raised by the Sun destroyed this moment of momentum, and contributed positive moment of momentum until the rotation and the revolution of the mass were made in the same period. But when the rotation in the forward direction became faster than the revolution as the mass contracted, then the tides acted in the opposite direction and decreased the moment of momentum. Consequently, *the Saturnian system had its maximum moment of momentum with respect to the center of Saturn when its rotation was forward, with a period exactly equaling the period of Saturn's revolution around the Sun.* When the mass had shrunk down to the size of the orbit of *Japetus*, it must have rotated at the rate *Japetus* now moves, and we must find, if the theory is true, that its moment of momentum had been decreased by the retarding effects of the tides raised by the Sun.

Let us first compute an upper limit to the maximum moment of

momentum. We do not know the dimensions of the mass, except that it was somewhere between the orbits of *Phoebe* and *Japetus*, when it had its maximum. But the greater its radius was, the greater was its maximum moment of momentum. We shall certainly get something *too large*, if we suppose that this condition was realized when it extended entirely out to the orbit of *Phoebe*. We do not know the law of density, but the mass was certainly densest at its center. If we assume that it was homogeneous, we shall certainly get *too large* a result. Since the mass rotated on its axis but once in 29.5 years, it cannot have been sensibly flattened at its poles. Hence we shall certainly have an upper limit to the maximum moment of momentum, if we compute it under the assumptions that the mass extended out to the orbit of *Phoebe* at this time, and that it was homogeneous.

Consider the case when the mass had shrunk to the dimensions of the orbit of *Japetus*. It rotated in the period of revolution of this satellite, and may have been considerably flattened at the poles. If we assume that it was not flattened, we shall get a result which is *certainly too small*. If we assume that it is homogeneous, we shall get a result which is, so far as this factor is concerned, too large. We shall undoubtedly be nearer the truth if we assume that it obeyed the Laplacian law of density, and that its surface density was very small compared with the mean density.

With the assumptions made, the formula for the upper limit of the maximum moment of momentum is

$$M_M = \frac{2}{5} m R_P^2 \omega_S \ ,$$

where m is the mass of the Saturnian system inside of the orbit of *Phoebe*, R_P the mean radius of the orbit of *Phoebe*, and ω_S the mean angular velocity of *Saturn* around the Sun. The moment of momentum at the time the mass extended to the orbit of *Japetus* was less than, or perhaps about equal to,

$$M_J = \frac{2}{3}\left[1 - \frac{6}{\pi^2}\right] m R_J^2 \omega_J = 0.2614\, m R_J^2 \omega_J \ ,$$

where R_J represents the radius of the orbit of *Japetus* and ω_J is its angular rate of revolution.

Hence we find for the ratio of the upper limit of the maximum moment of momentum to that at the time the mass extended to the orbit of *Japetus*,

$$\frac{M_M}{M_J}=\frac{0.4R_P^2\omega_S}{0.2614R_J^2\omega_J}=\frac{1}{0.6535}\left(\frac{R_P}{R_J}\right)^2\left(\frac{T_J}{T_S}\right),$$

where T_J and T_S are the periods of *Japetus* around *Saturn* and *Saturn* around the Sun respectively. It is known that

$$R_J=2{,}225{,}000 \text{ miles},$$
$$T_J=79.3 \text{ days},$$
$$T_S=29.46\times 365.25 \text{ days},$$

and from Pickering's elements of the orbit of *Phoebe* that

$$R_P=7{,}996{,}000 \text{ miles}.$$

Consequently

$$\frac{M_M}{M_J}=0.1456\ .$$

That is, we have found that the upper limit to the maximum moment of momentum is only one-seventh of something certainly less than the maximum. The only explanation of this contradiction is that the supposed development of the Saturnian system upon which the computation was based is erroneous. It follows that this retrograde revolution of *Phoebe* is actually, as well as apparently, squarely contradictory to the ring theory.

13. *Conclusions.*—While only abstracts of a portion of the discussions have been made in this paper,[1] enough has been said to show that the spiral theory is even now a good working hypothesis. It explains all the phenomena upon which the ring theory rested, and many others which are contradictory to the ring theory. Nothing has yet been found which seems seriously to question its validity.

The spiral theory raises a whole series of new and very difficult questions in celestial mechanics. These are the immediate effects of the tidal forces which are developed by the near approach of two suns, the perturbations of the orbits of matter which has been ejected by one of them under a variety of conditions, and the secular evolution of the orbits of this ejected material. A large amount of labor

[1] More details are given in the second volume of the *Geology* by Chamberlin and Salisbury, and in the writer's *Introduction to Astronomy*, both of which are to appear soon.

will be required to carry the discussion of these questions to a successful conclusion.

The spiral theory is fertile in suggesting new considerations for interpreting the immense variety of special phenomena of the system. It is not too much to expect that it may suggest new questions for observational investigation. It affords geologists new conceptions of the early history of the Earth. But perhaps its most interesting contribution is to our general philosophy of nature. Heretofore we have regarded the cosmical processes as forever aggregating matter into larger and still larger bodies, and dissipating energy more and more uniformly. Now we recognize important tendencies for the dispersion of matter.[1] This idea has introduced an element of possible cyclical character in the evolution of the heavenly bodies, though the question of the source of the requisite energy is serious.[2] There is hope that the difficulties of this question may soon be relieved, for recent discoveries respecting the internal energies of atoms suggest the possibility that the Helmholtzian contraction theory explains the origin of only a part of the energy given up by the stars.

UNIVERSITY OF CHICAGO,
August 30, 1905,

[1] This idea was first sharply worked out in Chamberlin's paper "On a Possible Function of Disruptive Approach in the Formation of Meteorites, Comets, and Nebulae," *Astrophysical Journal*, **14**, 17–40, 1901. The phenomena of radioactivity point in the same direction, though in quite a different way.

[2] This trouble is equally serious under the ring theory. See Chamberlin's "On Lord Kelvin's Address on the Age of the Earth as an Abode Fitted for Life," *Smithsonian Report* for 1899, p. 240.

VARIABILITY OF WAVE-LENGTH IN THE LINES OF SPARK SPECTRA

By NORTON A. KENT

HISTORICAL SURVEY AND STATEMENT OF THE OBJECT OF THE INVESTIGATION

In 1901 Haschek[1] stated that the lines of the spark spectrum of titanium exhibit considerable shift as compared with the same lines in the arc, a shift in one instance as great as 0.13 tenth-meter.

In 1903 the writer of the present paper made a careful experimental investigation of the subject.[2] No such large shifts were discovered; but it was shown that in all probability the wave-lengths of certain lines of the titanium spark spectrum are sensitive to various conditions of the electric circuit, such, for instance, as the amount of capacity, self-induction, and ohmic resistance employed, and the presence or absence of an auxiliary spark-gap in series with that under investigation.

The instrument used was a 10-foot Rowland grating. It seemed advisable to use greater dispersion. Hence the present work was undertaken with a 21-foot grating. During its progress, Eder and Valenta,[3] using a 15-foot instrument, have published a paper in which they state that in the case of neither pure zinc nor brass wire do the lines studied by Haschek show any shift. They attribute the results of these investigators to a photographic displacement of the center of gravity of the lines due to overexposure—the lines in question being those which exhibit very strongly the phenomenon of unsymmetrical broadening; and they suggest also the possibility of a pseudo-shift due to poor focus. The conclusion drawn by Eder and Valenta is that, at minimum photographic exposure, the coincidence of arc and spark lines is perfect. Moreover, Middlekauff[4] has lately studied the iron spectrum and finds no shifts in the region investigated. The focal length of his grating was 21.5 feet.

Such an inconsistency in the results of different investigators

[1] *Astrophysical Journal*, **14**, 181, 1901.

[2] *Ibid.*, **17**, 286, 1903.

[3] *Ibid.*, **19**, 251, 1904.

[4] *Ibid.*, **21**, 116, 1905.

emphasizes the importance of taking extreme care in all the details of the experiment, guarding against any possible false shifts, and stating with exactness the conditions under which the investigation is made.

The purpose of the present series of experiments is, then, to attempt to settle definitely the question of the coincidence or non-coincidence of arc and spark lines, to discover the determining conditions in case shifts do exist, and to harmonize, if possible, the results obtained by previous investigators.

EXPERIMENTAL CONDITIONS; CONSTRUCTION AND ADJUSTMENT OF THE GRATING-MOUNT

The conditions under which the present investigation has been carried out are as follows: The science hall of the college is a substantial building located not less than seventy-five yards from a roadway upon which there is heavy traffic; the foundations of the building are laid solidly upon glacial drift; and the room in which the spectroscope is mounted is situated in the cellar. The grating is a 21-foot Rowland concave, of 15,000 lines to the inch, having a ruled surface of 6 by $2\frac{1}{2}$ inches. The grating-holder is of brass, and is solidly built. The entire mount is of steel or iron. Seven-inch channel beams form two sides of the triangle; the third is a beam of 5 inches. The tracks are of $2\frac{3}{4}$ by $\frac{1}{2}$-inch rolled steel. The carriages are massive; the truss-rod connecting the two is of $2\frac{1}{4}$-inch piping; and three draw-bars conduce to great rigidity. The camera box is of iron and brass, designed by the author, and is especially massive. The photographic plate is held in its place by brass screws. The slit is connected rigidly to the channel beams, and the whole mount is supported by rubber disks on three substantial brick piers. After adjusting the instrument for any definite region of the spectrum, the bases of the grating-holder and camera box were always clamped to the steel tracks by nine iron clamps of various sizes.

The camera shutter, being supported from the floor, is entirely free from the camera box, and is so arranged that there is absolutely no chance of mechanical jar from the operation of changing the shutter between exposures. The slit mechanism is surrounded by cardboard cylinders, which are held free from the slit by supports

connected rigidly to the walls of the room. Around these cylinders is tied the black cloth which separates the dark spectrometer room from the light room in which are placed the spark and arc. The entire mount, including channel beams, steel tracks, trucks, truss-rod, draw-bars, and all possible parts of grating and camera box, is carefully wrapped in several layers of paper or cloth to facilitate obtaining constant temperature conditions. The steam pipes in the room are protected thoroughly by asbestos covering or several layers of paper.

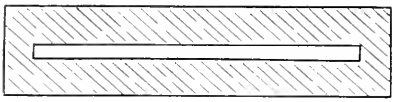


Fig. A

In adjusting the instrument the slit and grating rulings were made parallel by the use of a plumb bob. The coincidence of the normals to the grating and to the plane of the camera box was determined by using an illuminated slit 1 mm wide and adjusting until the image was horizontally coincident with the object. The ordinary method of superposing the image of a candle flame upon the flame itself was not regarded as sufficiently delicate.

Fig. B

DURATION AND METHOD OF EXPOSURE

In general the exposures were short, ranging from three to ten minutes, and all the quantitative work of the investigation was carried on when temperature conditions were most favorable, the change in temperature as given by one or more thermometers whose bulbs were placed underneath the cover of the mount usually being absolutely unreadable, and never more than a half of a tenth of a degree Centigrade. Moreover, the method of exposure was such that pseudo-shifts, due either to mechanical jar or temperature change, must have been eliminated. The camera shutter was arranged as shown in the accompanying cuts. Usually the arc was exposed first, its spectrum being placed along the center of the photographic plate—this by use of the slit as given in Fig. A; then the spark image was thrown upon the slit of the spectroscope and its spectrum placed on the edges of the plate—this by the use of the slits shown in Fig. B; then

Fig. C

the first operation was repeated, and the arc spectrum again thrown upon the center. But in most cases, always during the latter part of the investigation, the shutter was so arranged that the vertical superposition of the first and third exposures was not perfect, so that the central portion of the plate containing the arc spectrum presents some such appearance as that shown in Fig. C. A mechanical shift may thus be easily detected.

PRODUCTION OF THE SOURCES OF LIGHT

The spark was produced in the following manner: The city current, of 110 volts and 133 cycles per second, was led into a 5-kilowatt transformer (110 to 15,000, 30,000 or 60,000 volts) manufactured by the Central Laboratory Supply Co., of Lafayette, Ind., in series with which was a home-made water rheostat. The secondary voltage ranged from approximately 1600 to 14,000 volts, as the ratio of transformation which was employed was 30,000 to 110. The high potential current was led into glass-paraffin-tin condensers giving a range of capacity varying from about 0.008 to 0.08 microfarads (see Table I). Both the charging and discharging high potential wires were very heavy (about No. 8 standard wire gauge).

TABLE I

Capacity of Condensers

No. of Condenser	Approximate Area of One Element in cm^2	Mean Thickness of Glass in cm	No. of Elements	Total Capacity in Microfarads
1	900	0.245	4	0.0076
2	900	0.249	12	0.0226
3	900	0.241	10	0.0195
4	730	0.246	20	0.0315
5	730	0.479	8	0.0067

The self-induction coil employed consisted of three concentric cylindrical coils of dimensions given below (see Table II). Generally, when photographing the spark, there was placed in the rear of it, at a distance of 80 cm, a 12-inch Westinghouse fan, the strong air current from which was directed upon the spark-gap by a megaphone 70 cm long, the smaller opening (4 cm in diameter) being about 2 cm distant from the spark. When a secondary spark was used in series with that under investigation, the side draft from the fan was allowed

to play upon it also. The fan undoubtedly produces greater disruptiveness and increases the potential of the discharge; but its steadying influence is very desirable, especially when self-induction is used in the discharge circuit. The luminous image was formed on the slit by a glass lens (aperture 3 inches, focal length 25 inches, made by Bausch & Lomb) situated about 110 cm from the source of light and 152 cm from the slit. This lens was held in a metal casing clamped to a marble slab which was set on an iron pipe imbedded in a cement foundation.

TABLE II

DIMENSIONS OF SELF-INDUCTION COIL IN CM. GAUGE NUMBER OF WIRE = 14 S.W.G.; EXTERNAL DIAMETER WITH INSULATION = 0.54 CM

Coil	Length	External Diameter	Total No. of Turns
Inner	66.0	5.22	124
Central	65.3	8.59	122
Outer	64.7	9.95	122

In all cases the spark which was being photographed was placed in a horizontal position, in view of the possibility that different regions of the spark might show different spectra.

In the primary circuit of the transformer were placed a Thomson indicating watt-meter of maximum voltage 150 volts and maximum current 50 amperes, graduated in hectowatts, and a Thomson alternating ammeter of maximum current 100 amperes. The primary voltage was read by a Weston direct reading, alternating and direct current voltmeter, a two-scale instrument, 15 and 7½ volts, with 1 to 10 multiplier.

The arc was developed by a 133-cycle alternating circuit, the current used being about 12 amperes, measured by a 50-ampere Ayrton and Perry spring ammeter. The discharge took place between one-half inch carbons, the lower one of which was of the cored type and was hollowed out a trifle to receive the small pieces of metal used. The distance between the terminals was always in the neighborhood of 3 mm. That the energy for the spark and arc discharges could be obtained from the city circuit was a distinct advantage, for under these conditions there was no possibility of mechanical vibration due to the presence of heavy electrical machinery.

REDUCTION OF ASTIGMATISM, METHOD OF MEASURING PLATES, ETC.

The investigation was confined to the second spectrum, the dispersion being one Ångström unit to 0.75 mm. The length of the rulings of the grating surface was diminished by reducing the vertical aperture to about 2 cm (using black paper for the purpose); this process, together with stopping down the length of the slit to 5 or 6 mm, so much reduced the astigmatism that the tips of the spectral lines could be seen in the field of the micrometer used in measurement. We thus avoid a shift due to overexposure, for, by setting the cross-hair upon the tip of the spark image, where it is just visible to the eye, we are certainly making use of minimum exposure conditions.

The filar micrometer, manufactured by Bausch & Lomb, was reinforced by adjustable trusses to produce greater rigidity. A test of the accuracy of the instrument showed that the errors of measurement were greater than those of the screw. The magnification employed was about seven diameters, and a single cross-hair was found to give the best results. The plates were placed violet left, and four settings were made on the external or spark spectrum, then two upon the internal or arc spectrum; then the same set of measurements was repeated with the violet to the right. The plates used were the regular Seed "Gilt Edge" 27, 5×7, cut into strips $1\frac{1}{4}\times7$ inches.

The developer generally used was that described by Wallace,[1] as giving with the Seed 27 plate a definite and regular silver grain deposit.

Several plates were developed with Jewell's solution,[2] to which was added an excess of bromide (roughly 0.5 gram of potassium bromide to a 3 oz. solution).

[1] *Astrophysical Journal*, **20**, 118, 1904. The formula was kindly sent the author by Mr. Wallace, and is as follows:

A.	Grams	B.	Grams
Hydrochinon	1	KBr	1
Metol	1	$NaOH$	12
Adurol	2	H_2O	175
Na_2SO_3 (crystals)	24		
H_2O	175		

Take equal parts of A and B.

[2]

A.	Grams	B.	Grams	C.	Grams
Hydrochinon	30	Potas. ferrocyanide	30	K_2CO_3	30
Na_2SO_3 (crystals)	150	H_2O	180	H_2O	180
H_2O	750				
Alcohol	10 cc.				

Take one part of B, one of C, and six of A. Normal amount of bromide, 15 drops (10 per cent. solution) to 100 cu. cm of developer.

RESULTS

The results are contained in the following tables, concerning the data given in which a few general remarks may be made:

The time of the two arc exposures varied from 10 to 60 seconds, the short exposures predominating.

The length of the slit, unless otherwise stated, was either 5 or 6 mm, its width between 0.025 and 0.050 mm.

The arc spectrum was generally thrown on the center of the plate.

The values of current, power, and voltage given in the tables are the means of from two to six (average three) readings taken at fairly regular intervals during the exposures. The conditions of the circuit were quite stable, the three elements seldom varying more than 1 ampere, 20 watts, and 0.3 volts respectively.

The metals used were: titanium carbide or "cast titanium," made by Eimer & Amend of New York, 85 per cent. titanium, 15 per cent. carbon; a titanium-iron alloy, 20 per cent. titanium, 80 per cent. iron (Eimer & Amend); zinc battery rods; brass wire, 30.2 per cent. zinc, 69.8 per cent. copper; ordinary iron rod.

Unless otherwise stated, the part of the spark image used was that near one terminal, and the fan was employed to steady the discharge.

The secondary spark took place between brass rods 4 mm in diameter.

The focus, to an accuracy of 1 mm, was determined repeatedly from week to week as the temperature of the room changed. This was a precautionary measure: no change in focus was noticed.

The given values of the shifts represent generally but one set of measurements on each plate, never more than two. This course was pursued because the different plates agreed so well that numerous measurements upon the same plate were deemed unnecessary. The average deviation from the mean of the measurements on the different plates in all the sets was about 0.0027 tenth-meters for titanium and iron, and 0.0070 for zinc. The average deviation from the mean of different measurements of a line on any one plate was about 0.0025 tenth-meters for titanium, 0.0018 for iron, and 0.0060 for zinc.

To gain in another manner an idea of the degree of accuracy of

PLATE VI

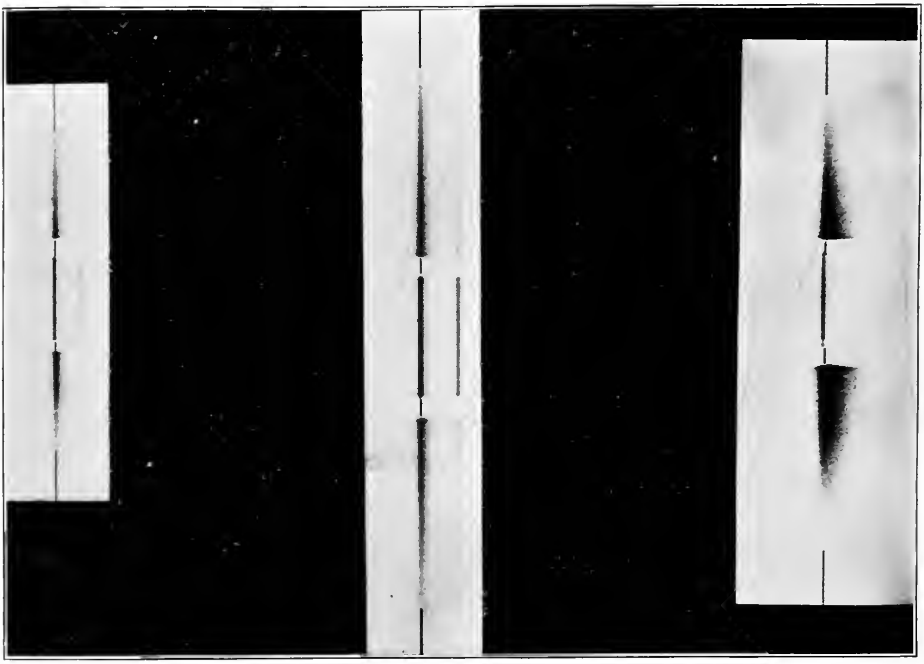


FIG. 1 FIG. 2 FIG. 3

FIG. 1.—Negative of *Ti* λ 3913.58 as apparent on plate No. 22. Mean shift as given by plates Nos. 22 to 25, 0.031 t.-m. Scale of enlargement about 5 : 1.

FIG. 2.—Negative of *Ti* λ 3913.58 as apparent on plate No. 27. Influence of self-induction shown. Mean shift as given by plates Nos. 26 and 27, 0.005 t.-m. Scale of enlargement about 7 : 1.

FIG. 3.—Negative of *Zn* λ 4722.26 as apparent on plate No. 47. Mean shift as given by plates Nos. 47, 48, and 43, 0.065 t.-m. Scale of enlargement about 5.5 : 1.

measurement, four settings, two red-right and two red-left, were made[1] on each of three lines which appeared on a focus plate: *Ti*, λλ 3898.645, 3901.114, and 3904.926, as given in Rowland's table of solar wave-lengths. The first line only was reversed; all were somewhat too dense for very accurate settings. Assuming the values of the wave-lengths of the first and third lines as given, that of the second was calculated. The first determination gave 3901.112, the second 3901.114; the mean being 3901.113, but 0.001 tenth-meter different from the wave-length as given above.

The spark lines which were studied most thoroughly—namely, *Ti*, λλ 3900 and 3913; *Zn*, λλ 4680, 4722, and 4810—generally appear enhanced, diffuse, and unsymmetrically broadened. Many iron lines also show unsymmetrical broadening to some extent.

Figures 1, 2, and 3 of the accompanying plate are illustrative of the lines studied. The scratches, appearing at the tips of the spark and between the arc and spark segments, were drawn by pricking two holes in the film of a positive of an original negative at points which seemed to lie in the "center of gravity" of the very tips of the spark lines. These two points were then joined by a ruler and the film carefully scratched away along its edge in the four regions above mentioned. A negative of the positive was then made.

The arc line lies considerably to the left, or violet, in all cases but that of Fig. 2, which is a "self-induction line." It will be noticed that the double arc exposure sketched in Fig. C appears only in Fig. 3 of the plate. As stated previously, this non-superposition method was not used at the very first of the investigation.

Let us discuss first the results as given by the titanium plates in Table III:

1. The members of the set of six plates, set 1 (see next to last column of the table), agree in showing that with large capacity shifts exist (about 0.023 tenth-meters), even in the absence of a secondary gap.

2. Set 2 (5 plates) shows that, with a secondary gap of 4 mm and lessened capacity, considerable shifts exist (about 0.037 tenth-meters) The effect of a secondary gap is to increase the potential of the discharge, and this gives the same result as an increase of capacity.

[1] A suggestion due to Professor Frost.

TABLE III

SHIFT OF SPARK LINE TOWARD RED IN

No. of Plate	Date	Length of Spark Exposure in Minutes	Constants of Electric Circuit: Primary: Amps.	Primary: Watts	Primary: Volts	Secondary: Capacity in Microfarads	Secondary: Length of Gap in mm: Primary	Secondary: Length of Gap in mm: Sec'dary	Secondary: Part of Image Used	Character of Lines S=Sharp D=Diffuse
6	Dec. 15....	15	40.6	193	6.8	0.0812	1.0	0	End	D
7	" 15 ...	15	41.1	180	7.0	0.0812				
8	" 16....	15	40.4	188	6.5	0.0812				
9	" 16....	22	40.9	172	5.8	0.0812				
10	" 17....	26	39.7	188	6.3	0.0812				
28	" 17....	15	40.0	160	7.2	0.0803				
12	" 17....	5	39.0	390	12.0	0.0195	1.5	4.0	End	D
13			37.8	390	12.5		2.0			
14			38.3	400	13.0		1.7			
15			38.5	380	11.3		2.1			
16			38.5	380	12.2		2.5			
22	Jan. 14....	6	35.5	1010	46.0	0.0067	2.0	31	End	D
23	" 17....	5	34.1	910	37.0		1.0	27		
24	" 17....	6	34.4	910	36.0		2.0	27		
25	" 17....	5	34.0	883	38.3		2.0	27		
26	" 17....	5	35.0	717	35.0	0.0067	2.0	27	End	S
27	" 21....	7	33.6	775	41.0			25		
70	April 15....	5	41.5	415	11.6	0.0226	2.0	5	End	D
72	" 15....	3	42.4	400	11.3					
93	May 5....	8	41.4	395	10.9					
99	" 5....	11	41.2	400	11.2					
77	April 15....	5	41.7	397	11.1	0.0226	2.0	5	End	S
78	" 15....	9	41.2	420	11.3					
73	" 15....	4	41.5	440	12.0	0.0226	9.0	0	End	D
74	" 15....	3	41.1	410	11.5					
97	May 6....	9	39.8	417	11.2					
101	" 6....	10	41.2	410	11.5					
102	" 6....	10	41.0	427	11.5					
75	April 15....	4	41.6	425	11.7	0.0226	9.0	0	Center	D
76	" 15....	3	41.5	435	11.5					
96	May 5....	6	41.5	425	11.4					
98	" 6....	9	41.0	425	11.1					
80	April 29....	10	41.3	297	9.1	0.0226	4.0	0	End	S
81	" 29....	15	41.9	313	9.0					
82	" 29....	15	41.9	313	9.0					
83	" 29....	5	42.1	320	8.9	0.0226	4.0	0	End	D
84	" 29....	4	41.5	315	9.0					
87	May 2....	8	41.8	300	9.2					
88	" 2....	7	42.0	300	9.1					
103	" 10....	10	40.7	307	8.7					
104	" 10....	10	40.2	300	8.5					
105	" 10....	8	40.9	315	8.6					
106	" 10....	8	41.3	310	8.4					

TITANIUM

TENTH-METERS: POSITIVE UNLESS OTHERWISE STATED

Metal Used	Fan	Shift: λ 3000.68 Separate Measurements	Shift: λ 3000.68 Mean	Shift: λ 3013.58 Separate Measurements	Shift: λ 3013.58 Mean	No. of Set	Remarks
85% *Ti* 15% *C*	Yes	0.036* 25 21 18* 21 18	*0.023*	0.031 24 20 22 24 22	*0.024*	1	Plate No. 8, spark inside; arc outside
85% *Ti* 15% *C*	Yes	0.042 35 38 37 39	*0.038*	0.048 30 39 31 34	*0.036*	2	Plate No. 16, spark inside; arc outside
85% *Ti* 15% *C*	Yes	0.045 36 26 32	*0.035*	0.041 25 28 30	*0.031*	3	Plate No. 25, spark inside; arc outside
85% *Ti* 15% *C*	Yes	0.002 8	*0.005*	0.004 7	*0.005*	4	Triple coil of self-induction used
85% *Ti* 15% *C*	Yes	0.027* 15* 21 17	*0.020*	0.026* 23 16* 24	*0.022*	5	Developed with Jewell solution
20% *Ti* 80% *Fe*	Yes	0.001 4	*0.002*	0.003	*0.003*	6	
85% *Ti* 15% *C*	No	0.020 14* 22 21 17	*0.019*	0.017* 17* 19 23 15	*0.018*	7	Developed with Jewell solution
85% *Ti* 15% *C*	No	0.006 6 6 7	*0.006*	0.003 6 5 7	*0.005*	8	Developed with Jewell solution
85% *Ti* 15% *C*	No	—0.001 — 7 10	*0.001*	—0.001 — 6 3	*—0.001*	9	Two meters of fine wire, resistance 2.7 ohms, in discharge circuit of condenser
85% *Ti* 15% *C*	No	0.020* 18* 16* 14* 13 20 18 14	*0.017*	0.023* 18* 12* 15* 15 20 17 17	*0.017*	10	Developed with Jewell solution

* Mean of two measurements.

3. A condition (set 3) given by an increase of secondary gap and further reduction of capacity still presents a shift (about 0.033 tenth-meters). It may also be seen that plates Nos. 8, 16, and 25 (sets 1, 2, and 3), on which the arc spectrum was photographed outside of the spark spectrum, show no marked or consistent difference from those obtained in the ordinary manner. Care was taken that the spark exposure be short.

4. Introducing the self-induction coil into the discharge circuit of the condenser (set 4) unquestionably conduces to coincidence of arc and spark line (mean value of shift 0.005 tenth-meters).

5. Sets 5 and 6 show that the use of the alloy, 20 per cent. titanium, 80 per cent. iron, gives much less displacement (0.002 tenth-meter) than that given by titanium carbide, 85 per cent. titanium, 15 per cent. carbon (about 0.02 tenth-meter).

6. The part of the spark image used is extremely important; set 7, given by that region of the image near the terminal, shows a shift of 0.018 tenth-meter, while the central portion of the spark (set 8) 4.5 mm from the terminal shows but about 0.006 tenth-meter.

7. Sets 9 and 10 show that ohmic resistance (2.7 ohms) in the discharge circuit of the condenser produces an effect exactly similar to that of self-induction (see result No. 4), reducing the shift from about 0.017 tenth-meter to practically zero.

8. Sets 7 to 10, in which no fan was used, show that the same is not a *sine qua non* in the matter of shift.

9. Further, the developer used appears to have no effect upon the shift. Therefore in sets 5, 7, 8, and 10 the measurements made on plates developed with the Jewell solution are averaged in with the others. Indeed, it is difficult to see how there could be a difference, provided the tip of the line be used in measurement. If, however, the setting be made upon the body of the line, a difference might very easily be introduced (see below, result No. 14), a "softer" developer, such as Wallace's, tending to displace the apparent center of gravity of the line toward the side of greater broadening.

Passing to zinc, Table IV, we have:

10. Sets 11, 12 and 13 give results comparable with those of 3, 1, and 2 for titanium, and sets 14 and 15 confirm the alloy effect of 6 and 7.

TABLE IV. ZINC

Shift of Spark Line toward Red in Tenth-Meters

No. of Plate	Date	Length of Spark Exposure in Minutes	Constants of Electric Circuit: Primary: Amps.	Primary: Watts	Primary: Volts	Secondary: Capacity in Micro-farads	Secondary: Length of Gap in mm.: Pri.	Secondary: Length of Gap in mm.: Sec.	Secondary: Part of Image Used	Secondary: Shape of Terminals	Metal Used	Fan	Shift: λ 4680.38: Separate Measurements	λ 4680.38: Mean	λ 4722.26: Separate Measurements	λ 4722.26: Mean	λ 4810.71: Separate Measurements	λ 4810.71: Mean	No. of Set
31	Jan. 27	10	33.4	055	36.0		?	25					......		0.066		0.048		
33	Jan 28	10	32.5	040	38.8		2	24		Flat	*Zn*		0.019		43		30		
34	Jan. 28	15	32.8	965	41.0	0.0067	2	21	End	3×6	battery	Yes	37	*0.028*	40	*0.042*	30	*0.039*	11
36	Feb. 3	15	34.8	983	41.0		2	21		mm	rods		24		36		48		
37	Feb. 3	15	33.2	993	42.5		2	21					32		26		..		
38	Feb. 3	30	41.1	193	7.4	0.0736	1	0	End	"	"	"		*0.025*		*0.047*		*0.039*	12
41	Mar. 4	4	41.0	440	12.8	0.0195	1.5	3	End	Round 3 mm diam.	"	"	0.034	*0.033*	0.047	*0.047*	0.071	*0.065*	13
42	Mar. 4	6	40.8	445	13.2								33		48		59		
45	Mar. 11	20	41.2	343	9.6	0.0226	2	2.5	End	"	30.2% *Zn*	"	0.003	*0.003*	0.000	*0.003*	0.017	*0.012*	14
46	Mar. 11	18	41.7	350	9.7					Flat 1×3 mm	69.8% *Cu*		......		6		8		
47	Mar. 11	10	40.7	490	14.2			2	End		*Zn*		0.029*		0.059*		0.084*		
48	Mar. 11	10	40.8	463	13.8	0.0226	2	2		"	battery	"	......	*0.032*	63	*0.065*	125	*0.102*	15
43	Mar. 4	5	42.0	385	11.1			2.5			rods		35		74		97		

Set 14: Filings from zinc battery rod used in arc as in other cases. Lines sharp.
Plate No. 48 developed with Jewell solution and normal amount of bromide.
* Mean of two measurements.

In the case of iron, Table V, we find:

11. Among the lines investigated, the shifts are in general very small, but are nevertheless present.

Further facts are as follows:

12. A plate on which three exposures were made in the usual manner, the first and third being superposed, and the source of light being the titanium arc, shows that no shift is introduced by throwing a diffuse image on the slit, destroying the focus by moving the arc

TABLE V. IRON[1]

SHIFT OF SPARK LINE TOWARD RED IN TENTH-METERS

Wave-Length	Shift	Wave-Length	Shift
3688.58	0.004	3788.01	0.001
3705.10	2	95.13	3
09.37	3	99.68	3
20.07	3	3815.97	10
27.78	3	27.96	6
48.39	5	34.37	7
58.36	5	40.58	2
63.93	6	41.19	8
67.34	5		

two inches nearer the lens. This proves that even if either arc or spark were out of focus to a small extent, the shift would not be altered.

13. A plate exposed to the carbon and graphite arcs shows that no shift occurs between lines which are due to the same impurities in both. The currents used were 33 and 75 amperes respectively. The graphite arc is produced with considerable difficulty, a large current being required, and the discharge is quite explosive. It was thought that this explosive effect might produce a small shift toward the red.[2] Evidently the disruptive effect is of too great a period and not sufficiently violent.

14. A plate was taken under conditions similar to those of Nos.

[1] The conditions under which the iron plates were obtained are as follows: six plates taken at various times; average spark exposure, 12 minutes; average current, power, and voltage, 41.3 amperes, 420 watts, 11.5 volts; capacity, 0.0226 microfarads; length of primary gap, 9 mm; no secondary gap and no fan; region of spark image used, that near terminal. The above values of the shift are means of measurements made on from two to six plates.

[2] The suggestion that a comparison of these two spectra be made is due to Professor Crew.

83 and 84, except that the slit-length was extended to about 1 cm, the whole vertical aperture of the grating was used, and the exposure lengthened to ten minutes. Under these conditions there results a greatly overexposed spark spectrum. Upon careful measurement, setting on what appears to be the maximum of intensity of the body of the line, shifts as great as 0.031 and 0.025 tenth-meter appear; whereas set 10 gives values of 0.017 and 0.017. It is evident, then,

TABLE VI. TITANIUM

DATA TRANSFERRED FROM TABLE III

Mean Current in Amperes	Mean Power in Watts	Mean Potential Difference in Volts	Mean Shifts for λλ 3900, 3913	Set	Effect of
40.5	180	6.6	0.023	1	
38.4	388	12.5	37	2	
34.5	928	39.3	33	3	Self-induction
34.3	746	38.0	5	4	
41.6	402	11.3	21	5	Alloy
41.5	409	11.2	2	6	
40.9	421	11.5	18	7	Part of spark image
41.4	428	11.4	5	8	
41.7	308	9.0	0	9	Ohmic resistance
41.3	308	8.8	17	10	

as Eder and Valenta suggest, that by overexposure a false displacement may be introduced in a line marked by unsymmetrical broadening.

15. To confirm the data given in Table III, which concern only the lines *Ti*, λλ 3900, 3913, a series of plates was taken in other regions of the titanium spectrum. A study of the lines λλ 4443.97, 4468.65, 4489.24, 4533.42, 4534.15, 4549.79, 4563.94, and 4572.15 showed the presence of the alloy and self-induction effects of sets 6 and 7, also 3 and 4, Table III, respectively.

It was found also that large shifts are developed in other lines than λλ 3900, and 3913, namely, 0.024 tenth-meter and 0.036 tenth-meter for λ 4827.74 and λ 4836.25 respectively.

Further, when both spark and arc lines appear reversed, the central portions of the reversals are coincident (e. g., λλ 3223.1 and 3237.5).[1]

16. By referring to Table VI, the data given in which are taken

[1] Approximate wave-lengths.

from Table III, it will be seen that the energy involved is not the only element in the shift, for sets 3 and 4, in which nearly the same power (928 and 746 watts) was used, show a large difference in shift. This is due to self-induction, which causes less disruptiveness in the discharge.

Ohmic resistance (sets 9 and 10) gives the same effect.

The influence of a low-percentage alloy appears in sets 5 and 6, and the difference between the center and end regions of the spark is shown in sets 7 and 8.

It may be remarked that set 3 (power 928 watts) shows less shift, 0.033, than set 2 (power 388 watts), 0.037; but the difference is very small, and, moreover, the disruptiveness in the latter case may have been greater.

SUMMATION OF RESULTS

In conclusion it may be said that the writer's former results have been qualitatively and, to a certain degree, quantitatively confirmed. It is difficult to see how we can avoid the following deduction: Under certain conditions, namely, when self-induction and ohmic resistance are absent, large capacity or long secondary gap are used, and the terminals are of a high-percentage alloy, the part of the spark which lies near the terminal gives a spectrum, the wave-lengths of the lines of which are greater than those of the arc spectrum. The shifts occur when the energy involved in each discharge is great, and the discharge is disruptive in nature. These conditions mean the creation of a large amount of metallic vapor in a short time—a highly explosive phenomenon. That an increase of wave-length should result is rather to be expected. This may be rendered more evident from the following discussion:

In addition to those theoretical considerations (mentioned in the writer's former paper)[1] which would lead us to expect an increase in wave-length, namely those dealing with the pressure, p, in a gaseous medium as a function of the number of particles per c.cm, n, the mass of each, m, and the mean squared velocity, u^2, postulating the truth of the equation $p=\frac{1}{3}\, m\, n\, u^2$, it may be stated that such an increase in pressure may come from the fact that, at the beginning of the discharge, the air acts as an incompressible medium (compare the downward explosive effect of dynamite placed on the surface of

[1] *Astrophysical Journal*, **17**, 296, 1903.

the Earth). The development of the metallic vapor takes place so quickly that the air has no time to move. The lessened shift in the center of the spark can then be easily explained, for it is known that some time elapses before the metallic particles, advancing from the terminal, reach the central region of the spark. When they have finally reached this region, the air has yielded and the pressure fallen. An unusual amount of vapor is disengaged in the discharge between electrodes of titanium and zinc in water (not so prominently the case with iron). That the pressure developed is also great was noticed by the writer in his work with Professor Hale on the spark under water.[1] The terminals were badly shattered; indeed, there is good reason to expect that the lines of the spectrum given by the discharge in air pass through a series of changes which resembles that in water. Certain titanium lines appear reversed on some of the present series of plates, and not so on others. The reversal comes up unsymmetrically on the violet edge of the center of gravity of the line, a common occurrence in the spectra of the spark under water.

POSSIBLE EXPLANATION OF THE INCONSISTENT RESULTS OF HASCHEK, EDER AND VALENTA, AND MIDDLEKAUFF

Haschek undoubtedly dealt with overexposed lines and increased the true shift by a photographic displacement of the region of maximum intensity.

Eder and Valenta probably used a vertical spark and set the central region of the same on the slit of the spectroscope. Attention is called to the fact that the lines studied by the writer, *Zn*, λλ 4680, 4722, and 4810, are among those investigated by Eder and Valenta. We are assuming that the same effect obtains in the case of zinc as with titanium (sets 7 and 8, deduction No. 6 above) with reference to the central and end portions of the spark. That no shift exists under the conditions employed between the absorption regions of reversed arc and spark lines, as stated by Eder and Valenta for zinc, was found to be true in the case of titanium.

Lastly, that Middlekauff found no shift with iron may easily be explained by the fact that few lines show a shift, and these generally in but a small degree, under highly disruptive conditions and near the region of the terminal. Middlekauff probably used a compara-

[1] *Astrophysical Journal*, **17**, 154, 1903.

tively weak spark, set its image vertically on the slit, and integrated with his spectrometer the spectra of the central regions.[1] Moreover, Middlekauff and Eder and Valenta may have used small wires in the discharge circuit of the condenser and so have introduced ohmic resistance.

INFLUENCE OF RESULTS UPON THE MAPPING OF SPARK SPECTRA AND UPON STELLAR LINE-OF-SIGHT WORK

The bearing of the above results upon radial-velocity investigations in which a titanium spark is employed to give the comparison spectrum is such as to emphasize the fact that low-energy and non-disruptive conditions should be employed, if the lines of the resultant spectra are assumed to be severally of the same wave-length as those of the arc or the absorption and bright lines of the stars. Furthermore, if a highly disruptive spark be used, the same circuit conditions should be employed at all times in any one observatory, to make the plates there taken mutually comparable; or, finally, the same conditions should prevail at all observatories if mutually consistent results are desired.

The general bearing upon the mapping of spark spectra may be seen in the fact that an agreement between the wave-lengths of spark lines as measured by various investigators is impossible unless the conditions of the electric circuit be the same in all cases.

In conclusion, I wish to acknowledge my indebtedness to the Rumford Committee of the American Academy of Arts and Sciences (to which this paper has been presented) for the two grants which made this work possible, and to Professors Hale and Frost of the Yerkes Observatory, for the loan of the grating used. I am also under obligation to Dr. George S. Isham for the gift of the filar micrometer, and to Mr. Cornelius Vanderbilt for generous financial aid. My assistants, Messrs. Hartley and Miller, have proved invaluable at various times; and to Mr. Boord I am indebted for the analysis of the brass wire.

Wabash College, Crawfordsville, Indiana,
June, 1905

[1] The statement is made that the spark was 8 or 9 mm in length and its image on the slit about 6 mm.

VARIATION OF ARC SPECTRA WITH PHASE OF THE CURRENT PRODUCING THEM[1]

By H. CREW and B. J. SPENCE

In 1897 the radiation of an alternating current arc in which the current was interrupted during alternate half-periods was examined with some care by Professor Basquin[2] and one of us. The intervals during which the current was cut off were utilized for the examination of the arc. This was accomplished by means of an occulting screen such as that employed by Fleming and Petavel[3] for comparing, at any instant, the photometric intensity of the arc with its power absorption.

The upshot of this experiment was to show that the characteristic radiation of the arc completely disappears before the arc has been interrupted as long as $\frac{1}{1000}$ second, and that there remains, hovering over the arc-gap, a relatively long-lived, bright, detached cloud. The attempt to photograph the spectrum of this luminous cloud met with inferior success, because we could not operate the large occulting screen and generator in the concave-grating room then at our disposal. Having recently obtained a satisfactory outfit, through the generosity of the Carnegie Institution, the problem was again taken up.

An occulting screen three inches in diameter, rotating on the shaft of a synchronous motor, is provided with four radial slots 2° wide and 180° apart in current phase. The hub of the occulting screen (shown in Fig. 1) can be clamped to the shaft of the motor armature in any desired phase so as to allow one to photograph, or observe the arc visually, in any desired phase, or at least to within 2° on each side of the desired phase. Let us define "zero phase" as that phase of the current in which the characteristic radiation is a

[1] Read before the American Physical Society, April 21, 1905.

[2] *Proc. Amer. Acad.*, **32**, 1898.

[3] *Phil. Mag.*, **41**, 315–360, 1896. The work of Fleming and Petavel was not mentioned in our previous paper, because we did not then know of it.

minimum. Then, when the ordinary continuous alternating arc is observed at zero phase, it is at once evident to the eye that nothing remains of the arc except the detached luminous cloud mentioned above. It is not, therefore, necessary to use the intermittent current previously employed; for, without this, the continuous alternating current will give practically every phase of the arc from complete interruption to maximum radiation.

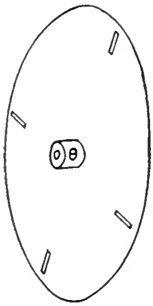


FIG. 1.

In the meantime (i. e., between these two experiments) de Wattville[1] has suggested a very clever modification of this occulting screen which will allow one to photograph several phases of a spectrum at once; but practically his device is limited to non-astigmatic spectrographs and to sources in which the spectrum does not appreciably vary from one part of the source to another.

He employs a screen, shown in Fig. 2, provided with several slots, each at a different phase and each at a different distance from the axis of rotation, thus obtaining on a single photographic plate and in immediate juxtaposition as many phases as there are slots; de Wattville has commented upon the similarity between flame spectra and the spectrum of the arc at zero phase; he has also noted some interesting differences between the behavior of banded and linear spectra.

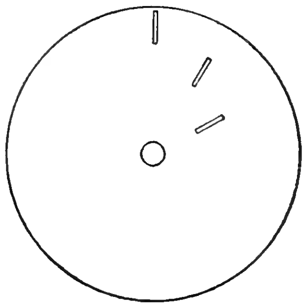


FIG. 2

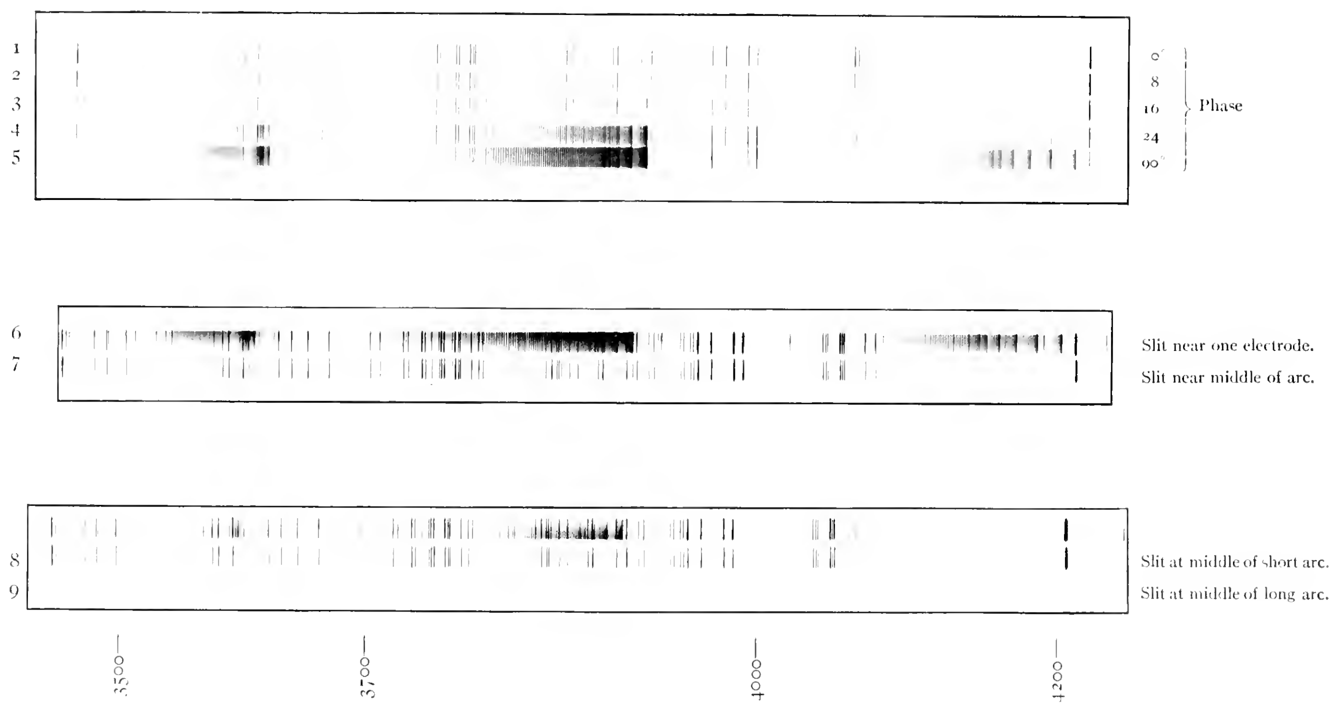
The results which we here describe are, so far as we know, new and additional to those of de Wattville; part of them were obtained before the appearance of his paper. Incidentally we have verified all the results which de Wattville describes.

ORDINARY CARBON ARC

1. The profound difference between the spectra of carbon at maximum and at zero phases is illustrated by the strips numbered 1 and 5 on Plate VII. It is the difference between something and

[1] *Comptes Rendus*, **138**, 485–486, 1904.

PLATE VII



1
2
3
4
5
0°
8
16
24
90°
Phase
6
7
Slit near one electrode.
Slit near middle of arc.
8
9
Slit at middle of short arc.
Slit at middle of long arc.
3500
3700
4000
4200

nothing. In the first strip, which was taken when the current was at zero phase, there is not a trace of any carbon band remaining; in the fifth strip, taken at a phase of 90° (maximum) we have the fully developed carbon spectrum. It is an easy matter to obtain as many phases as one may desire between these two extremes.

2. Near zero phase the cyanogen bands are very weak at the middle of the arc, and comparatively strong near the electrodes. This effect is shown in strips 6 and 7, Plate VII. As a result of this, we observe that when a very short arc, say less than 2 mm, is used the cyanogen bands are present at all phases; but when a long arc, say 6 mm or over, is employed, the cyanogen flutings entirely disappear from the middle of the arc. This phenomenon is illustrated in strips 8 and 9 of Plate VII.

3. At zero phase, the aluminium bands, as well as the *Al* pair between H and K, have their relative intensity greatly increased. In fact, the aluminium flutings appear under precisely the conditions which cause the cyanogen flutings to disappear. Is this in some way connected with the fact that cyanogen is strongly endothermic, and accordingly very stable at high temperatures; while aluminium oxide is fiercely exothermic, and therefore especially stable at low temperatures?

The different behavior of two impurities under identical conditions is well illustrated by the *Al* pair between the *Ca* pair at H and K. The *Ca* pair diminishes in intensity with the phase; and so does the *Al* pair, but at a rate so much slower than that of the *Ca* that one could easily tell from the relative intensity of the two pairs whether the photograph was taken near the phase of maximum or near that of minimum current. This is quite in harmony with the enormous intensity of H and K at the temperature of the Sun; for increase of phase in these experiments undoubtedly means increase of temperature.

4. Between the *K* pair at λ 4044 and λ 4047 lies an iron line whose intensity diminishes with phase so much more rapidly than that of the *K* pair that almost anyone could estimate, from their relative intensities, within 10° of the phase at which the photograph was made. This triplet behaves in precisely the same manner in

the *spark*[1] passed between *heated* carbon electrodes whose temperature is falling.

GRAPHITE ARC

We have not been able to work the graphite arc with alternating currents in the ordinary way. In any event, fifty amperes on a 110-volt circuit does not maintain an arc of sufficient length to project upon the slit of the spectrograph. Accordingly, we used a rotating graphite electrode and found that, at zero phase, the arc gives practically zero light, the only lines appearing between λ 3700 and λ 5300 being three calcium lines, a trace of the cyanogen band at λ 3883, and traces of a few ultra-violet iron lines.

To digress for a moment: A very curious thing happens with this rotating graphite arc, viz., the appearance of the well-known carbon spark line at λ 4267, a *Hauptlinie* of Eder and Valenta. Few lines are so sensitive to rapid changes in E. M. F. This line is strong in the spark between amorphous carbon poles, *provided they are cold;* if, however, the carbon tips have been heated, the line disappears even from a powerful transformer spark. On the other hand, this line which does not appear in the ordinary carbon arc comes out strong in the rotating graphite arc, where the natural quickness of break which characterizes the graphite arc is accentuated by the motion of the electrode. In other words, this line disappears with great readiness from its normal source (the carbon spark) and appears, on slight provocation, in a source where it might be least expected, the graphite arc. Such capricious behavior might lead one to hesitate in assigning this line to carbon; but its presence in pure Acheson graphite leaves little room for doubt as to its identity.

Returning from this digression, it is, perhaps, worth noting that, as the phase increases, all the carbon flutings appear *on the photographic plate* in the order of their intensities, which would be exactly the case if they all made their actual appearance at the same instant. If, therefore, the so-called cyanogen bands are due to a radiating system different from that which emits the so-called carbon bands, the evidence here furnished would seem to indicate, at least, that both systems radiate under similar conditions.

[1] Crew and Baker, *Proc. Amer. Acad.*, **38**, 397–406, 1902.

ALUMINIUM ARC

At zero phase nothing is left of the *line* spectrum of aluminium except the pair between H and K; even an hour and a half's exposure does not leave the slightest trace of the heavy pair at λ 3100. The manner in which the *Al* flutings are intensified, at zero phase, as compared with the line spectrum, has already been mentioned.

Arons and Hemsalech appear to have proved (by working the *Al* arc and spark in hydrogen) that these flutings are due, not to an oxide, but to metallic aluminium. If this be true, the opposite behavior of these two spectra—banded and linear—would seem to indicate that they originate, at least, in a different vibrating system.

MAGNESIUM ARC

The effect at zero phase of magnesium in cored carbons is merely to introduce the *Mg* fluting at λ 5007, or at least to make it visible by removing the green carbon fluting. The *Mg* arc between moving metallic electrodes is reduced to the fluting at λ 5007 and traces of the stronger lines.

IRON ARC

The iron spectrum at zero phase consists essentially of about 300 lines lying between λ 2700 and λ 5600. It is distinctly shorter than the flame spectrum of iron which de Wattville[1] has measured up to λ 2300. On the other hand, it is distinctly longer toward the blue, and therefore presumably hotter, than the spectrum obtained by A. S. King[2] in his electric oven. And what is true of the iron spectrum is probably true of most of the metallic elements, namely, the spectrum of the arc at zero phase is essentially a flame spectrum. The upshot of the whole matter is, therefore, that an occulting screen on the shaft of an alternating generator or on a synchronous motor furnishes a means of obtaining flame spectra, and all the intermediate steps between flame spectra and arc spectra.

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[1] *Phil. Trans.*, **204**, 139–168, 1905.

[2] *Annalen der Physik*, **16**, 360–381, 1905.

ON THE SPECTRUM OF THE SPONTANEOUS LUMINOUS RADIATION OF RADIUM

PART III. RADIATION IN HYDROGEN[1]

By SIR WILLIAM HUGGINS AND LADY HUGGINS

As soon as we found[2] that the glow of radium bromide consisted mainly of light from nitrogen stimulated into luminosity by the radium, and giving the negative-pole spectrum, we formed the intention of photographing the spectrum of the glow when the radium bromide was placed in an atmosphere of hydrogen, primarily in the hope of finding an answer to the question raised in our first paper, "Have we to do with occluded or with atmospheric nitrogen?" and, in the second place, to determine whether the radium is able to render hydrogen luminous.

In these experiments some unexpected results came out, which made it desirable to repeat them many times. This circumstance, together with the long exposures necessary—from ten to fourteen days—and the slow changes which we found to take place in the radium when allowed to remain in the hydrogen for long periods, reckoned in months, have necessarily delayed the publication of this paper. The investigation is still in progress, but it seems desirable not to delay any longer the publication of the results which have been already obtained.

An account of each experiment would make the paper long and unnecessarily tedious. It will be sufficient to give the results of each group of experiments made under similar conditions. The same form of apparatus was used for all the experiments. Small glass vessels were prepared consisting of a round cell with flat base, to hold the radium, into which on opposite sides, tubes of small bore were blown. The walls of the cell were ground flat on the top to receive a thin microscopic cover-glass, or a thin plate of quartz,

[1] From advance proofs of an article to appear in the *Proceedings of the Royal Society*.

[2] *Astrophysical Journal*, **18**, 151, 390, 1903; *Proc. R. S.*, **72**, 196, 409, 1903.

which was cemented down after the radium salt had been placed on an ebonite support within the cell.

As the glow of radium takes place at atmospheric pressure, it was desirable that experiments with hydrogen should be made under like conditions. A current of hydrogen was allowed to flow through the glass vessel for some minutes, until all traces of air must have been carried out; the two tubes were then sealed up, leaving the radium in hydrogen at atmospheric pressure.

Afterwards, a second series of experiments was made with hydrogen at reduced pressure. The glass vessel was connected with a vacuum pump and exhausted to below 1 mm of mercury; hydrogen was then allowed to enter. The vessel was again exhausted and refilled with hydrogen several times, and was then sealed up with the contained hydrogen at the pressure of about 1 mm of mercury.

Portions of the same two specimens of radium bromide which had been used in our former experiments were employed, namely one from Buchler & Co., Brunswick, and the other from the Société Centrale de Produits Chimiques, Paris.

The results of repeated experiments made in hydrogen at atmospheric, and also at reduced, pressure, and with both samples of radium bromide, were uniformly similar. The glow became sensibly fainter to the eye when the radium had remained for a few days in hydrogen; perhaps the diminution of the brightness took place sooner in hydrogen at reduced pressure. Photographic plates, exposed in the spectroscope to radium in hydrogen for the same time as to the same radium in air, showed a feebler spectrum, which was that of nitrogen without any traces of the lines of hydrogen.

From these experiments we may assume either the existence of occluded or combined nitrogen, or that the spectrum was due to minute traces of air which had remained within the vessel. If, however, the increasing feebleness of the glow was due to the latter cause, we should expect that, on unsealing the tubes and admitting air, the glow would at once recover its original brightness. On April 15, 1904, when the radium had remained twenty-six days immersed in hydrogen, the tubes were opened and air blown through, but no recovery of brightness, as estimated by the eye, took place at the time. Then a photographic plate was exposed in the spec-

troscope for seven days, on which, when developed, the nitrogen spectrum was even feebler than on a similar plate which had been exposed for the same time before the air was admitted. A few days later, however, a small increase of brightness was detected by the eye, which continued until the radium slowly recovered its original brightness.

When radium was allowed to remain for months in an atmosphere of hydrogen some unlooked-for results were observed.

Experiment 1.—A portion of the Brunswick radium bromide was sealed up in hydrogen at reduced pressure on June 24, 1904.

a) As in former experiments, the brightness of the radium bromide, as estimated by the eye, gradually diminished.

b) Photographs taken with the spectroscope showed the nitrogen spectrum only, and with increasing feebleness as time went on, until the strongest of the nitrogen bands only were just suspected upon the plate.

c) Some days after sealing up in hydrogen, the radium bromide, which was originally of a yellowish-cream color, began slowly to become darker, until by August 9, 1904, it had reached a dark russet-brown.

d) On March 17, 1905, the radium, which had now been sealed up for nearly eight months, was observed by eye to have become much brighter, indeed nearly as bright as the French radium which had remained in air. Unfortunately, the radium in the hydrogen had slipped out of the ebonite support to the bottom of the cell, and therefore could not be brought before the slit of the spectroscope. It was then decided to open the vessel and remove the radium in order to photograph its spectrum while in this brighter condition. Before placing it in the spectroscope it was thought desirable to compare it again at night, about eight hours after it was taken out of hydrogen, with radium which had remained in air. To our great surprise, the radium removed from the hydrogen had completely lost its light; it was now quite dark, without any sensible glow. It retained its very dark russet-brown color.

e) Before the vessel was opened, while the radium was bright in hydrogen, its radio-active power was measured with an electroscope; after it had been removed from the hydrogen and had become dark

and glowless, its radio-activity was again measured. The amount of the induced leak of the charged leaves was found to be the same as before, showing that the sudden change from a bright condition to one without any sensible glow had not been accompanied by an alteration in the intensity of the β- and γ-rays.

f) The glowless radium was examined in the dark at intervals of a few days. By May 9, 1905, a very faint glowing was perceived, and at the same time the dark brown color was observed to have become less intense. These changes proceeded slowly until, by August 13, the radium had regained its original creamy color and nearly its original brightness. During these three months its radiation, as measured by the electroscope, remained the same.

Experiment 2.—Fortunately, we have for the purpose of comparison a portion of the French radium which has been sealed up in hydrogen at reduced pressure since September 12, 1904, about eleven months. Many photographs of the spectrum of this sealed-up radium have been taken at intervals from last September to the present time, showing with similar exposures, increasing feebleness, but always, when any action could be detected upon the plate, some of the stronger bands of the nitrogen spectrum. Recently, however, a band has appeared in the green part of the spectrum, for which the plate is but feebly sensitive, without any action being discernible on the plate in the blue and violet regions, for which the photographic film is greatly more sensitive. Fortunately, on one plate the chief bands of the nitrogen spectrum, though excessively faint, can be just detected, while at the same time the new band, falling in a much less sensitive region photographically, is relatively strong.

The defined line which begins the band on the less refrangible side is a little more refrangible than the brighter edge of the green band of the Swan spectrum at λ 5165. The band has not yet been identified.

The band is accompanied by a faint continuous spectrum which runs back to D.

The radium bromide has turned to a dark russet-brown color, as in the former experiment. To the eye the brightness of the radium has remained greatly diminished, until within the last few days, when we suspect that, as in the preceding experiment, a slow increase of brightness has set in.

On re-examining all the photographs of the spectrum of the glow of radium which we have taken, a plate was found, developed on August 23, 1904, of the spectrum of a portion of the French radium which had been sealed up in hydrogen for a few days only, but when its light had faded to about one-half, which shows very faintly, but unmistakably, the new band.

The suggestion presents itself to the mind whether in both experiments, when the radium had almost ceased to glow with nitrogen light, it was able to stimulate the molecules of the substance producing the band into luminescence. On this supposition an explanation of the sudden going out of the bright glow, when the radium was taken out of the atmosphere of hydrogen in the first experiment, would be found in the absence in the atmosphere of the needful molecules, the radium having lost temporarily the power of exciting nitrogen; unless we take the view that, to be stimulated into luminescence, nitrogen must be not merely in outward contact with the radium, but in a more intimate connection with it, which time is required to bring about.

The suggestion was considered in a former paper whether the operative cause of the glow was to be found in the β-rays, which are known to be analogous to the cathode corpuscles, upon the nitrogen of the air. In these recent experiments the electroscope showed that these rays, and the γ-rays, were being radiated with undiminished force at the time that the radium bromide remained glowless in air. It may be mentioned here that the cathode discharge is efficient in bringing out easily the first spectrum of hydrogen,[1] but the radiations of radium appear to be unable to do this. Our experiments seem rather to support the view, suggested in our first paper, that the spontaneous light of radium may not be produced by any form of its radiations acting upon the nitrogen of the air outside it, but by a more direct action through encounters with nitrogen molecules, in some way associated with the radium, of those molecules of the radium which are undergoing active changes.

[1] Liveing, *Proc. Camb. Phil. Soc.*, **12**, 338, 1904.

AN OBSERVATION OF THE ZODIACAL LIGHT TO THE NORTH OF THE SUN

By SIMON NEWCOMB

The zodiacal light is commonly conceived and described as a phenomenon extending on both sides of the Sun, in or near the plane of the ecliptic, its possible breadth being left out of consideration, except as implied in the term "lens-shaped," sometimes used to designate its form. So far as I am aware, the possible thickness of the lens has never been considered, nor do I know of any attempt having been made to see the light north or south of the Sun. Such an observation, being necessary to the delineation of its complete outline, must be regarded as of prime importance in defining the zodiacal light itself. The observation requires a rare combination of conditions. One of these is that the Sun shall be more than 18° below the horizon at midnight, but as little in excess of 18° as practicable. The obvious reason for this is that, at a less depression than this, there might be some effect of twilight; while the farther it is below the limit, the less likely is the light to be seen. In fact, unless the minor semi-axis of the light considerably exceeds 18°, it may be forever impossible to distinguish it from twilight itself. The condition in question can be fulfilled only in middle or northern latitudes, not much south of 46°.

The other condition is that the observation shall be made from as elevated a point as possible above the Earth's surface. The light is known to be so susceptible to atmospheric absorption at ordinary elevations, and near the horizon, that we never see it, even in the zodiac, during those summer months when the ecliptic makes a small angle with the horizon. To what height it might be necessary to ascend could be determined only by trial.

During several summers past I have endeavored to make this observation from elevated points in the White Mountains and elsewhere, but have invariably been defeated by the density, smokiness, or haziness of the air near the horizon. Better stations could doubtless be found on the mountains of the Pacific coast; but not having

visited that region, Switzerland seemed to be the next best attainable region. It happens, however, that even in that country nearly every point of observation which could be readily occupied throughout the night has yet more elevated points to the north of it, which cut off the view of the horizon. This is particularly the case at the Gorner Grat and the base of the Matterhorn, which have respectively the Mischabel and the Weisshorn to the south of them.

Speaking of the problem to Dr. Frei, a Swiss geographer, he suggested that the Brienzer Rothorn, a mountain to the north of Lake Brienz, might be a suitable and practicable point, and was accessible by a rack-and-pinion railway. The position is: $\phi = 46° 47'$; $\lambda = 8° 3'$ E. The observation would be much facilitated by the existence of a hotel only three hundred feet below the summit of the mountain. The altitude of the mountain, 7700 feet, although not as great as could be desired, yet seemed to offer sufficient hope of success to justify a trial; I therefore repaired thither on July 26, 1905.

The condition of the air on Lake Lucerne, as I passed over it, was far from satisfactory, it being more hazy than I had ever before seen it. On reaching the mountain, it was found that the upper surface of the haze was slightly below the astronomical horizon, so that the prospects for an observation, if the light were bright enough to be seen, were fairly good. The following are notes of the observations made during my stay on the mountain:

Wednesday, July 26, 1905.—Reached Hotel Rothornkulm at $6^h\ 45^m$, Local M. T., about an hour before sunset. Air below very hazy. Sharply defined upper surface of denser yellow haze well marked, about $10'$ or $15'$ below the astronomical horizon; but air not satisfactorily transparent, even above this line, as shown by the Sun's lurid color in approaching the haze. On entering the latter it turned deep red, and became quite invisible before reaching the sensible horizon marked out by the distant mountain tops.

$9^h\ 0^m$. Still bright twilight.

10^h. Twilight seems to have completely passed. A faint glow is now visible over the northwestern horizon, the central part of which is on a vertical passing near or between the pointers, and which can be traced about $12°$ or more on each side of the central region.

$10^h\ 20^m$. The central part of the glow now seems to be north of the vertical through α *Ursae Majoris*, and about $20°$ west of north.

$11^h\ 30^m$. Now see that above the western horizon, and below *Arcturus*,

the sky is whiter than elsewhere at the same altitude. Sweeping the eye around the horizon, a well-marked glow is visible through the entire quadrant from north to west, rising to an altitude of some 20° or 25°.

About midnight.—Looking out of the window toward north, a slight glow visible around the northern horizon does not seem to be anything more than a phenomenon generally visible during the night at any point. The haze seems to have so thickened as to preclude any decision as to what is actually to be seen.

July 27 and 28.—Bad weather.

July 29.—Enveloped in cloud most of the day and early evening.

Midnight.—Quite clear, except a stratum of cloudlike haze around the northern horizon, with thick atmosphere immediately above it. But an illumination of the low northern sky is distinctly visible from north window, and observations from the Kulm are decided upon.

$11^h\ 45^m$ to $12^h\ 5^m$ M. T. While walking up to the Kulm, from occasional glimpses, the maximum of brightness seemed to be just below *Capella*.

$12^h\ 5^m$ to $12^h\ 10^m$ M. T. The characteristic zodiacal glow distinct and unmistakable—not so bright as ordinarily seen east or west of the Sun, yet several grades brighter than the limit of doubt. It extends from a little east of *Capella* to a region below the pointers. The maximum of brightness is midway between *Capella* and the north point, say between 10° and 15° east azimuth, and at 10° of altitude. The appearance of maximum brightness below *Capella* was evidently due to the Milky Way.

$12^h\ 45^m$ M. T. Carefully scanning the sky, especially in the region of the ecliptic and the horizon. At one time had a strong impression of an illumination in the south much fainter than that in the north; but this was supposed to be the Gegenschein, fainter and more diffused than I had ever before seen it. But the impression itself soon faded, though the sky seemed in the minutest degree brighter in the low south than elsewhere at the same altitude.

I now see that the glow in the north does not extend quite to the vertical of the pointers. South of *Capella* it is merged in the Milky Way. Tried to trace it between the *Pleiades*, now well above the horizon, and *Aldebaran*, now upon it, but the presence of *Jupiter* rendered a conclusion impossible, though a glow was well seen. Farther south no trace could be made out.

Later.—*Venus* was seen rising toward the horizon; and it was evident that the combination of its light with that of *Jupiter* would prevent any positive ascertainment of the continuity of the glow in the north with the familiar phenomenon of the zodiacal light.

The conclusion reached from these observations is that, in the direction of the Sun's axis, the zodiacal light is bright enough to be plainly seen to a distance of about 35° on each side of the Sun. Of course, the actual observations are made only on the north side, but there can be no reasonable doubt of the symmetrical character of

the phenomenon. This conclusion is quite in accord with what anyone may notice who carefully examines the light, as near the horizon as it can be seen, on a suitable evening in February, or a morning of October. It will then be seen that the light rapidly broadens toward the horizon, although, in our latitudes, it cannot be seen near the actual horizon, owing to atmospheric absorption. Of course the boundary of the phenomenon does not admit of precise delineation, the fading off being gradual and continuous. The limit of 35° which I have set, nevertheless seems to me much more precise than any limit that has been, or can be, set in the plane of the ecliptic.

I suggest that the zodiacal light be hereafter described as a luminosity surrounding the Sun on all sides, of which the boundary is nowhere less than 35° from the Sun, and which is greatly elongated in the direction of the ecliptic.

I may also invite attention to my inability to see the Gegenschein under circumstances on the whole quite favorable. The depression of the Sun was 25°, and the conditions seemed extremely favorable.

SPECTROGRAPHIC OBSERVATIONS OF CERTAIN VARIABLE STARS

By EDWIN B. FROST

During the past year I have added to the observing program for the Bruce spectrograph a few variable stars (chiefly of the *Algol* type) for which a knowledge of the radial velocity would be likely to be of interest. Most of these stars are faint, and in many instances the spectral lines are so diffuse that the plates are not susceptible of precise measurement. All of the plates were obtained with a dispersion of one prism and with a camera of 607 mm focus. The results as here communicated are to be regarded only as provisional, but the measures clearly show that *R Canis Majoris*, *Z Herculis*, and *U Sagittae* are spectroscopic binaries, and their velocities correspond in sense to what would be expected from the phase in the light-variation. On account of the long exposures required the accumulation of plates in a season is necessarily small.

I have added to the usual data the length of exposure, since it constitutes in some cases quite a fraction of the half of the star's period of variation. The interval after (*a*) or before (*b*) the nearest minimum is also given. The times of minimum are taken from Hartwig's ephemerides in the *Vierteljahrsschrift der Astronomischen Gesellschaft*.

R Canis Majoris ($\alpha=7^h\ 15^m$; $\delta=-16°\ 12'$; Mag. $=5.9$ to 6.7)

Plate	Date	G. M. T.	Exposure	No. Lines	Radial Velocity	Interval from Minimum
IB 492....	1905 January 27	$17^h\ 5^m$	80^m	4	-66^{km}	*a* $6^h\ 30^m$
499....	February 3	17 12	80	7	-54	*a* 11 2
530....	April 7	13 59	70	3	-13	*b* 6 54

The period of the star's light-variation is $27^h\ 16^m$, with which the above observations would suggest an orbital velocity of about 27 km and a velocity of translation of the system of about -40 km, a circular orbit being assumed.

The spectrum is between Vogel's types Ia2 and Ia3, but the

lines are rather broad. Those best for measurement are at λ 4481 and λ 4549 and they receive the greatest weight. The three plates so far obtained do not show indications of the spectrum of the second star.

Z Herculis ($\alpha=17^h\ 54^m$; $\delta=+15°\ 9'$; Mag.=7.0 to 8.0)

Plate	Date	G. M. T.	Exposure	No. Lines	Radial Velocity	Interval from Minimum
IB 543....	1905 June 2	$16^h\ 36^m$	200^m	4	-75^{km}	a $10^h\ 19^m$
551....	June 19	17 31	300	—	—	a 35 56
578....	August 28	15 51	230	3	− 2	b 10 37

The double period of light-variation is $95^h\ 50^m$ (from even to even minimum, from which the above intervals are reckoned). Plate 551 was taken through a thick sky and is hardly measurable. It clearly indicates a moderate negative velocity, however, in accordance with the direction to be expected. A negative velocity of the system is implied by the fact that the last value is slightly negative. The velocities given may be regarded as uncertain by several kilometers.

The spectrum is much like that of the preceding star, but the lines are all faint, as none of the plates is fully exposed. Except for a faint suggestion of duplicity at $H\gamma$ on the last plate, there is no indication of the spectrum of the second component.

U Sagittae ($\alpha=19^h\ 14^m$; $\delta=+19°\ 26'$; Mag.=6.5 to 9.1)

Plate	Date	G. M. T.	Exposure	No. Lines	Radial Velocity	Interval from Minimum
IB 542....	1905 May 19	20 6^m	160^m	3	-49^{km}	a $17^h\ 56^m$
555....	July 14	16 39	199	2	+69	b 20 52

The star's period of light-variation is $3^d\ 9^h\ 8^m$.

I have to thank Mr. Barrett for taking the second plate. The spectrum is of the *Orion* type, and the measurements depend chiefly on the helium line at λ 4472, and the magnesium line at λ 4481. Evidences of lines due to the second star do not appear on these plates.

U Ophiuchi.—Of this well-known *Algol* variable I have obtained five spectrograms, three in 1904 and two in this season. The spectrum is of the *Orion* type, but the helium lines are faint. The

hydrogen lines and λ 4481 are exceedingly broad and diffuse, and the measurement of the plates will be very difficult. There are strong indications of two component spectra, and there are evidently variable displacements of the lines, but a definite statement of results must be deferred until measurements can be made on the best of these plates and on others yet to be secured.

RX Herculis.—Two spectrograms have been secured this season of this star (magnitude 7.0 to 7.8), but the diffuseness of the lines almost prohibits measurements. The helium lines are probably present but exceedingly faint. On the first plate λ 4481 seems widely double, but single on the second. Enhanced lines of titanium and iron are present but faint. Their displacements are appreciably different on the two plates. We hope to secure another spectrogram this season.

Y Cygni.—I obtained a first plate of this star on July 19, 1904, with an exposure of 202 minutes; and a second on June 12, 1905, in 183 minutes, with the assistance of Mr. Barrett. The helium lines are very faintly represented. The appearance of $H\gamma$ is peculiar, and its duplicity is suggested on the first plate, unless an enhanced titanium line at λ 4338 is unduly strong. The lines are diffuse, and quantitative results probably cannot be derived from the first plate. Definite statements as to the star's radial velocity are reserved until other plates can be obtained and measured.

R Coronae.—In view of the interest that the recent light-variations of this long-period variable have aroused, it may be well to communicate the measurements of two plates I have secured of this spectrographically faint object.

R Coronae ($\alpha = 15^h\ 44^m$; $\delta = +28°\ 28'$; Mag. = 5.5 to 10)

Plate	Date	G. M. T.	Exposure	No. Lines	Radial Velocity
I B 58.......	1903 July 24	$17^h\ 5^m$	240^m	11	$+14^{km}$
575......	1905 August 25	15 38	200	11	+13
				Mean	+13

Three days after the first date the magnitude was measured by Mr. Parkhurst and found to be 7.2. On the second date he estimated the magnitude to be rather brighter than 7.

A very wide slit (0.1 mm) was necessary on account of the star's

faintness, and the lines in the stellar and comparison spectrum are broad. The second spectrogram is the better, but neither can be measured with much refinement, although settings can be made on numerous lines, as the spectrum is of the early solar type like that of *a Persei*. In view of the difficulty of measurement, the two results may be considered in accidentally close agreement, and as indicating no change in the star's radial velocity. The above values were derived from the eleven best-defined lines measured on both plates.

No qualitative change can be detected in the appearance of the spectrum on the two plates.

I acknowledge with pleasure the assistance received during these rather tedious exposures from Mr. Sullivan, who as usual shared in all the guiding.

YERKES OBSERVATORY,
September 16, 1905.

THE EFFECT OF A PRESSURE OF 37 ATMOSPHERES ON CERTAIN LINES OF THE ARC SPECTRUM OF IRON

By W. J. HUMPHREYS

After several years of gradual preparation, I recently came into a position to continue experimental work on the effect of pressure on spectra. The equipment consisted in part of a Rowland concave grating of 14,438 lines per inch (568 per mm) and 21.5 feet (6.5 meters) focal length, with a ruled surface approximately 6×2 inches (15×5 cm), a four-stage Norwalk compressor, and a specially designed large, forged-steel bottle in which an electric arc was inclosed. The excessive difficulty of making this forging caused much delay, as a number of good firms declined undertaking it, and I have to thank Professor Stratton for giving me the address of The Janney Steinmetz Co., of Philadelphia, who furnished a most satisfactory piece of work. The bottle and its attachments will be described in another paper.

After several attempts, a fair negative was secured of a portion of the iron spectrum, due to an electric arc produced by a 220-volt direct current, under a pressure of 37 atmospheres. But at this time the experiments were unavoidably discontinued, and therefore a report on the results obtained is made at once, in the hope that they may be of value to others interested in spectrum work. It is my purpose, however, to take up this investigation again at the earliest practicable date.

An inspection of this negative showed at once that the tendency to reverse is much greater under a pressure of 37 atmospheres than it is under that of only one. Many lines at this high pressure spread out considerably, while others, like λ 4315.21 and λ 4337.14, remain fairly clean and sharp. Several, of which λ 4233.76 and λ 4236.09 are good examples, become somewhat hazy and show large displacements toward the red end of the spectrum. At least one line, λ 4227.60, not only shows a marked shift, but also indicates its gradual disintegration as a line under increasing pressure. It seems as though a somewhat higher pressure than that used would cause it to disappear as a broad faint haze, entirely to the red side of its normal position.

The lines measured are given in the table. The wave-length λ is taken from values given by Kayser and Runge. Under "intensity and character" R means reversed, while the numbers show roughly the relative intensities; 1 indicating the faintest line measured, 2 referring to a line about double this intensity, 3 to one of three times that value, and so on for higher numbers. Under Δλ is given in Ångström units the observed increase in wave-length of the respective lines as formed under a pressure of 37 atmospheres over their values when formed under that of but one atmosphere.

On the whole, these values of Δλ are reasonably well proportionable to those obtained some years ago,[1] at much lower pressures.

IRON LINES AND THEIR INCREASE IN WAVE-LENGTH (Δλ) PRODUCED BY AN INCREASE IN PRESSURE ABOUT THE ARC FROM 1 TO 37 ATMOSPHERES

Wave-Length λ	Intensity and Character at 37 Atmospheres	Intensity and Character at 1 Atmosphere	Δλ	Wave-Length λ	Intensity and Character of 37 Atmospheres	Intensity and Character at 1 Atmosphere	Δλ
3903.06	R 4	1	0.121	4156.88	2	1	0.118
3948.87	2	1	.090	4181.85	2	1	.095
3950.05	2	1	.072	4202.15	R 6	2	.076
3951.25	2	1	.133	4219.47	3	2	.114
3956.77	R 2	R 1	.120	4222.32	2	1	.202
3969.34	R 6	R 2	.114	4227.60*	2	2	.365
3977.83	2	1	.065	4233.76	4	2	.213
3984.08	2	1	.104	4236.09	6	2	.183
3997.49	3	1	.071	4250.28	6	2	.215
3998.16	2	1	.130	4250.93	R 6	2	.076
4005.33	R 6	2	.115	4260.64	R 8	2	.197
4009.80	2	1	.060	4271.93	R 12	R 3	.148
4013.91	2	1	.087	4282.58	3	1	.070
4021.96	2	1	.091	4294.26	4	1	.101
4045.90	R 12	R 4	.131	4299.42	4	2	.193
4062.51	R 10	R 3	.148	4307.96	R 10	R 3	.147
4071.79	R 8	R 2	.161	4315.21	2	1	.070
4118.62	2	1	.093	4325.92	R 12	R 3	.131
4143.96	R 6	R 2	.129	4337.14	2	1	.095
4154.57	2	1	.099				

* The position of this line cannot be determined with much certainty. It is greatly displaced, but very hazy and ill-defined.

The darkest parts of reversals and the most intense portions of the unreversed lines—the positions of the lines when due to a small amount of material—were the places measured. It was impossible, however, to measure them with great accuracy, as they are not

[1] *Astrophysical Journal*, **6**, 200, 1897.

sufficiently narrow and well defined. Therefore, hoping to reduce errors as much as practicable, the settings were made by moving the plate first in one direction, so that increasing scale-numbers meant increasing wave-lengths; and then, after reversing the plate, by moving it in the opposite direction, so that increasing numbers meant decreasing wave-lengths. In all, five measurements were made in each direction, and the values given are the average of these ten reasonably concordant (usually to within 10 per cent. of each other, though sometimes differing by as much as 20 per cent.) separate settings. The dividing engine used for this purpose was one of the large type furnished by the Geneva Society. No corrections were made for the errors of the screw, as the errors of setting did not justify this refinement.

This work was done in the physical laboratory of the University of Virginia, during my connection with that institution, and I wish to thank Professor Smith for his constant support in my efforts to secure the necessary equipment. My thanks are also due the Rumford Committee of the American Academy of Arts and Sciences, one of whose grants materially aided me, particularly in getting the pressure bottle and fitting it to the needs of the investigation.

U. S. Weather Bureau Research Observatory,
Mount Weather, Bluemont, Va.,
September, 1905.

REVERSAL OF BANDS

By W. J. HUMPHREYS

Liveing and Dewar,[1] by feeding a considerable amount of cyanide of titanium to an electric arc surrounded by a magnesium crucible, obtained, of the cyanogen bands, "complete reversals of the five bands near L, of the two strong bands near N, and a less complete reversal of the six bands beginning at about λ 4215." Negative results were given by the cyanides of other metals.

Petavel and Hutton,[2] while working with an electric arc between carbons surrounded by air at a pressure of forty atmospheres, secured a "marked reversal of the five heads of the cyanogen band beginning at λ 3883."

The heads of this band are also nicely reversed on one of my own negatives, recently taken under a pressure of only sixteen atmospheres.

The conditions in each of the above were unusual, and it may therefore be worth while to call attention to a number of bands that reverse, more or less easily, in the open arc at atmospheric pressure. A direct 110-volt current was used, and the light analyzed by a Rowland concave grating of 6.5 meters focal length. The lower or positive carbon was bored axially and filled with the substance under examination.

The most easily obtained and the most beautiful of these reversing bands are two due to the fluoride of calcium, one at about λ 6030, the other about λ 6060. Their reversal is an unusually beautiful spectroscopic phenomenon. On watching one may see them, as the quantity of material in the arc fluctuates, suddenly seem to head first one way and then the other; indicating a reversal, not only of the heads themselves, but also of many of the next succeeding fine lines; with, however, a gradual falling off in the distinctness of the reversal, and finally only an increasing intensification of the more and more distant and ordinarily fainter tail lines. However, as these lines are exceedingly close together, it is not easy to deter-

[1] *Proc. R. S.*, **33**, 3–4, 1881. [2] *Phil. Mag.*, (6) **6**, 571, 1903.

mine what happens to them individually; only the general effect is conspicuous.

The fluoride of strontium gave five reversed bands, and the fluoride of barium two. All these are listed in the subjoined table, in which the wave-length does not refer to any one of the heads, but only to that place in the spectrum about which the band in question occurs; the object of the table is simply to locate each band as a whole.

The positions of the heads of bands due to the fluorides of calcium, strontium, and barium have been given by Fabry.[1]

REVERSING BANDS

Substance	λ
Calcium fluoride (CaF_2)	6030
Calcium fluoride (CaF_2)	6060
Strontium fluoride (SrF_2)	5780
Strontium fluoride (SrF_2)	6300
Strontium fluoride (SrF_2)	6400
Strontium fluoride (SrF_2)	6515
Strontium fluoride (SrF_2)	6625
Barium fluoride (BaF_2)	4960
Barium fluoride (BaF_2)	5015

This work, by the kindness of Professor Smith, for which I wish to thank him, was done in the physical laboratory of the University of Virginia.

U. S. Weather Bureau Research Observatory,
Mount Weather, Bluemont, Va.,
August, 1905.

[1] *Journal de Physique*, (4) **4**, 245, 1905.

Minor Contributions and Notes

ON THE ENHANCED SERIES OF TITANIUM, IRON, AND NICKEL

Mr. F. E. Baxandall,[1] replying recently to a paper by the writer[2] on the subject of enhanced lines, has criticised the results reported in that paper. Probably the main difference between Mr. Baxandall and the writer is merely a difference in judgment as to what constitutes enhancement. It was pointed out in the original article that in comparing the relative intensities of arc and spark lines, the question of the proper exposure time assumes the greatest importance and is not easily settled. The Kensing on observers, although they recognize the fact that some lines are more enhanced than others, seem to divide all lines into two definite classes, enhanced lines and lines that are not enhanced. Yet Mr. Baxandall expresses surprise that on the author's plates only one iron line was found stronger in the arc than in the spark, plainly implying that this was contrary to his own experience. The inevitable conclusion is that there is room for at least a third class of lines; and, in fact, there are many gradations from the most strongly enhanced lines to those that are just the reverse.

The facts in the case seem to be as follows: The relative intensity of lines in spark spectra is by no means the same as that of the same lines in arc spectra. Moreover, if it is possible to divide these lines into groups such that the relative intensity of all the lines in one gr up is the same for arc as for spark, but that the relative intensity differs as we pass from one group to another, any such division is extremely rough, and must depend very greatly upon the individual's judgment. Then obviously the only procedure for an investigator is so to choose his exposure times that as many lines as possible shall have about the same intensity in the two cases. Here is where differences in judgment can enter to a great extent, and the difficulty is still further complicated by the fact that an increase or decrease in the exposure times of both spectra (without changing their ratio) can change the contrast between the photographic images of individual lines to a considerable degree. Hence it seems to the author not at all remark-

[1] *Astrophysical Journal*, **21**, 337, 1905. [2] *Ibid.*, **19**, 322, 1904.

able that the results obtained at Kensington should indicate a much smaller number of enhanced lines than he himself found.

A minor point of discussion is the inclusion in the original paper of a number of lines which were marked as of doubtful origin, inasmuch as they could not be identified in any published lists. These Mr. Baxandall rejects as probably due to impurities, but it is the belief of the author (and also of Director E. B. Frost, at whose request the work was done) that most of them belong to the elements under consideration, since every effort to identify them with other elements was unsuccessful.

There remain a few cases where lines listed as enhanced in the Kensington publications were not found to be so by the writer. It is difficult to account for these cases, except on the assumption that the difference was caused by the different conditions under which the spark was generated.

H. M. REESE.

UNIVERSITY OF MISSOURI,
September 8, 1905.

Reviews

The Dynamical Theory of Gases. By J. H. Jeans. Cambridge: University Press; New York: The Macmillan Co., 1904. Pp. viii + 352. $4.50, net.

In this work the writer aims "to develop the theory of gases upon as exact a mathematical basis as possible." The introductory chapter is followed by a discussion of "The Law of Distribution of Velocities," including also the H-theorem, and Boltzmann's theorem of the equipartition of energy. Then follow two chapters on the "Physical Properties of a Gas," including Boyle's law, van der Waals's equation, the virial of Clausius, and entropy and its interpretation. These are followed by chapters on a "Non-Conservative Gas," discussing the problem of the dissipation of the energy of a gas by radiation in the ether, and an attempt to co-ordinate the kinetic theory with the known facts regarding gaseous spectra, and the obvious deductions therefrom regarding molecular and atomic vibrations. Six chapters treat of viscosity, and other phenomena depending on mean free path, Maxwell's assumption of a repellent force between molecules varying as the inverse fifth-power, and the propagation of sound. Of the remaining chapters, XVII deals with "Planetary Atmospheres;" XVIII, with "Molecular Aggregation and Dissociation;" and XIX, with "Numerical Values," especially as to the size of molecules.

In an atmosphere, subject like ours to rapid disturbances, the conditions of pressure and temperature approximate more nearly to convective (adiabatic) than to isothermal equilibrium. The temperature is then a linear function of the vertical height, and the formula suggests a free surface at the height of about 29 kilometers (18 miles). The composition of such an atmosphere is uniform. The outer atmosphere must consist of molecules so scattered that there are practically no collisions, but describing orbits under the Earth's gravitation. Because of the lack of collisions, its condition is most aptly described by the equations for isothermal equilibrium, which simply deal with the averages of a fortuitous aggregation. The constitution is no longer uniform, but the lighter gases must be present in relatively greater proportions. The assumption of the temperature of 17°, absolute, for the outer atmosphere, leads to the conclu-

sion that beyond a distance of 80 kilometers (50 miles) from the Earth's surface the atmosphere must be pure hydrogen.

The author deduces the formulæ connecting the escape of molecules from the outer atmosphere with the temperature and gravitational constant. In the case of the Earth, at any temperature of the outer atmosphere less than 230° C. the loss of atmosphere is inappreciable, much more so at actual temperatures; while, if this temperature was ever greater than 740° C., the hydrogen atmosphere must then have been lost, and the hydrogen of the present atmosphere is to be explained as a later addition, probably from chemical action at or near the Earth's surface. The superior planets, except *Mars*, would even more than the Earth be able to retain their atmospheres. From *Mars*, hydrogen and helium would probably escape, while the heavier gases would be retained. For *Venus*, a temperature of 130° C. would be required for the escape of hydrogen, so that she probably retains an atmosphere. *Mercury*, smaller and hotter, is probably devoid of atmosphere, as also the Moon. At the Sun, a temperature of 1,530,000° C. would be required for any appreciable loss, even of hydrogen.

Misprints and minor errors are delightfully few. On p. 118, § 135, is the statement: "Equation (281) admits of negative values for p, whereas an examination of the physical conditions shows that p is necessarily positive." While p is *usually* positive, the negative values given by the equation (that of van der Waals) apply to the *liquid* state, in which negative pressures have been observed, as when the mercury "sticks" to the top of a barometer tube. Also on p. 313, § 375: "When there is no limit to the height of the atmosphere, the neglect of rotation is impossible." In the immediate context it is shown that for distances from the Earth's surface of less than an Earth-radius the rotational effect, even if the Earth is able to drag the air with it, is very small as compared with the gravitational, while the mean free path, at that height, is many times the Earth-radius, so that at greater distances the drag-effect, and hence the rotation, would be practically non-existent.

The book is a piece of conscientious work by one who has already made valuable contributions to this subject. It belongs rather with the classical work of Burbury and of Boltzmann than with any of the attempts at a simple or more popular presentation. Competent criticism of the validity of its conclusions on disputed points can be made only by the greater masters of the subject.

W. P. BOYNTON.

Atlas of Emission Spectra of Most of the Elements. By A. HAGENBACH and H. KONEN. Authorized English edition by A. S. King. Jena: Fischer; London: Wesley, 1905. Pp. vii+70; with 28 plates. £1 7*s*.

Many volumes dealing with subjects in science are addressed to readers of a particular level. Such a book may be uninteresting to one who has pushed the subject to a higher level on account of its elementary character; at the same time, it may not attract a younger student, because it lies beyond his ken. But this *Atlas of Spectra* is a work which will probably be of interest to all students of spectroscopy, containing as it does much that is easily comprehended by the beginner, and something that will be new to nearly every scholar.

The volume contains no fewer than 280 photogravures exhibiting the spectra of 68 out of a list of 79 elements. With the exception of fluorine, the 11 elements omitted all belong to the class described as "extremely rare;" so that we have before us what is by all odds the most comprehensive survey of spectra ever published.

The scale of the maps is uniform, being approximately 16 Ångström units (tenth-meters) to the millimeter The sources employed are principally the metallic arc, the carbon arc, the carbon spark, the vacuum tube, and the coalgas-oxygen flame. Each element has been studied under various conditions, so that the maps are well adapted to the illustration of nearly all the principles of modern spectroscopy. Among interesting features of this kind are the differences between arc, spark, and flame spectra, the positive and negative pole spectra of nitrogen, the red and blue spectra of argon, the effect of self-induction, common impurities running through nearly all spectra, the phenomenon of air lines in practically all spark spectra, ghosts, reversals, the distribution of series lines, etc.

The value of the map is greatly enhanced by the accompanying notes which point out the more important lines, their wave-lengths and physical characteristics, the principal impurities, etc. In addition to these details concerning particular spectra, there are ten pages of "Spectroscopic Notes" giving a discussion of laboratory methods and of the variability of spectra. These notes really constitute a summary of present spectroscopic practice.

The authors would perhaps have anticipated some questions if they had told us why they reproduce the positive instead of the negative of their photographs. Black lines on a white ground, as illustrated by the maps of Eder and Valenta and Higgs, would seem to be more easily read.

Possibly the positive was chosen, in the present case, because it brings out the reversals so beautifully.

The clear and idiomatic English of Mr. King adds much to the value of the volume for American readers, and even possibly for that large group of young men who "read German quite as easily as English."

HENRY CREW.

Lehrbuch der Physik. Von O. D. CHWOLSON. Band II. Pp. xxii+1056. Figs. 658 and 3 stereoscopic pictures. Translated from the Russian into the German by H. PFLAUM. Braunschweig: Vieweg & Sohn, 1904. 18 marks; bound, 20 marks.

The favorable impression which was created by the first volume of this compendious textbook of physics is maintained in the second volume. It treats of sound in eleven chapters, covering 141 pages; while the remaining 900 pages (in eighteen chapters) are devoted to radiant energy, dealing with the subject in a highly satisfactory manner. We may note as special features the great wealth of illustration and the very full references to the literature of the subject, which are collected at the close of each chapter, whence the reader's attention is not diverted by footnotes. The references are given in chronological order for each of the sections into which the chapters are subdivided. Only elementary mathematics are used, but the dimensions of the book would be too greatly extended if much space had also been given to purely theoretical considerations.

After an introductory chapter, the part dealing with radiant energy treats of the transformation of heat energy into radiant energy (under which Kirchhoff's law is discussed), the velocity of propagation, the reflection, refraction, dispersion, transformation, and measurement of radiant energy, in nine chapters. The seventh chapter, treating of dispersion, gives over a hundred pages to what would be included under spectroscopy. The methods and results described are entirely up to date, including in many cases researches as late as 1903. The ground is well covered, although, of course, many important topics receive but limited attention. Celestial spectroscopy is not neglected, occupying more space than might be expected in a work on physics. It is interesting to see the cut of the flash spectrum (Fig. 292) obtained by W. K. Lebedinski at the total eclipse of July 28, 1896, or on the occasion when Schackleton obtained his well-known photograph of the flash spectrum. Gratings and interference methods in spectroscopy receive an excellent discussion in Chapter XIII, on "Interference." The different varieties of photometers are fully dis-

cussed in the ninth chapter, more completely and with better illustrations than in ordinary textbooks on physics. Interesting figures are given of the author's pyrheliometer and actinometer in use in Russian observatories.

Optical instruments are discussed in the tenth chapter, rather in the conventional method. Some points in physiological optics are taken up in the eleventh chapter, and optical phenomena of the atmosphere in the twelfth. The illustrated description of Michelson's work on the comparison of the meter with the wave-length of the three cadmium lines will be welcomed by those who have not the original memoir at hand. The chapters on double refraction and on polarization appear to follow the conventional lines.

On the whole, the volume will be found very useful for reference, as has been confirmed by the reviewer's experience during the several months that it has been at hand. It is likely that it will be of special interest to the readers of this *Journal*, as it deals so largely with radiant energy, and it will presumably be obtained by many who do not feel able to purchase the whole work.

E. B. F.

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SELECTIVE REFLECTION IN THE INFRA-RED SPECTRUM

By JAMES T. PORTER

Certain substances which transmit a large percentage of the incident radiation in the visible portion of the spectrum, reflect strongly in the infra-red. Of these, one of the most noteworthy is quartz. It has been shown by Professor E. F. Nichols[1] that in the neighborhood of wave-length $8.5\,\mu$ the reflection from a quartz surface is 20 or 30 times greater than in the other parts of the spectrum, and that, consequently, in the spectrum of rays after three successive reflections these waves will lose little in intensity, whereas those lying on either side of this value will be reduced in the ratio of $(20)^3$ or $(30)^3$ to 1. The spectrum, then, after three reflections will contain practically only the radiation of wave-length $\lambda = 8.5\,\mu$, and this in measurable quantity. Rubens and Nichols,[2] and Rubens and Aschkinass[3] have employed this method, commonly known as the method of "Reststrahlen," for the detection of waves of great length in the infra-red.

In the work which has been done by these investigators it has been assumed that the positions of the reflection maxima and the absorption maxima coincide, and dispersion formulæ have been used in order to predict the positions of the reflection maxima. This

[1] *Physical Review*, **4**, 297, 1897.

[2] *Annalen der Physik*, **60**, 418, 430, 1897. [3] *Ibid.*, **65**, 241, 1898.

assumption is not entirely justifiable, and the agreement between the calculated and the observed values for the very long waves is not sufficiently conclusive. One of the objects of this research was to accumulate facts which might possibly throw some light upon the problem. The data, however, in regard to the dispersion of substances available for experiment are too limited to enable one to draw any conclusions. It is hoped, nevertheless, that the few facts here added may at some time prove useful in the solution of the problem.

The following is a list of the substances which have been studied by others and the wave-lengths of their *Reststrahlen* measured: quartz, fluorite, sodium chloride, sylvine, mica, marble, sodium bromide and calcium bromide. A detailed discussion of these is to be found in the papers referred to above. Each substance is characterized by well-marked maxima, which occur in the grating spectrum after three or more successive reflections. Fourteen other crystalline compounds have been examined by the writer. Seven of them, viz., potassium dichromate, copper sulphate, tartaric acid, ammonium chloride, potassium sulphate, potassium bisulphite, and potassium ferrocyanide, show unmistakable maxima at various parts of the spectrum. A few words with reference to each of these will be said after the apparatus has been described.

APPARATUS

Radiometer.—The radiometer was selected for this work because of the great difficulty in working in this laboratory with an instrument which is highly sensitive to small changes in the electric or magnetic conditions. This reason has barred the use of bolometer and thermopile, while the radiomicrometer, the only other instrument which can be used in work of this kind, for equal sensitiveness, is not so reliable as the radiometer. The latter instrument, undoubtedly the best for infra-red work in this laboratory, is objectionable on account of the absorption due to the fluorite plate. Above 11 μ practically no radiation gets through. With it, therefore, measurements cannot be made on waves whose wave-lengths exceed this value.

The instrument here described is in almost every respect similar to the one used by Professor Nichols at the Yerkes Observatory in the summer of 1900.[1] Two vertical sections at right angles to each

[1] *Astrophysical Journal*, 13, 110, 1901.

other are shown on the following page. The scale of the diagram is three-eighths natural size for all parts except the suspension, *H*, which is approximately three-fourths natural size. Tube *A* was cemented to

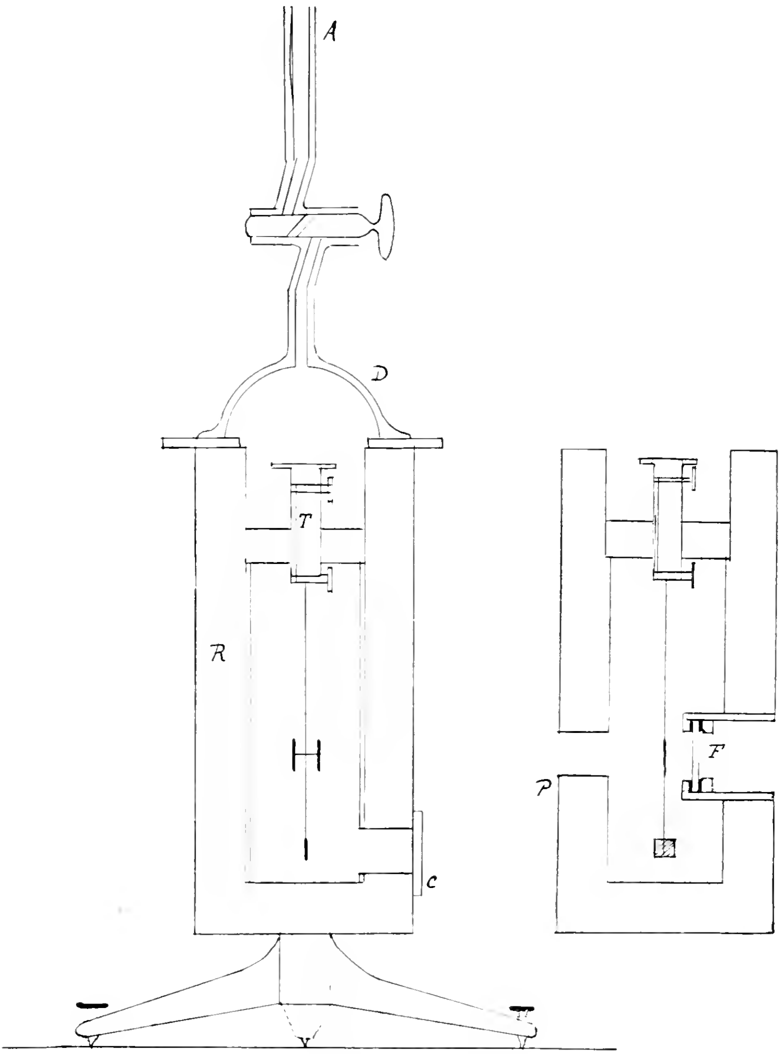


FIG. 1.—Diagram of Radiometer.

a drying tube containing phosphorus pentoxide with a Toepler mercury pump. On the frame supporting the pump was also placed a McLeod gauge which gave readings consistent to 0.001 of a milli-

meter of mercury. The openings at the lower end of the brass case R, three in number, were made air-tight in the following way: Over C and P glass plates were cemented by means of the rubber preparation ordinarily used for making stop-cocks air-tight. The window F was closed by means of a circular fluorite plate about 2 cm in diameter and 2 mm thick. This plate was placed between rubber washers, which had been previously smeared with the rubber preparation, and lowered into place at the inner end of a metal tube extending almost to the center of the case. A brass ring was then screwed down so as to hold the plate in place and secure an air-tight fit. The dome D was held in place by the same rubber preparation, and cemented to the glass connecting tube with Khotinsky cement. The rate of leak of the entire system, which contained besides these openings two stopcocks, amounted to about 0.003 mm in 24 hours, which is not large considering the number of possibilities for leakage.

The suspension.—The suspension was made in the following way: a very thin rod of glass about 3 cm long carried a cross-arm near its upper end. To the extremities of this arm were cemented, by means of hard shellac, rectangular mica vanes covered with lamp-black. The lower end of the vertical rod carried a small mirror placed at right angles to the plane of the vanes. A diagram and the exact dimensions are given below.

FIG. 2

$mn = 32$ mm

Vane on the right of the diagram

Length = 0.5313 cm (mean)

Width = 0.0685 cm (mean)

Area = 3.637 sq. mm

Vane on the left of the diagram

Length = 0.5305 cm

Width = 0.0687 cm (mean)

Area = 3.641 sq. mm

ab (outside measurements) = 0.5401 cm

cd (outside measurements) = 0.5427 cm

Mean = 0.5414 cm

$\frac{1}{2} \times 0.5414 = 0.2707$ cm

OH (measured) = 0.2728 cm

Size of mirror = 3×3 sq. mm

Total weight = $6\frac{3}{4}$ mg.

A quartz fiber attached the suspension to the torsion head.

Sensitiveness.—In the radiometer it is well known that there is a critical pressure at which the sensitiveness is a maximum. The accompanying curve shows the relation existing between sensibility and pressure for the instrument under consideration. Abscissæ are pressures, ordinates are deflections. In plotting this curve there was used a 76-volt, direct current Nernst filament supplied by a storage battery with a constant current of 0.32 of an ampere. At the time

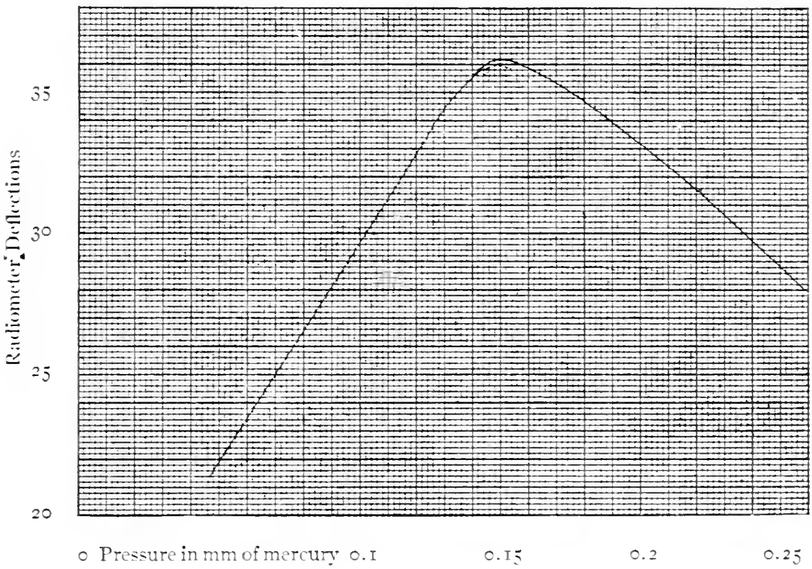


FIG. 3.—Sensibility Curve of Radiometer.

the observations were made the battery was being used for no other purpose. It will be observed that the maximum sensibility falls at about 0.15 of a millimeter. This critical pressure varies greatly in different instruments. Values ranging from 0.05 mm in Nichols' radiometer to 0.15 mm in this case, are to be found. It has been suggested that the McLeod gauges which have been used to measure the pressure are responsible for the discrepancy.

The sensitiveness was also tested by means of a paraffin candle in order to compare it with that of other instruments of a similar construction. The three radiometers compared in the following table are: radiometer used by Nichols[1] at the Yerkes Observatory

[1] E. F. Nichols, *loc. cit.*

in 1900; one used by Stewart[1] at Cornell in 1901; radiometer used in this investigation. The deflections have been reduced so that the numbers correspond to deflections on a scale one meter from the mirror due to a candle one meter from the vane.

COMPARISON OF SENSIBILITIES OF VARIOUS RADIOMETERS

COMPARISON SOURCE—A PARAFFIN CANDLE

By Whom Constructed	Vanes	Area of Vane in sq. mm	Deflection (Candle and Scale 1m off)	Deflection per sq. mm	Relative Sensibilities	Time Required for Maximum Deflection in Seconds
Nichols............	Mica	3.14	395	126	1.0	5.5
Stewart............	Platinum	30.00	1467	49	0.4	40.0
Stewart............	Mica	30.00	....	...	1.5	300.0
Porter.............	Mica	3.64	1000	275	2.0	45.0

Difficulties.—The difficulties met with in working with a radiometer have several times been enumerated, but a few words with reference to them in the present case may not be out of place. In general troubles arise from four sources: namely, (1) unsteadiness of the zero position, (2) mechanical jarring, (3) leakage of the radiometer case and connections, and (4) static charges on the vanes. Leakage and charges on the vanes gave practically no trouble after the instrument was finally gotten into working condition. The rate of leak, as has been seen, was too small to be in the least annoying, and the size of the vanes permitted them to be placed at such a distance from the fluorite window that, while still fulfilling the conditions of sensitiveness, they could turn completely round without striking against it and consequently becoming charged, as frequently happens in the case of larger suspensions.

Unsteadiness of the zero position has given far more trouble. In so sensitive an instrument it is to be expected that such a difficulty will arise. This variation of the zero in the radiometer may be reduced in two ways: (1) by care in the construction of the suspension, and (2) by guarding against irregular distribution of the radiant energy reflected from objects situated obliquely in front of the fluorite window.

If the suspension were perfectly symmetrical with reference to the

[1] *Physical Review*, **13**, 263, 1901.

axis of rotation, any source of radiation, no matter how intense, to which the vanes are equally exposed, should produce no effect. It is, however, not possible to secure perfect symmetry; therefore deflections may arise due either to the unequal absorption of radiant energy by the vanes, or to inequality in the length of the two arms, or to both these causes combined.

Furthermore, the vanes are located at the inner end of a comparatively long tube; consequently, if the objects situated obliquely in front of this tube radiate unequally, it is possible that both vanes may not be at the same time exposed to the action of equal forces. There will therefore result a rotation. This difficulty may be overcome by resorting to screens which will assist in securing a uniform distribution of the energy in the neighborhood of the radiometer. The figures given above give some idea in regard to the symmetry of the suspension, and the diagram of the apparatus shows the positions of the sheet-iron screens about the radiometer.

The source of the greatest annoyance has been mechanical jarring. The radiometer as well as other parts of the apparatus was supported upon marble slabs resting upon iron bars built into the wall of the laboratory. Nevertheless it was impossible to make observations during the day time. In the experiments on the *Reststrahlen* the readings were all taken between the hours of 8 P. M. and 12 P. M., when there was no other person in the building. Even under these conditions much time was lost in waiting for the effects of passing cabs to subside. I have found that when perfect quiet is obtained the deflections of this instrument are entirely reliable to one-tenth of a millimeter on a scale one meter off.

The spectrometer.—The divided circle used is one which has been employed in this laboratory for testing small plane gratings. This circle was set upon a heavy iron base and a steel arm so mounted that it, together with the circle, could be rotated in a horizontal plane about an axis passing through the center of the circle. This arm carried three things: (1) a small concave mirror of 52 cm radius of curvature, made by Bausch & Lomb, (2) a Nernst filament, the ballast being placed on the wall, and (3) a wire grating so mounted that the axis of rotation lay in the plane of the wires. The second mirror of the spectrometer, which was of the same size and make as

the one mentioned above, was fixed. The arrangement of the apparatus is shown by the diagram on the following page, which represents a horizontal section.

The letters indicate the following:

A, spectrometer table.

n, Nernst filament.

Q, sheet-iron screens.

W, steel bar.

m, *m*, silver-on-glass concave mirrors.

G, grating.

K, movable screen operated by a string in the hand of the observer at *H*.

V, *V*, verniers.

S, slit.

S_1, S_2, S_3, surface under consideration.

M, large silver-on-glass concave mirror.

R, radiometer.

C and *D*, switches in the incandescent lamp circuits.

F, a resistance box.

L, a brass rod by means of which the bar *W* can be turned about *O*.

T, a telescope for reading the radiometer deflections.

T′, a telescope for reading the vernier.

B, an ammeter indicating the strength of the current through the Nernst filament.

I, a sheet-iron screen surrounding the radiometer and mirror.

P, a plane silver-on-glass mirror.

The large mirror *M* was made by Bausch & Lomb, and has a diameter of 12 cm and a radius of curvature of 26 cm.

In order to read the vernier *V* by means of the telescope *T′*, a small plane mirror was mounted above it making an angle of 45° to the vertical. A miniature incandescent lamp, which could be turned on and off by means of a switch near the observer at *H*, illuminated the scale and vernier. It can be seen from the arrangement of the apparatus that while taking a series of readings it is entirely unnecessary for the observer at *H* to change his position and this rarely occurred.

The source.—The source was a 76-volt, 0.44 of an ampere direct

current Nernst filament supplied by a storage battery, which, while readings were being taken, was in use for no other purpose save for supplying the 14-volt incandescent lamps which illuminated the

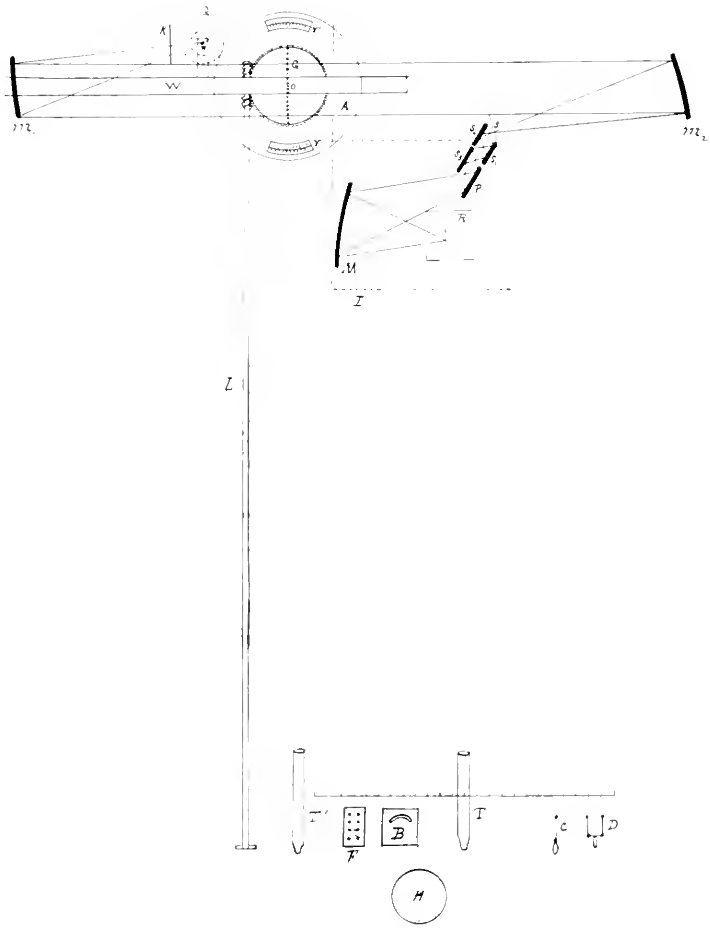


FIG. 4.—Diagram of Apparatus.

scale and vernier. These, as a rule, were kept burning continuously during a series of observations.

The source circuit contained in series a resistance box and milliammeter so placed that the current could be watched, and if necessary controlled, while readings were being taken without the observer's

moving from position. The current variation was very small, never exceeding a hundredth of an ampere and frequently absolutely no change in the reading of the ammeter could be detected.

The grating.—The grating was made in a way similar to that described by Rubens and DuBois in *Naturwissenschaftliche Rundschau*, **8** (No. 36), 1893. A heavy brass frame, represented in the diagram, was placed in a lathe, the ends of two wires of as nearly the same diameter as possible soldered at *A*, and wound under tension about *M* and *N*. After perhaps five centimeters of the length of the frame had been covered in this way, the wires were soldered at *B* and the whole stretched by means of nuts *H* and *K* provided for the purpose. One of the wires was then cut and carefully unwound, while the remaining one was made fast to the brass pieces *M* and *N* by depositing electrolytically upon them a comparatively heavy coating of copper. This done, the wire on one side was cut away. The spacings between the wires of a grating made in this way are very nearly equal to the diameter of the wire. The grating constant was determined by means of a dividing engine as follows: A setting was made on the edge of each wire and the value of each space determined for the whole grating. The mean of these values was then averaged with the value of the constant found by taking the first and last readings and dividing the difference between them by the number of spaces. The value of the constant for the grating used in the wavelength determination was found to be 0.2414 of a millimeter.

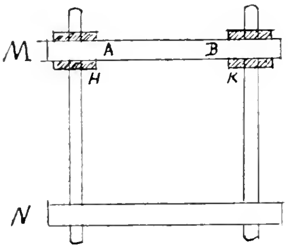


FIG. 5.

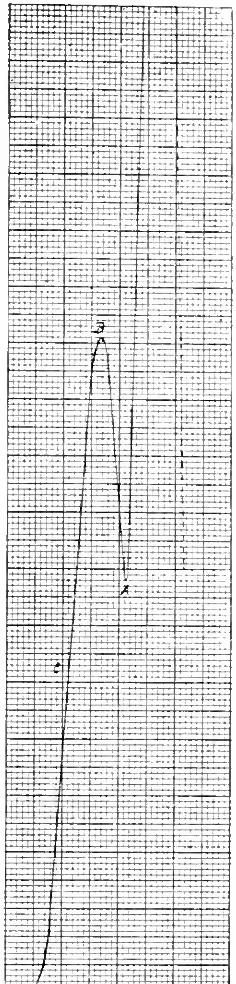
Adjustments were made in the following way. After having first placed the grating *C* so that the plane of the wires included the prolongation of the axis of rotation, as nearly as this direction could be determined, the mirror *m* was moved along *W* until the reflected beam became parallel, then rotated about a vertical axis until normal incidence upon the grating was secured. By means of *m* the spectrum was brought to a focus in the plane of the slit *S*. These adjustments having been made, it was then only necessary to turn the rod *L* in order to bring any desired portion of the spectrum upon the

slit. If the slightest change in the angle of deviation was to be indicated by the radiometer, it was necessary that the image of the filament and the slit *S* should have exactly the same width, and since the mirrors m_1 and m_2 had the same focal lengths, *S* had to have a width equal to the diameter of the filament in order to secure this result. This width was generally slightly less than one millimeter.

As soon as the current begins to flow through a Nernst filament the filament twists out of its original position. Observation showed, however, that after the current had once started and the filament had been adjusted parallel to the slit there was no relative shift of the image and slit during a series of observations.

The surfaces S_1, S_2, S_3, supported on comparatively heavy iron blocks, resting on a wooden platform, were placed at about the angle represented in the diagram, no special care, however, being taken to secure accuracy in this respect. The mirror *M* focused the image of the filament on the radiometer vane which was slightly smaller than the image itself.

The method of measurement is this: note the spectrometer reading when the central image is focused on the vane, pass the spectra



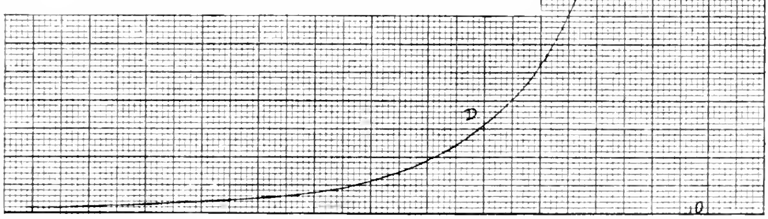


FIG. 6.—Curve showing the distribution of energy in the grating spectra of a 76-volt, direct current, Nernst glower. Abscissae are angles of deviation; ordinates are radiometer deflections.

Scale: Abscissae, 1 small division = 2′;
Ordinates, 1 small division = 1 mm on radiometer scale.

across the slit by turning L, and if the surfaces show selective reflection for a particular wave-length, the deflections of the radiometer will rise to a maximum as that portion of the spectrum falls on the slit. The difference between the spectrometer readings in the two cases is the angular deviation. The sine of this angle times the grating constant, if the spectrum be the first order, gives the wave-length. Before making any new measurements it was thought best to repeat some which had already been made. For this purpose four quartz plates, 4×4 cm, were placed at the points indicated in the diagram. The sensibility of the radiometer was such that the aperture of the mirror m had to be reduced considerably in order to bring the deflections within the range of the scale. The value of the wave-length determined from the position of the first maxima was 8.14 μ. Since this value was much too low according to the measurements of Rubens and Nichols, and since no special care had been exercised in procuring normal incidence upon the

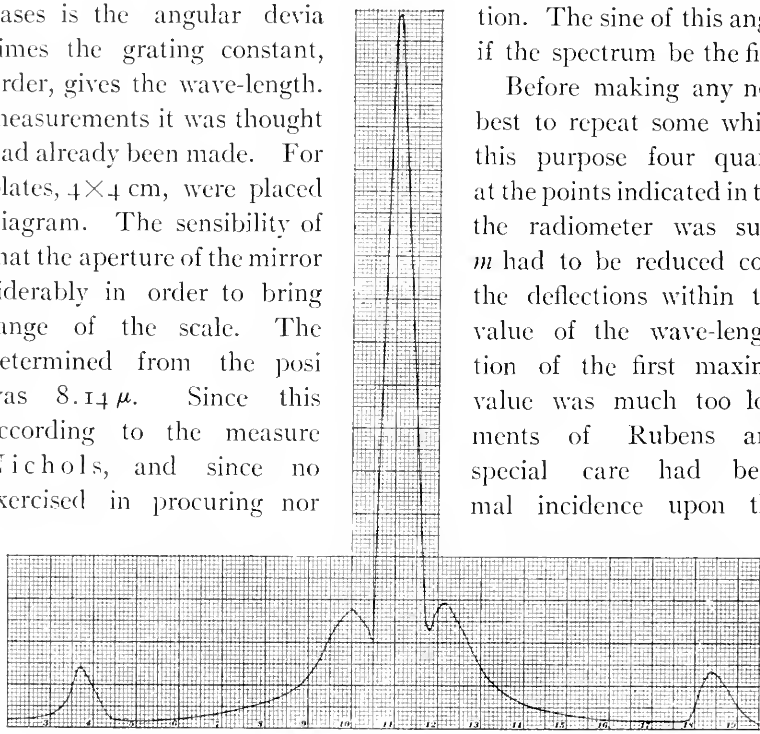


FIG. 7.—Potassium Dichromate.

Scale: Abscissae, 1 small division = 2′;
Ordinates, 1 small division = 1 mm on radiometer scale.

grating, the adjustments were all made again in as careful a manner as possible. The mirror m was moved until the diameter of the beam reflected across the room remained constant. In order to secure normal incidence a long narrow mirror with the silvered side next to the wires was placed carefully on the grating above the portion which was being used, and the mirror m turned about a vertical axis until the reflected image of the filament lay in the prolongation of the filament itself. The wires of the grating and slit S were

arranged parallel to each other by means of a fine, silk-thread plumb-line viewed through a telescope. The filament was then adjusted parallel to S. These adjustments were of course all made after the spectrometer table had been leveled. The quartz surfaces were then put in place, and the positions of the first maxima on either side of the central image determined as follows. The observer at H moved the spectrum across the slit S by turning the rod L until he was sure he had passed beyond the position of the maximum in the first order spectrum. L was then turned slowly in the reverse direction as the observer watched the action of the radiometer through the telescope T. When it became evident that the maximum was approaching the slit of the spectrometer, the screen K was lowered and when the steady conditions were reached K was raised and the deflection noted. K was then again lowered and another setting made, and so on until the first spectrum on the opposite side of the central image was reached. In the neighborhood of the position of maximum deflection settings at

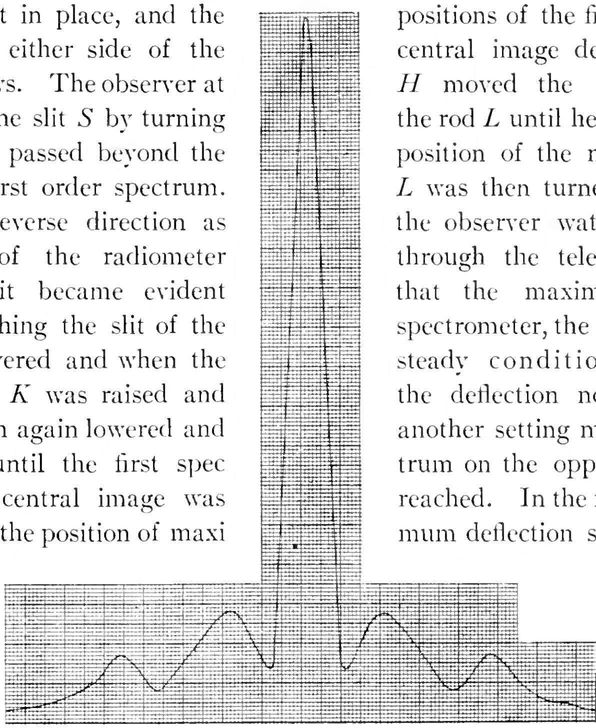


Fig. 8.—Copper Sulphate.

Scale: Abscissae, 1 division = 1′;
Ordinates, 1 division = 1 mm of deflection.

every minute of arc on the spectrometer were made, care being taken always to make the setting by turning L in the same direction. In order not to bias the judgment no differences between spectrommeter readings were taken until the observations were completed. Regarding the position of the central image as the zero on the spectrometer, the positions of the maxima were on the one side $1° 58'$, on the other $1° 58'$. For a grating constant equal to 0.2414 mm the value of the wave-length for this angle is $8.28\,\mu$.

Although two different gratings and four different sources, namely, a Nernst filament 110-V., A. C., a Nernst filament 76-V., D. C., a Welsbach mantle, and a hot platinum wire have been used, I have been totally unable to obtain a value higher than this. It is with some hesitation that I give these results, my only excuse being that I have sought diligently for a source of error and have been able to find none. Objection might be raised to the use of a Nernst filament without a slit. To this it seems only necessary to say that the value of the wave-length obtained when the filament was replaced by a Welsbach and slit was $8.25\ \mu$. Frequently after having determined the positions of the three maxima, i. e., the first order on the left, the central image and the first order on the right, the spectra were again shifted across the slit and the positions of the three maxima again determined, with the same result within the limits of error of observation, thus always showing that the apparatus underwent no change during a series of observations.

Aschkinass[1] has found that marble (white) reflects strongly in the neighborhood of wave-length $6.7\ \mu$, which value he obtained by the method of *Reststrahlen*. The value I have obtained for white marble is $6.77\ \mu$.

The remainder of this paper will be devoted to a consideration of substances whose *Reststrahlen* have not been determined before. It was not possible to cut the crystals with any reference to the optic axis, nor does

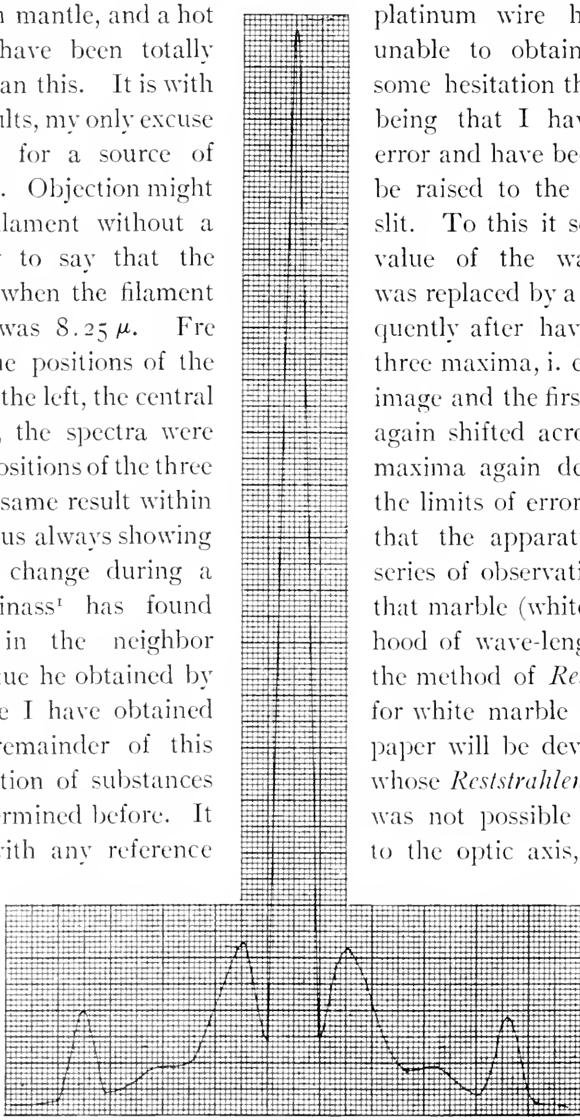


FIG. 9.—Tartaric Acid.

Scale: Abscissae, 1 division = 1′;
Ordinates, 1 division = 2 mm of deflection.

[1] *Annalen der Physik.*, **65**, 241, 1898.

this seem necessary, for the experiments with quartz have shown that the phenomena are independent of the direction in which the faces are cut. Furthermore, owing to the small size of the crystal, the number of surfaces used has been uniformly only three.

Potassium dichromate.—Crystals of this substance readily take a high polish. In area the surfaces obtained varied from three to six square centimeters. The curve is plotted in the way usually employed in measurements of this kind with spectrometer readings as abscissæ and deflections of the radiometer as ordinates. The curve is therefore symmetrical about the maximum deflection corresponding to the central image.

In order to make clear the meaning of this as well as the curves that follow, I have added another which shows the distribution of the energy from a 76-volt, direct current Nernst glower in the grating spectra, from the central image out beyond 10 μ in the first-order spectrum. This curve was obtained by substituting for S_1, S_2, S_3, silver-on-glass mirrors. The maximum deflection in the central image is not given because it was too large to be read on the scale, notwithstanding the fact that the aperture of the mirror

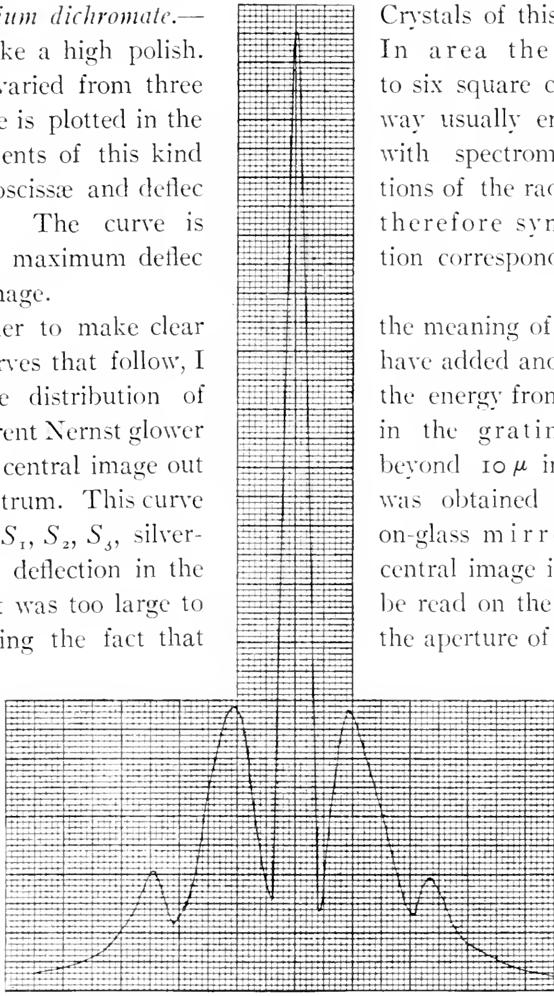


FIG. 10.—Ammonium Chloride.

Scale: Abscissae, 1 division = 2′;
Ordinates, 1 division = ; mm of deflection.

The central maximum is drawn to one-half scale.

m was cut down to the size of a pin head. *ABCD* shows the energy distribution in the first-order spectrum. *B* is the point of maximum emission of energy from the source. Its angle of deviation is approximately 20′, which corresponds to a wave-length $= 1.4\ \mu$. Assuming the law $\lambda_m \theta =$ constant, we get for the temperature of the glower 2062° C. Abs.

In all the curves given herewith we find these emission maxima on either side of the central image. An examination of the curves, however, will show that there is some variation in their position relative to the central image. This shifting may be due to several causes. In the first place the current through the filament was not the same for any two curves;

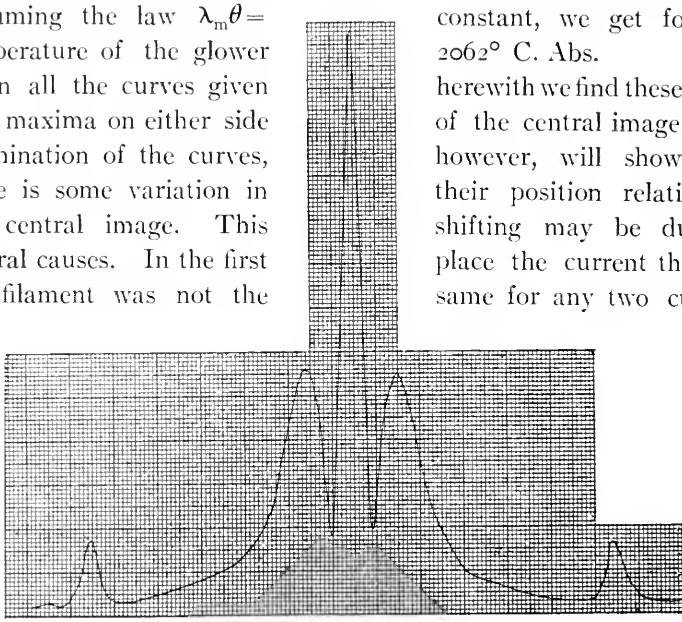


FIG. 11.—Potassium Sulphate.

Scale: Abscissae, 1 division = 2′;
Ordinates, 1 division = 1 mm of deflection.

The central maximum is drawn to one-third scale.

and in the second, if the substance should possess any strong reflecting power in the neighborhood of this maximum, the result would be an apparent shifting of the energy maximum, in either one direction or the other.

We will return to the consideration of the potassium dichromate curve. *D* and *E*, then, are the maxima due solely to the energy emission of the source. By increasing the number of surfaces these could doubtless be cut out, as was found to be the case with quartz. With three quartz surfaces these maxima occur in approximately the same position as in all the curves here given, but by increasing the number of surfaces to four they disappear, although the curve

slopes off gradually at the base on either side of the central image and does not fall to zero abruptly as does the one given by Rubens and Nichols for four surfaces. The cause of this difference is undoubtedly to be found in the greater sensitiveness of the instrument used in this work. *A* and *C* are the first-order spectra of the waves most strongly reflected by the substance, or what has been termed the *Reststrahlen*. In comparing this curve with the energy-curve of the source it is to be remembered that the aperture of the mirror *m* in this case was the total aperture, while in the case of the energy-curve obtained with the silver mirrors it was not over a millimeter.

The angles of deviation of *A* and *C* are 2° 28′ and 2° 26′ respectively. The wave-length corresponding to 2° 27′ is 10.31 μ.

Copper sulphate.—The quality and size of these surfaces were similar to those of the dichromate. The curve presents the same peculiarities, only appearing different because of the difference in the scales to which the two are drawn.

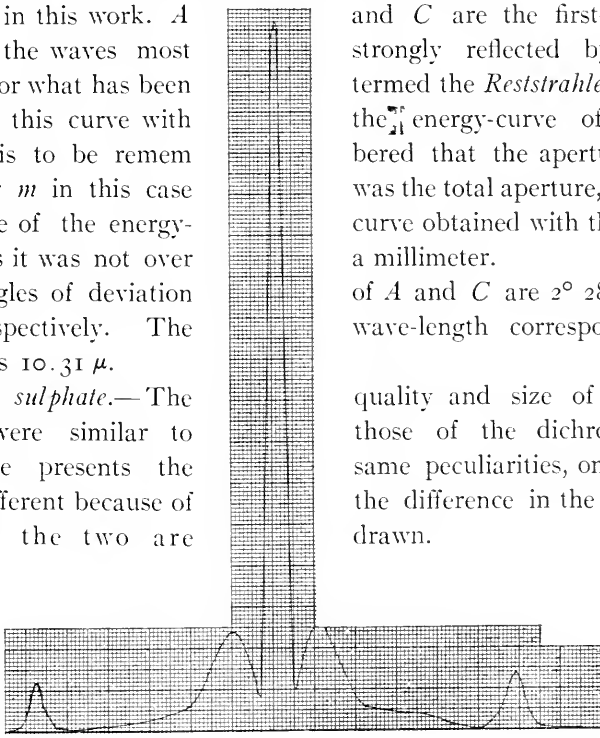


FIG. 12.—Potassium Bisulphite.

Scale: Abscissae, 1 division = 2′;
Ordinates, 1 division = 2 mm of deflection.

Positions of the maxima:

Energy maxima,	$18\frac{1}{2}'$ and $20\frac{1}{2}'$
Reststrahlen maxima,	$47\frac{1}{2}'$ and $46\frac{1}{2}'$

The wave-length corresponding to 47′ is 2.30 μ.

Tartaric acid.—The reflecting surfaces obtained in this case were exceptionally good, but the areas of the surfaces were very small, not over 3 sq. cm for the largest S_3.

Positions of the maxima:

Energy maxima,	19′ and 20′
Reststrahlen maxima,	1° 21′ and 1° 22′

Wave-length corresponding to 1° 21½′ is 5.72 μ.

Ammonium chloride.—The surfaces of ammonium chloride were not so good, but were much larger than in the above cases.

Positions of the maxima:

Energy maxima,	19′ and 21′
Reststrahlen maxima,	47′ and 48′

Wave-length corresponding to 47½′ is 3.44 μ.

Potassium sulphate.—Large crystals of this substance were not at my disposal. Aggregates of small ones, however, were found which were so compact that surfaces were readily polished on them. Although the surfaces obtained in this way were somewhat discontinuous, the amount of energy reflected was amply sufficient for the measurements. The area of the largest was not over 4 sq. cm.

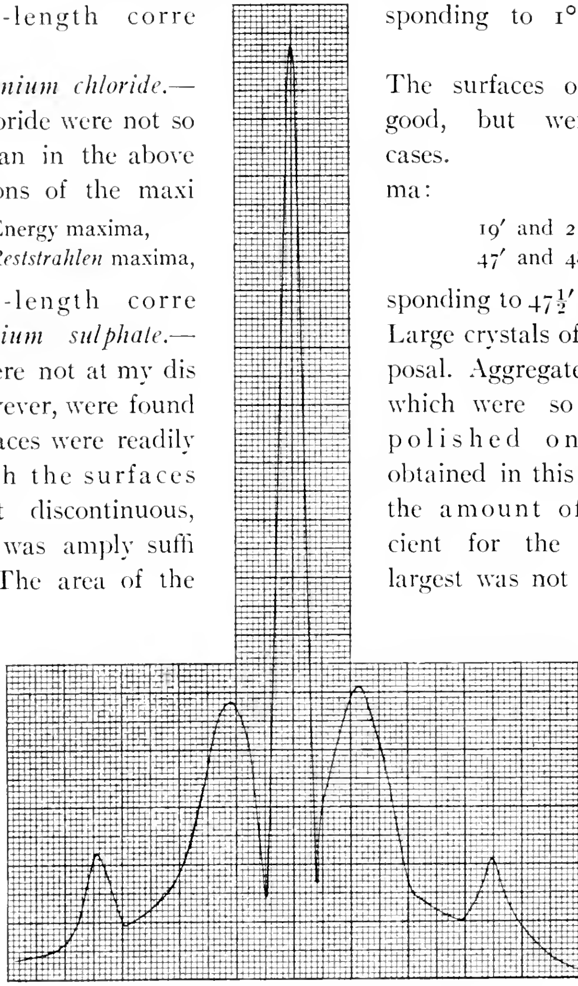


FIG. 13.—Potassium Ferrocyanide.

Scale: Abscissae, 1 division = 2′;
Ordinates, 1 division = 1 mm of deflection.

The central maximum is drawn to one-third scale.

Positions of the maxima:

Energy maxima,	21′ and 21′
Reststrahlen maxima,	2° and 2°

Wave-length corresponding to 2° is 8.42 μ.

Potassium bisulphite.—These surfaces were also obtained by polishing aggregates of small crystals as in the last case.

Positions of the maxima:

Energy maxima,	20′ and 21½′
Reststrahlen maxima,	1° 56′ and 1° 58′

Wave-length corresponding to 1° 57′ is 8.21 μ.

Potassium ferrocyanide.—The somewhat cheesy nature of potassium ferrocyanide rendered it difficult to obtain surfaces of sufficient reflecting power. Success was finally obtained and the accompanying curve shows the results.

Positions of the maxima:

Energy maxima,	22′ and 24′
Reststrahlen maxima,	1° 9′ and 1° 9′

Wave-length corresponding to 1° 9′ is 4.84 μ.

SUMMARY OF RESULTS

Wave-lengths of the *Reststrahlen* from various substances below 11 μ determined from measurements on first-order spectra.

Substance	Source	Wave-Lengths
Quartz	76 V., D. C. Nernst Filament	8.28 μ
Marble (white)	76 V., D. C. Nernst Filament	6.77 μ
Potassium dichromate	110 A. C. Nernst Filament	10.31 μ
Copper sulphate	110 A. C. Nernst Filament	2.30 μ
Tartaric acid	76 V., D. C. NernstFilament	5.72 μ
Ammonium chloride	" " "	3.44 μ
Potassium sulphate	" " "	8.42 μ
Potassium bisulphite	" " "	8.21 μ
Potassium ferrocyanide	" " "	4.84 μ

The following table, giving the substances whose *Reststrahlen* have been determined by others, is added for the sake of completeness.

Substance	By Whom Measured	Wave-Lengths in μ
Quartz	Rubens and Nichols	8.50, 9.02, 20.75
Mica	Rubens and Nichols	9.20, 18.40, 21.25
Fluorite	Rubens and Nichols	24.4
Rock salt	Rubens and Aschkinass	51.2
Sylvine	Rubens and Aschkinass	61.1
Marble	Aschkinass	6.69, 29.4

Before concluding I wish to say a few words in regard to some experiments which have been made with a view to determining whether or not the waves in the neighborhood of 8.3μ can be elliptically polarized by reflection from a quartz surface. The fact that one is compelled to keep both the source and the radiometer in fixed positions has rendered the necessary adjustments very difficult. For polarizer and analyzer I have used two small fluorite plates about $2\frac{1}{2}$ centimeters square. At this wave-length fluorite reflects only about 2 per cent. of the energy incident upon it; consequently, by the time the radiations have suffered reflection at two fluorite, and three quartz surfaces there is very little energy left at one's disposal. Although I have not succeeded in carrying out the experiment to a definite conclusion the preliminary tests have shown that the radiometer is sufficiently sensitive to detect the small quantity of energy with which one has to deal. I therefore hope to obtain some results which will be conclusive on this point.

To Professor Joseph S. Ames, under whose direction this work has been done, I am greatly indebted, chiefly for his valuable suggestions and his unfailing interest.

Johns Hopkins University,
Baltimore, June 1905.

ON THE ANOMALOUS TAILS OF COMETS

By E. E. BARNARD

It seems to be the custom in general to give to the Sun the main credit for all the phenomena of a comet's tail, the supposition being that the comet itself merely supplies the material for the Sun to work upon. This idea could generally be reconciled with the facts known previous to the revelations made by the photographic plate. But the photographs of the last ten or twelve years have shown us new phenomena, many of which cannot be reconciled to the old ideas of a comet's tail. They are entirely of a revolutionary nature and are not easily explained. I have long suspected that other agencies besides the Sun entered into the making of these phenomena, and I now believe there are three main causes which often combine to produce the results shown in photographs of some of the recent comets. These are:

First, the Sun, which produces in the nucleus of the comet a disturbing action, and which influences the general direction of the tail-producing particles.

Second, the comet itself. A strong ejective power seems to be seated in the comet, by which the matter forming the various tails is ejected. That this is true is shown by the fact that straight minor tails or streams of particles are often seen to issue at large angles to the direction of the main tail. Such peculiarities are quite contrary to the effect produced by the Sun. It would seem that this power is great enough to overcome entirely the direct pressure of the Sun's light.

These small tails, as I have said, are generally straight. One would expect the pressure of the Sun's light to bend them more or less toward the radius vector, but this does not seem to be the case. It would therefore appear that the Sun has but little influence on these small divergent tails.

It is probable that if, after the nucleus becomes active, the effect of the Sun's repulsive force were removed, tails would be shot out

from the head of the comet in all directions. This is, of course, not opposed to the generally accepted theory of cometary tails.

Third, an outside influence that has no concern whatever with the comet itself and whose effect is purely an accident of circumstances. This is shown in the remarkable and rapid distortions and deflections of the tail or tails. There is clear evidence that this is some sort of resistance offered by some kind of medium not uniformly distributed in the planetary spaces. It may be a meteor swarm of sufficient density to break the comet's tail, or to distort or deflect it. There is no question but these sudden distortions or deflections are independent of any action of the Sun or of the comet itself. Their causes may not necessarily be due to meteor swarms or streams. As I have suggested elsewhere,[1] there may be currents of resistance of some kind moving about the Sun capable of producing these phenomena, of which we know nothing at present. If so, their presence would be indicated alone by the distortions they produce in comet tails. That these influences do not permanently exist throughout the solar system is shown by the fact that only occasionally does a comet show their effect. The average comet with a tail usually shows only phenomena that can be satisfactorily explained by the first two causes.

Encounters with such a medium would readily explain the sudden brightening up of some comets when long past their greatest theoretical brightness—such, for instance, as shown by Sawerthal's comet in May 1888; or it might account for the breaking up of such a comet as Biela's. Whatever the supposition may amount to, there is forced upon us the fact that some outside influence beyond the control of comet or Sun entered into the production of the peculiarities of the tail of Brooks' comet in October and November 1893.

I have already[2] called attention to the remarkable disruption of the tail of this comet, as shown on the photographs between October 20 and 23, 1893, but the phenomena of November 2 and 3 of that year have not yet been studied. The photographs made on these last two dates perhaps bear as strikingly as those of October on the question at issue, and they more clearly show the deflection of the tail. At the Astronomical Congress in September 1904, at St. Louis, I exhibited a lantern slide made by superposing the two orig-

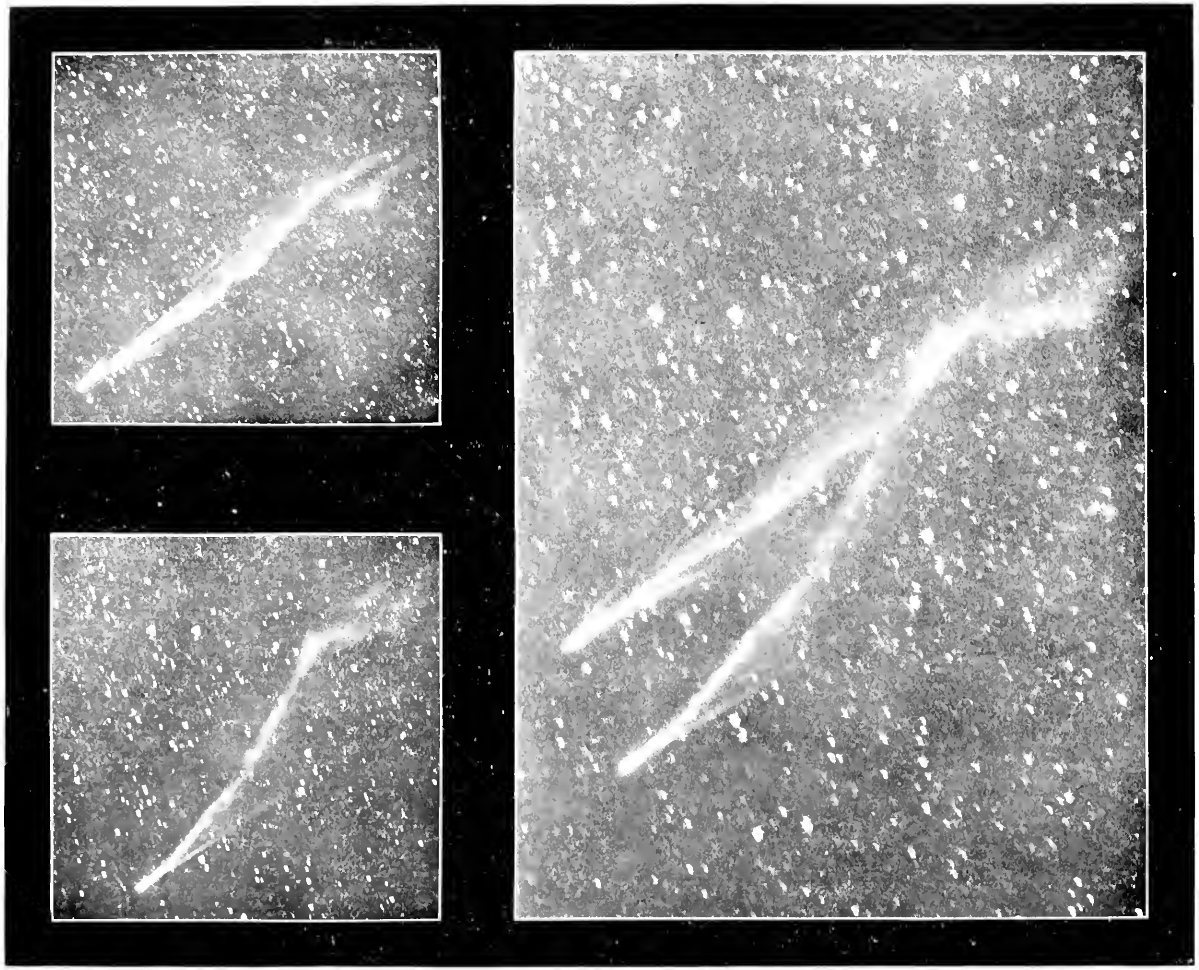
[1] *Monthly Notices*, **59**, 355, 1899. [2] *Popular Astronomy*, **12**, 1, 1904.

PLATE VIII

North

II
1893, Nov. 3,
$16^h\ 0^m$ to $17^h\ 30^m$
P. S. T.

I
1893, Nov. 2,
$16^h\ 0^m$ to $17^h\ 25^m$
P. S. T.



Combination
of I and II

Scale: 1 mm = 3′

COMET 1893 IV (BROOKS), SHOWING ANOMALOUS TAILS

inal negatives, star for star, which would thus show on one plate the two pictures of the comet separated by the amount of motion in the one day's interval. This method does not appear to have been used before for showing the changes in a comet's tail. As the two plates covered essentially the same region of the sky, the stars of the last plate were easily superposed on those of the first night; and when an exact register was made, the effect was the same as if two instantaneous exposures of the comet on the two dates had been made on the same plate, using the same guiding star for both exposures. If one examines thus these two photographs of this comet, he is forced to the conclusion that the deflection of the tail in the interval between the two photographs cannot be explained by any of the ordinary causes that go to produce the phenomena of a comet's tail. In this case the computed position angle of the radius vector for the two dates was nearly the same, 324°. In the photographs the two tails should therefore be parallel to each other. But there is a difference of some 15 degrees between the two. (Observed position angle: November 2, 322°; November 3, 306°; the deflection comes from the direction in which the comet was moving.) The strange thing is that the ends of the tails seem not to have changed their position in space while the head has advanced in the comet's path. This phenomenon is almost exactly reproduced in the photographs of November 6 and November 7. There was little change in the computed position angle of the radius vector for some time, and if at about this date any other two successive plates of the comet are superposed, it will be seen that the tail has moved essentially parallel to its first position.

In the accompanying plate the large picture is a composite made by superposing the two negatives of November 2 and November 3. This exhibits well the peculiarities I have mentioned of the lagging of the end of the tail, as if it were delayed by a resistance of some kind. In this picture there are two regions of greater brightness where the tails combine. The first of these is at α 12^h $35^m.2$, $\delta + 28° 12'$. This one is mainly due to the tail of November 2. The second is in α 12^h $30^m.4$, $\delta + 28° 24'$ (1855). It is due mainly to the tail of November 3.

A few points with respect to the individual pictures which are also given on Plate VIII may aid in the study of the photographs.

Photograph of 1893, November 2.—The length of the tail to the abrupt end is $3^\circ.7$; the extreme length of the tail, $5^\circ.4$; while the small narrow tail which runs northwest from the head is 2° long.

Photograph of 1893, November 3.—The extreme length of the tail is 5°. There is a large detached mass $5^\circ.0$ from the head, the position of the center of which is $\alpha\ 12^h\ 32^m.0$, $\delta + 28^\circ\ 25'$ (1855).

The following are the positions of the head of the comet as taken from the *B.D.* charts:

1893, November 2, $16^h\ 42^m$, Pacific Standard Time
$\alpha\ 12^h\ 45^m.2$, $\delta + 25^\circ\ 14'$ (1855)

1893, November 3, $16^h\ 45^m$, P. S. T.
$\alpha\ 12^h\ 46^m.8$, $\delta + 26^\circ\ 6'$ (1855)

In comparing these two pictures, I am not able to identify with certainty any one feature common to both photographs. The changes have doubtless been so rapid that nothing forming the tail in the first case remains as a part of it in the second case. I do not think that even the small, narrow tail running to the northwest is the same in both photographs. The detached mass at the end of the tail on November 3 might be supposed to be the bright mass at the end of the tail of November 2, but I do not believe this is so, for the changes were entirely too rapid for it to have lasted twenty-four hours. Yet one is tempted to believe them the same, for the end of the tail seemed to be detained in that region. I do not think it necessary to call attention to any other details, for the photographs will speak for themselves in this matter.

A list of the brighter stars shown on the photographs may be of use in the study of the peculiarities of the tails.

These are quite easily identified on the photographs. Referring to the picture of November 2, No. 1 is $\frac{1}{2}^\circ$ west of the abrupt end of the tail. Nos. 2 and 3 are the two stars $\frac{1}{2}^\circ$ west of the middle of the tail. Nos. 4 and 5 are the two 1° west and south of the head. No. 6 is the one close west of the tail, 1° back from the head. Nos. 7 and 8 are the two $1\frac{1}{2}^\circ$ east of the end of the tail, No. 8 being the eastern of the two.

I have previously shown[1] in the photographs of Borrelly's comet

[1] *Astrophysical Journal*, **18**, 210, 1903.

B. D. No.	Mag.	α (1855)	δ	Current No.
+27°.2158	7.9	12ʰ 33ᵐ 47ˢ.3	+27° 56′.3	1
+26.2383	6.8	12 36 9.5	+26 55.2	2
+26.2385	7.5	12 36 27.5	+26 27.4	3
+24.2493	6.5	12 39 25.7	+24 56.5	4
+24.2495	7.1	12 40 51.6	+24 53.5	5
+25.2568	6.0	12 41 43.6	+25 38.6	6
+28.2153	6.2	12 42 15.8	+28 20.4	7
+28.2156	5.0	12 44 41.4	+28 20.0	8

of 1903 that in that comet, at least, the tail really moved bodily out from the comet at a comparatively slow velocity, small in comparison with the velocities often attributed to comet tails, at the same time partaking of the onward motion of the comet; and that, if disconnected from the comet at any moment, it would continue to drift in space for at least several hours as a visible body.

This all fits in closely with the idea of a disruptive effect due to the tail meeting with some resisting medium in the case of Brooks' comet.

I am aware of the theories necessary to account for the apparently extraordinary flight of the tails of the great comets of 1843 and 1882 around the Sun, which would absolutely preclude the idea of an actual onward motion of the tail for any considerable time. Such conditions were, however, evidently entirely anomalous, and were due to the extremely close approach of these bodies to the Sun. Perhaps the velocity with which the particles leave a comet under the influence of the Sun's light varies inversely as the squares of the distances from the Sun, and hence in the case of such comets as those just referred to the initial velocity of the particles might be many thousands of times that of the particles leaving Borrelly's comet. In the comet of 1843 the particles would probably have a velocity of as much as 100,000 miles (160,000 km) a second.

From these considerations it is evident that every active comet should be photographed as often as possible. Photographs made on the same night will be especially valuable, and these should be made as frequently as possible during the night. It is from such photographs alone that the progressive changes can be followed intelligently. The changes from day to day in a comet are so very great

that it is usually not possible to connect certainly any phenomena of one night with that of the night before, as no part of the tail would be likely to live over the interval. But from successive photographs on the same night—especially when the comet is bright enough to permit short exposures—the actual changes may be observed and measured with certainty.

If an active circumpolar comet were to appear here in the winter time, it would be possible to get five or six or more successive pictures with exposures of from one to two hours. From such photographs the actual history of the changes could be studied, and in all probability with the most remarkable results.

The day-to-day history of a comet has too great an interval, and the changes are not necessarily at all connected. It is the hour-to-hour history that must be studied to understand the changes taking place in the comet. In the case of a very bright comet, exposures at intervals of half an hour should be made as long and as continuously as the conditions will permit. By this means it will be possible to determine the exact value of the motion of the particles in the tails of various comets, or of the same comet, at different distances from the Sun. The true law of the velocities of these particles with respect to the Sun and comet will thus become known independently of any prevailing theory.

Is it not more likely that the different tails of a given comet are all made up of the same kind of particles, and that the cause of their different directions is not so much due to different densities as to the fact that they are ejected toward different parts of space by a force residing in the comet itself? This would seem to me to be a more reasonable supposition than that the repellent action of the Sun had sifted out various elements of the comet and shot them forth with velocities conformable to their densities. It is not improbable that different comets are much alike in their chemical composition. The spectroscope has certainly not shown them to be of very diverse elements.

The tail is doubtless made up of particles of matter that recede from the comet in a general direction away from the Sun. This is evidently due to an influence exerted by the Sun upon the particles of the comet. There may be many tails, some of which diverge from

the radius vector by considerable angles. They are straight streams of matter apparently shot out by the nucleus quite in defiance of the Sun's power to force them in the direction of the radius vector. If these were entirely due to the expulsion of matter from the comet by the pressure of the Sun's light, these widely diverging tails would not be possible. It would appear that the generally accepted theory of comet's tails at least needs much modification. More credit must be given to the comet for the production of its tails than has heretofore been accorded it.

YERKES OBSERVATORY,
October 13, 1905.

ON THE SPECTRUM OF SILICON; WITH A NOTE ON THE SPECTRUM OF FLUORINE[1]

By JOSEPH LUNT

In a recent paper[2] M. A. de Gramont questions the silicon origin of certain lines, viz., λ 4089.1, λ 4096.9, and λ 4116.4, grouped together as Group IV by Sir Norman Lockyer,[3] who ascribes them to the element named. He says: "J'ajouterai que les lignes du groupe IV, qui indiqueraient, d'après Lockyer, une température excessive, ont toujours, sur mes clichés, accompagné les raies de l'air et ont disparu avec lui. Elles coïncident avec des lignes de l'oxygène et de l'azote, et ces deux gaz ont été reconnus dans plusieurs étoiles d'*Orion* et dans *β Crucis*. Je crois donc le groupe IV attribuable à l'air."

Sir Norman Lockyer and Mr. Baxandall[4] have replied by bringing forward photographic evidence in support of their conclusions. While agreeing with the latter authors that the lines in question, *with the exception of* λ 4096.9, are really silicon lines, I consider that the evidence brought forward by them is in itself insufficient to establish their conclusions satisfactorily.

Nearly three years ago I prepared a paper, "On the Spectrum of Silicon from its Dissociated Compounds," for inclusion in volume 10 of the *Annals of the Cape Observatory*, but as this volume has not yet appeared, owing to other papers being still under preparation, it seems desirable to publish, in advance, an extract dealing particularly with the lines under present discussion, and to mention briefly some other important lines. The extract is as follows:

"HIGH-TEMPERATURE LINES λλ 4089.1 AND 4116.4

"These two lines were first recorded as silicon lines by Lockyer in his 'Note on the Spectrum of Silicon,'[5] and it is of great interest to notice their behavior under different conditions. Of these lines

[1] From advance proofs of a paper presented by Sir David Gill to the Royal Society.

[2] *Comptes Rendus*, **139**, 188, 1904.

[3] *Proc. R. S.*, **67**, 403, 1900.

[4] *Ibid.*, **74**, 296, 1904.

[5] *Ibid.*, **65**, 449, 1899.

Lockyer writes: 'The lines in Group IV[1] have never been seen in the spark spectrum of silicium when small coil and small jar capacity are used,[2] but with the spark given by the Spottiswoode coil and plate condenser they appear as weak lines. They are not, like the members of Groups II and III, seen in the spectrum from the bulb when a vacuum tube is used, but in that given by the capillary the strongest ones are very prominent, and vie in intensity with the lines in Group III.'

"At the outset it may be stated that a large number of experiments had to be made before the confirmation of the silicon origin of these lines was considered satisfactory, but there can now be no doubt that Lockyer's identification is correct. These lines are absent in the list of lines given by Exner and Haschek, and by Eder and Valenta, and, so far as I know, no other observers have recorded them.

"In my earlier experiments with argon tubes[3] I had obtained these lines from the glass capillary, not only of argon tubes, but also from those of other tubes containing various gases; and concluded that they may be obtained from glass vacuum tubes, whatever the gaseous contents may be, provided that sufficient jar capacity and a suitable spark-gap are employed to decompose the glass. That I then doubted their silicon origin, however, is shown by the fact that they were not included in the list of silicon lines discussed in my first paper. This was owing to the fact that the spark spectrum of silicon tetra-fluoride had only been examined in wide tubes at atmospheric pressure, under which conditions the lines in question are absent, unless the immediate vicinity of the platinum electrodes is examined.

"I find that even at a pressure of 12.5 mm the glass capillary of a silicon tetra-fluoride tube fails to give these lines when a small jar and gap are used, although the other silicon lines are very pronounced. If, however, the pressure be reduced to 3 mm, still using one small jar and gap, these lines come out strongly and are almost as strong

[1] Group IV consists of three lines, the two above, and one, λ 1996.9, which I do not obtain in my photographs, and regard its silicon origin as doubtful.

[2] I show later that small coil and jar capacity suffice to bring out these lines strongly in the spectrum from the capillary of tubes of the fluoride.

[3] *Proc. R. S.*, **66**, p. 44, 1900.

as the strongest lines in the whole spectrum. With a similar pressure of silicon tetra-chloride, however, using the same jar and gap, these lines are exceedingly weak, while the rest of the silicon lines are strong.

"It is thus evident that the silicon spectrum from a mixture of silicon and chlorine (dissociated silicon tetra-chloride) is very different from that obtained from a similar mixture of silicon and fluorine (dissociated silicon tetra-fluoride). The effect of the chlorine being, apparently, to lower the temperature of the gas, and so extinguish the lines which require the highest temperature for their production.

"Lockyer[1] found that the presence of the chlorine in the dissociated chlorides of various metals had the effect of extinguishing the short, and therefore presumably high-temperature, lines, for he writes: 'It was found, in all cases, that the difference between the spectrum of the chloride and the spectrum of the metal was, *that under the same spark conditions the short lines were obliterated, while the air lines remained unchanged in thickness.* Changing the spark conditions by throwing the jar out of the circuit, this change was shown in its strongest form, the final results being that only the very longest lines in the spectrum of the metal remained.'

"This pronounced difference between the behavior of silicon tetra-fluoride and silicon tetra-chloride had the effect of again throwing doubt on the silicon origin of the lines under discussion. On examining, however, the photographs taken for the purposes of the former paper, in which the spark spectrum had been taken in hydrogen from beads of sodium and potassium silicates made from rock-crystal, it was seen that these lines *did* occur as short lines close to the beads, but not extending throughout the spark, as did the other lines. This, in itself, was another evidence, not only of their silicon origin, but also of the high temperature requisite for their production.

"All further doubt was, however, set at rest by preparing other beads of potassium silicate from carefully purified silica, made from silicic acid precipitated from silicon tetra-fluoride by water. The spectrum of these beads showed these lines as short lines as in the

[1] *Phil. Trans.*, **163**, 258, 1873.

case of rock-crystal silicate, and their length was not much increased by sparking the beads in the fused state.

"Accordingly, the weakness or absence of these lines from the capillary of silicon tetra-chloride vacuum tubes was attributed to the above-mentioned effect of chlorine. These lines can, however, be obtained from silicon tetra-chloride tubes as strong lines, having much the same relative intensity as those obtained by Lockyer from the bromide, by increasing the number and size of jars and the width of the spark-gap; *but only at the expense of decomposing the glass of the tube itself.* This decomposition of the glass is evidenced by the appearance of a strong spectrum of oxygen and the almost complete obliteration of the chlorine spectrum, much in the same way as the spectra of argon and helium can be obliterated and replaced by those of silicon and oxygen. The spectrum thus obtained is, in fact, practically identical with that obtained under similar conditions from a tube filled with pure oxygen at low pressure (2 mm), residual air, or any other gas, and cannot in any way be regarded as a spectrum of dissociated silicon tetra-chloride.

"A consideration of these facts suggests a serious objection to the acceptance of the spectrum obtained by Lockyer from a silicium bromide capillary vacuum tube by the use of the large Spottiswoode coil and plate condenser, as evidence that the lines in question are silicon lines. It is clear that they may be, and probably are, obtained from the glass tube, and might equally well belong to some other material contained in it.

"For example, the H and K lines of calcium and the D lines of sodium, and even the strong triplet of manganese, often accompany such spectra, and one might equally well attribute the lines to some other and possibly unknown substance.

"The weakness of the lines, when obtained from silicon itself, a substance likely to contain impurities, as results show, and the fact that Exner and Haschek did not obtain them from the specimens of silicon with which they worked, would rather suggest that they were due to some impurity in Lockyer's specimen of silicon; the fact of obtaining them as such strong lines from a silicium bromide capillary vacuum tube, under the conditions of his experiments, is no evidence to the contrary.

"Such evidence must, in fact, be obtained from carefully prepared pure silicates, or other pure silicon compounds, *sparked under such conditions that the presence of glass cannot possibly vitiate the results.* These two lines are present in ε *Canis Majoris* and other helium stars, as strong lines, together with other silicon lines, but the low-temperature silicon lines are either absent or very weak, the only low-temperature lines present in ε *Canis Majoris* being the persistent pair 4128 and 4131, which are weak and indistinct lines."

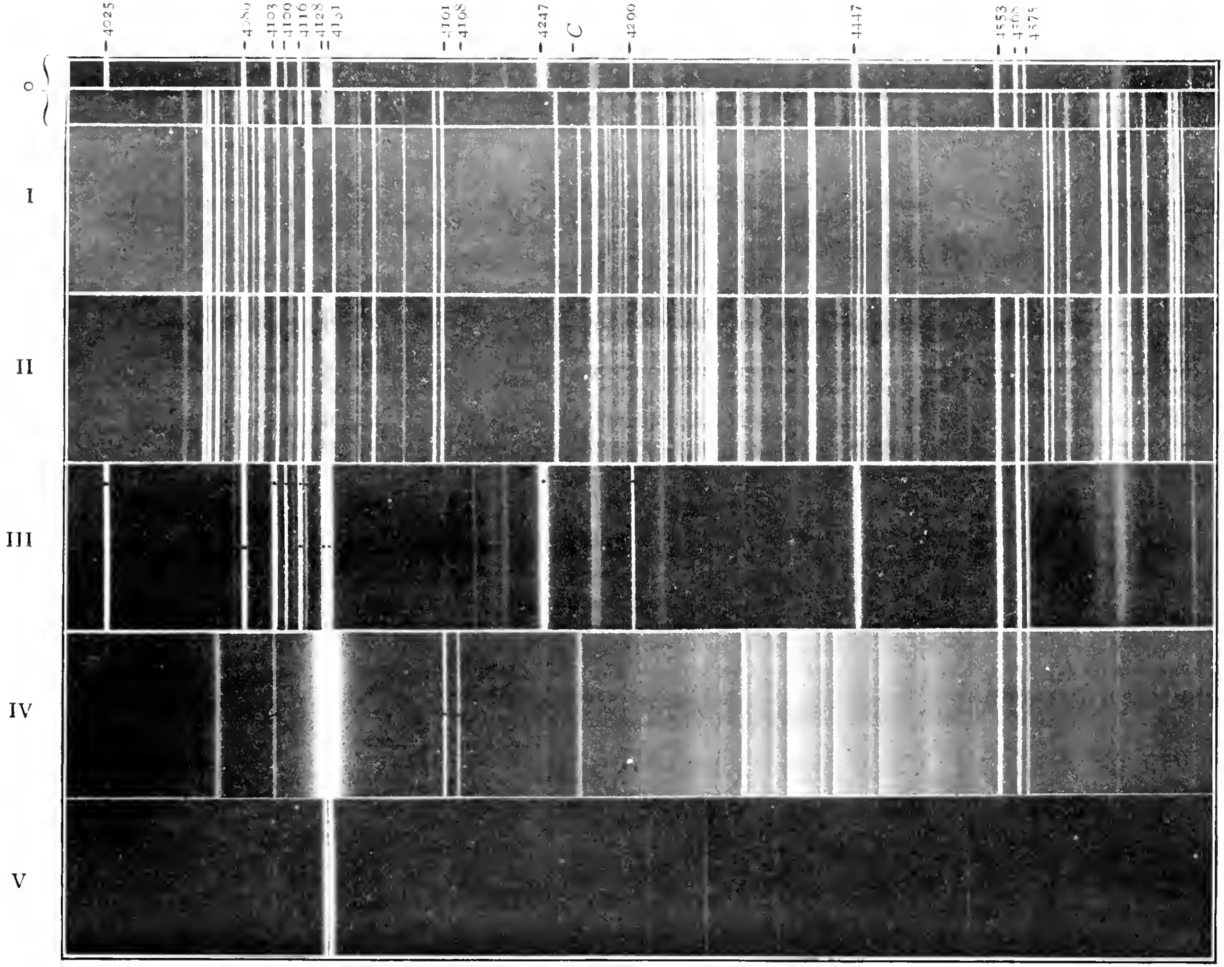
The photographs presented by Lockyer and Baxandall only serve to confirm the views expressed in the foregoing extract, viz.: that the silicon lines from their vacuum tubes filled with gaseous silicon compounds have their origin as much in the material of the glass capillary as in the gaseous compound introduced; and if we had no other evidence to the contrary, we might equally well say that the lines of calcium, sodium, and manganese, which appear in vacuum tubes so filled, belong to silicon and not to the metals named.

The spectra of silicon tetra-fluoride vacuum tubes reproduced in their paper show a very strong spectrum of oxygen, which is sufficient evidence that the spark conditions were such as to result in the decomposition of the glass of the tube, which introduces great uncertainty as to the nature of all the materials thus rendered incandescent. The oxygen lines cannot be due to contamination with atmospheric air, as the spectrum of nitrogen is absent.

The photographs of spectra which accompany this note show clearly the unimportant part played by the silicon tetra-fluoride in the production of the silicon lines in Lockyer and Baxandall's photographs, as their spectrum is practically identical, except for the presence of a few fluorine lines, with the second strip of the photographs sent herewith, which was produced from an oxygen tube, and could have been equally well obtained from a tube containing argon, helium, or other gases under suitable conditions (Plate IX).

The first strip shows the spectrum of oxygen for comparison. It was taken from the same tube as the second strip, but with small coil and small jar instead of the heavy disruptive discharge from the large coil and four large jars. The third strip shows, however, that a *true* spectrum of dissociated silicon tetra-fluoride may be obtained

PLATE IX

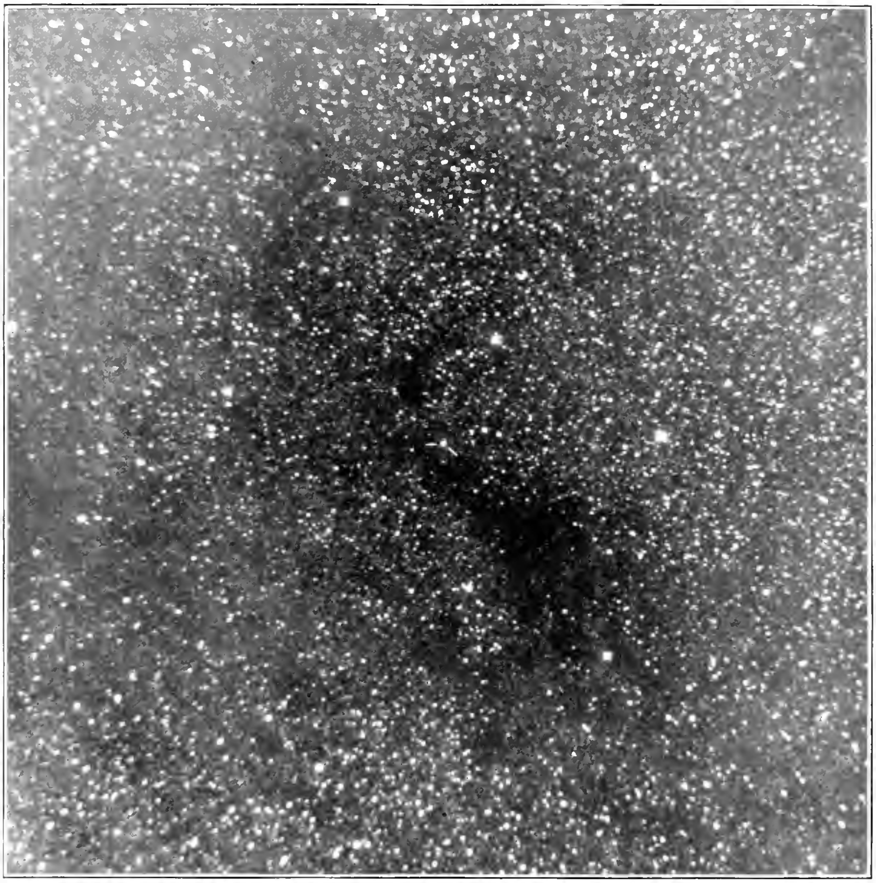


Spectra of Silicon and Fluorine

PLATE X

+ 1m North − 1m

E



W

+20

+10

0'

−10

−20'

Scale: 1 mm = 29″.7

NOVA AQUILAE OF 1905

24-Inch Reflector, 1905, September 21. Exposure, 3 hours

without simultaneously producing a strong oxygen spectrum. This is a spectrum of the gaseous materials purposely introduced into the tube, and not one of the glass of the tube itself, and therefore it has far more value as evidence, especially in view of M. de Gramont's statement that the lines of Group IV, in his experiments, appear and disappear with the air lines.

This photograph was produced with a small coil and a small jar, which gave a discharge insufficient to decompose the glass capillary. It will be seen that the spectrum of oxygen is almost completely eliminated, a trace only of the strongest lines, the strong triplet mentioned by Lockyer and Baxandall, being just perceivable.

The two strong lines of Lockyer's Group IV are, however, among the strongest in the spectrum, and they are accompanied by the well-known pair $\lambda\lambda$ 4128 and 4131 in Group II, and the strong triplet of Group III.[1]

Moreover, the spectrum of fluorine, considered later, is much stronger than in the Kensington spectra. The strongest fluorine lines are marked with one black dot at the upper end, while the silicon lines are marked by one black dot in the middle, the wide pair of silicon lines to the left being the two strong lines of Lockyer's Group IV, λ 4089 and λ 4116, the close pair on their right belonging to his Group II, λ 4128 and λ 4131, and the strong triplet on the right being Group III, λ 4553, λ 4568, and λ 4575.

Strip IV shows the central part of a negative taken from a silicon tetra-fluoride capillary under conditions which leave part of the SiF_4 undecomposed, as the band spectrum of this compound (mentioned also by Eberhard) shows. In this the fluorine lines are practically absent, although the silicon lines are very strong.

All the twelve lines[2] (Group IV being absent) of Lockyer's first three groups are exceedingly distinct, as well as two lines, the pair to the right of the very strong pair $\lambda\lambda$ 4128 and 4131, which I regard

[1] Sir Norman Lockyer and myself almost simultaneously and independently discovered these strong stellar lines (Group III) to be due to silicon, but both of us had not noticed that M. de Gramont had previously recorded them as silicon lines, which he found in the spectra of silicates (*Comptes Rendus*, **124**, 192, 1897).

[2] The green pair $\lambda\lambda$ 5042, 5057, the arc line λ 3906, and the Group II triplet $\lambda\lambda$ 3854, 3856, and 3863 are outside the limits of the strip sent for reproduction.

as two new silicon lines not hitherto recorded by any who have worked on the spectrum of silicon.

Strip V shows the pair λλ 4128 and 4131 as intense as in strip III, but without the lines of Groups III and IV. It is interesting as representing the low-temperature stellar spectrum of silicon reproduced in the laboratory. It is from a silicon tetra-chloride vacuum tube.

Herr G. Eberhard[1] has also made an important investigation of the spectra of silicon obtained from its halogen compounds. He says: "The arc lines λ 3905 and λ 4103 occur throughout the spark, but the lines λ 4089 and λ 4116, on the contrary, occur only in the immediate neighborhood of the points of the electrodes;" which agrees with the results of my experiments with silicates mentioned in the preceding extract, and probably explains why Exner and Haschek missed these lines, as Eberhard points out.

The wave-length of the silicon line λ 4116 cannot be accurately measured in presence of a strong fluorine spectrum, unless sufficient dispersion is employed to separate the slightly less refrangible fluorine line.

I have hitherto left out of account the middle line of Lockyer's Group IV, viz., λ 4096.9. It is a very important stellar line, as the following extract from Cannon and Pickering's[2] intensities show.

	29 *Canis Majoris*	τ *Canis Majoris*	ε *Orionis*	β *Centauri*	γ *Orionis*
4089.23	6	12	15	5	2
4096.9	18	6	4	2	1
4101.8 H_δ	25	25	25	35	40
4116.2	3	6	10	2	0

They say: "4096.9 is so near H_δ, wave-length 4101.8, that it gives the hydrogen line the appearance of being double, or of having a bright central line superposed on a broad dark line. By superposing this spectrum upon that of another star, it is easily seen that H_δ is not double. It then appears that H_δ of the superposed image

[1] *Zeitschrift für wissenschaftliche Photographie*, **1**, 349, 1903.

[2] *Annals of the Observatory of Harvard College*, **28**, part 2, pp. 233 and 235.

[3] The first and last of these are silicon lines. Cannon and Pickering assign no origin.

matches the line of greater wave-length in this spectrum, and the line 4096.9 is well separated from the hydrogen line. 4096.9 has not been seen in the preceding classes of spectra, and is strongest in spectra of this class (viz., Oe) declining in intensity in succeeding classes until B2A is reached, when it is not present."

It is doubtless the same line that Lockyer[1] records as λ 4097.3 in ϵ *Orionis*, ascribing its origin to *Si* (IV), and Hartmann[2] in δ *Orionis* as λ 4097.49, he also ascribing it to silicon, following Lockyer and Exner and Haschek as regards origin. Exner and Haschek find its wave-length as 4096.8, while A. de Gramont places a line at λ 4097.3, but ascribes it to air. Eberhard does not mention any silicon line here.

This line is certainly not present in the purest silicon spectra which I have been able to obtain, and which show the other two lines of Group IV strongly. In strip III of the accompanying photographs its place lies between the silicon line λ 4089 and the fluorine line λ 4103, a region destitute of lines of any kind.

In 29 *Canis Majoris*, according to Cannon and Pickering, this line is three times as intense as the stronger silicon line of Group IV, but—as is shown in the preceding extract from their intensities—it becomes weaker in stars showing the Group IV lines of silicon more strongly. I am convinced, therefore, not only because of the absence of this line in strip III and its relative weakness in the spectra of both Lockyer and Exner and Haschek, but also because of stellar evidence, that some other origin than silicon must be sought for this line.

There are both oxygen and nitrogen lines very close to this place, but neither of these elements accounts for the strong stellar line. These elements are sufficient, however, to account for the line in laboratory spectra of silicon showing air lines.

Exner and Haschek's lines $\lambda\lambda$ *3883.46, 4021.0, and 4764.20.*—These lines do not appear in the Cape photographs. Neither Lockyer nor Eberhard finds them, and M. A. de Gramont does not mention them. They also should be struck out from the list of silicon lines, as due to accidental impurities.

Line λ *4030.*—Similarly with line λ 4030, found both by Exner

[1] *Catalogue of 470 of the Brighter Stars* (1902), p. 52.

[2] *Astrophysical Journal*, **19**, 272, 1904.

and Haschek and Lockyer, although Lockyer states that it may be due to an impurity, and he does not include it in any of his four groups. This line is not obtained either by Eberhard or myself, and should also be struck out.

Lines λ 3854 and λ 4103.—The lines $\lambda\lambda$ 3853.9 and 4103.2 of Lockyer, which Eberhard does not obtain, are present in the Cape photographs, while Exner and Haschek record them as double lines. I do not doubt the silicon origin of these lines, but find them to be single and not double. The latter line is involved with both fluorine and oxygen lines when the spectra of those elements are present, but it is seen both in the spectra of the chloride and fluoride of silicon when examined under conditions which preclude the presence of the halogen spectra.

Mention may also be made of two pairs of silicon lines, one in the orange and another in the red, which were first noted by A. de Gramont. These I have confirmed, as well as the green pair also noted by A. de Gramont and confirmed by Lockyer.

New pair of silicon lines.—There is, however, another pair of lines which have not hitherto been recorded, which I regard as low-temperature silicon lines. Their wave-lengths are λ 4191.0 and λ 4198.5; they are well shown in strip IV. On the same negative are six other low-temperature lines, viz.: λ 3854, λ 3856, λ 3863, λ 3906, and the green pair $\lambda\lambda$ 5042 and 5057. The pair $\lambda\lambda$ 4128 and 4131 is very strong in the photograph (strip IV), while λ 4103 is a weak line.

The banded spectrum of the undecomposed fluoride is also well shown. This *partial* dissociation of the gas is evidence of the low-temperature condition, and the absence of the fluorine lines is well marked.

This new pair of lines was also obtained in the chloride, both in capillary tubes, at pressures of from 3.5 to 12.5 mm, and also in the spark between platinum electrodes in a bulb filled with the vapor of the chloride at atmospheric pressure.

NOTE ON THE SPECTRUM OF FLUORINE

It is a remarkable fact that none of the observers who have worked with the spectrum of silicon tetra-fluoride have attempted to assign a definite spectrum to fluorine, although its lines must have accom-

panied most of their silicon spectra. It is also remarkable that Lockyer and Baxandall's Plate 11, strip A, shows the strongest fluorine lines clearly differentiated from the oxygen and silicon lines by being thickened in the lower half of the strip. Notice particularly the line beneath the letter V in vacuum tube.

None of the silicon and oxygen lines have this appearance, and the following lines may be picked out in their photograph by mere inspection, viz.: λ 4103, λ 4109, λ 4246, λ 4299, and λ 4447.

The further elucidation of this interesting spectrum is suggested as a fruitful field for further research, which might preferably be undertaken outside an astronomical observatory.

It is evident that a complete knowledge of the spectrum of fluorine will help to increase our knowledge of the spectrum of silicon, and probably that of other elements which have volatile fluorides.

There is a fluorine line on the green side of the 4116 silicon line with a wave-length of $\pm\lambda$ 4116.8, stronger than the fluorine line λ 4113 and fainter than λ 4119, which in spectra of dissociated silicon tetra-fluoride gives the silicon line too high a value for wave-length, unless sufficient dispersion is employed to separate the two lines.

A list of lines which may be ascribed to fluorine is appended. There are other lines in this region which may be due either to silicon or fluorine, but further experiments are necessary before their origin can be determined satisfactorily.

SPECTRUM OF FLUORINE

	λ	Intensity		λ	Intensity
First triplet	3847.3	3	Group of five lines	4103.6	10
	3850.2	2		4109.3	5
	3851.8	1		4113.0	2
Second triplet	3899.0	2		4116.8	2
	3902.1	1		4119.3	3
	3904.0	< 1		4246.5	30
	4025.3	10 dup ?		4299.3	7
	4084.1	2		4446.8	20

ROYAL OBSERVATORY,
Cape of Good Hope,
January 23, 1905.

NOVA AQUILAE OF 1905

By J. A. PARKHURST

The region of this *Nova* was included on seven pairs of plates taken by Professor Barnard with the Bruce photographic telescope in 1904, and since the announcement of discovery five plates have been taken with the 24-inch reflector, one of which is reproduced in Plate X. From these plates the position of the *Nova* and a sequence of faint comparison stars have been measured; also the color of the *Nova* and the neighboring yellow star $-4^\circ.4663$. Photometric magnitudes of a sequence of comparison stars have been obtained with the 6-, 12-, and 40-inch telescopes.

Table I gives a list of the plates taken with the Bruce telescope,

TABLE I

PLATES TAKEN WITH BRUCE PHOTOGRAPHIC TELESCOPE

	Exposure	Limits	
		6-inch	10-inch
1904 April 15........	$2^h\ 25^m$	*g'*	*j*
20........	2 40	*g'*	..
May 10........	1 45	*h*	*j*
15........	3 40	*j*	*j*
June 9........	5 35	*g'*	*e'*
July 1........	1 10	*g'*	*g'*
15........	5 35	*j*	*d, e'*

simultaneous exposures being made with the 6- and 10-inch lenses. No trace of the *Nova* appears on any of these plates, but the last two columns give the faintest star distinctly visible. These stars can be identified on Plate X by the aid of the co-ordinates in Table V. From this table we conclude that the *Nova* was at least fainter than magnitude 15 in the spring and summer of 1904.

Table II shows the plates thus far obtained here with the 24-inch reflector. No. 233 was taken by Mr. F. C. Jordan; the others by the writer. Nos. 223 and 224, made on the same evening, show stars as faint as *k*, with disks suitable for diameter measures in order

TABLE II

PLATES TAKEN WITH 24-INCH REFLECTOR

No.	Date	Exposure: From	Exposure: To	Exposure: T	Aperture	Plate	Magnitudes: *Nova*	Magnitudes: A	Magnitudes: $-4^{\circ}.4660$
		C. S. T.	C. S. T.						
223	Sept. 2	$7^h\ 57^m$	$8^h\ 11^m$	14^m	12-in.	Seed 27	10.33	8.59	9.19
224	" 2	8 14	8 41	27	12	Cramer Iso.	10.24	8.32	9.16
226	" 20	9 25	9 55	30	24	Seed 27	.....	....	....
227	" 21	7 0	10 0	180	24	Seed 27	.....	....	....
233	" 27	7 10	11 8	238	24	Seed 27	.....	....	....

to find the magnitude. As the aperture was reduced to 12 inches, the field was flat enough to yield accurate positions. These two plates were accordingly measured and reduced in duplicate by Mr. Jordan and the writer, using as standards the four stars from the *Erstes Münchener Sternverzeichniss* given in Table III. The resulting position of the *Nova* for 1900 is

From plate No. 223, R.A. = $18^h\ 56^m\ 48^s.95$, Dec. = $-4^{\circ}\ 35'\ 20''.3$
From plate No. 224, R.A. = $18^h\ 56^m\ 48.98$, Dec. = $-4\ 35\ 20.3$
Mean, R.A. = $18^h\ 56^m\ 48.96$, Dec. = $-4\ 35\ 20.3$

TABLE III

STANDARD STARS FOR POSITION OF *Nova*

Star	*B. D.*	Munich No.	Place for 1900: Catalogue α	Place for 1900: Catalogue δ	Place for 1900: Plates α	Place for 1900: Plates δ
A.....	$-4^{\circ}.4663$	18367	$18^h\ 55^m\ 52^s.09$	$-4^{\circ}\ 34'\ 48''.9$	$55^m\ 52^s.08$	$34'\ 49''.4$
B.....	-4.4668	18425	18 56 35.42	4 28 32.9	56 35.38	28 34.1
F.....	-4.4665	18389	18 56 5.89	4 49 8.5	56 5.90	49 7.7
73....	-4.4673	18466	18 57 15.09	4 19 34.9	57 15.14	19 34.2

This position differs by $0^s.24$ and $6''.5$ from that given by Hartwig in *Astronomische Nachrichten* No. 4047, the latter place depending, however, on but one comparison star.

By the method of disk diameters, these two plates also yield magnitudes, and, by inter-comparison, star colors. The last three columns of Table II give these results for the *Nova* and the stars *A* (see Table III) and $-4^{\circ}.4660$. A comparison of these photographic

magnitudes with those given by Pickering[1] and Wolf[2] yields interesting results. Pickering gives the magnitudes of *A* as 6.98 visual and 8.67 photographic. Wolf gives the photographic magnitude of $-4^\circ.4660$ as 8.3, and of *A* as 8.4. Evidently stronger images of colored stars are given by a reflector than by a refractor, as the red and yellow rays, sharply focused by the mirror, are somewhat out of focus in an image formed by a photographic lens. The color effect given by comparison of the Seed and Isochromatic plates is 0.27 magnitude for star *A* and 0.09 for the *Nova*, showing the latter to be only slightly colored.

Table IV gives the photometric and photographic magnitudes of

TABLE IV

COMPARISON STARS FOR MAGNITUDE

Letter	B. D. No.	Photometric Magnitudes		Photographic Magnitudes		Harvard Circular No. 106		
		6-inch	12-inch	Seed	Iso.	Letter	Photometric	Photographic
E......	$-4^\circ.4650$	7.67	....	7.71	7.70	..	....	....
60.....	-4.4660	9.22	9.21	9.16	9.10	..	....	....
61.....	-4.4661	....	....	10.72	10.64	..	....	....
A......	-4.4663	7.30	....	8.59	8.32	*a*	6.98	8.67
F......	-4.4665	9.29	9.47	9.47	9.48	*c*	9.42	9.57
n......	-4.4666	....	10.52	....	....	..	....	....
B......	-4.4668	9.45	9.39	9.60	9.53	*b*	9.36	9.17

the brighter comparison stars for the *Nova*, the standards used being the stars *E* and *F*, which are assigned the magnitudes 7.55 and 9.40, respectively, in *Harvard Annals* **24**. An inspection of columns 5 and 6 will show that none of the stars in this table is appreciably colored except the star *A*. The last three columns give a comparison with the Harvard data in *Circular* No. 106.

Plate X reproduces the central portion of negative No. 227 on a scale of 29".7 to the millimeter. The *Nova* at the center of the plate is identified by the two short lines pointing toward it. This plate should be compared with the chart of the region given by Wolf in the *Nachrichten* above cited. Wolf states that the *Nova* is situated

[1] *Harvard College Observatory Circular* No. 106.

[2] *Astronomische Nachrichten*, **169**, 223, 1905.

between two remarkable starless regions. The impression given by our plate is somewhat different. The dark lane south preceding narrows as it approaches the *Nova*, which is situated just on the edge of the lane. Beyond the *Nova* the lane is bridged by a multitude of faint stars, reappearing farther north and curving toward the preceding side. The field is exceedingly rich, and is a striking instance of the connection of Novae with the Milky Way.

TABLE V

FAINT COMPARISON STARS

Letter	Co-ordinates from *Nova*			Photometric Magnitudes		Harvard Circul'r No. 106		
						Table III		Table IV Mag.
	$\Delta\alpha$		$\Delta\delta$	12-inch	40-inch	Letter	Mag.	
k..........	−217″.0	−14ˢ.5	+ 37″.2	11.30	11.31	*o*	12.10	....
i..........	−171.3	−11.4	+122.0	......	.....	..	.....	....
l..........	−159.8	−10.7	+ 54.2	10.82	10.96	*m*	11.28	....
h′.........	−150±	−10±	−150±	11.06	11.83	..	....	....
g..........	−115.3	− 7.7	+ 8.8	....	....	..	.....	13.5
h..........	−109.3	− 7.3	+ 50.0	....	....	..	.....	12.0
b..........	− 6.7	− 0.4	− 17.7	....	16±	*a*	.....	15.3
d..........	− 4.3	− 0.3	+ 18.0	....	14.61	*y*	.....	14.9
f..........	+ 2.7	+ 0.2	− 70.3	....	14.66	*x*	14.98	14.6
a..........	+ 28.2	+ 1.9	+ 22.1	....	....	..	.....	15.2
e..........	+ 46.7	+ 3.1	−113.8	....	15.5±	..	.....	14.8
c..........	+ 54.7	+ 3.7	+ 36.6	....	....	*z*	15.16	15.0
g′.........	+123.0	+ 8.2	− 15.8	13.7±	13.27	*t?*	14.17	13.7

A sequence of faint comparison stars was selected on negative No. 227, beginning with *a*, the faintest, and proceeding in alphabetical order to *l*, of about eleventh magnitude. The positions of these stars were measured on the negative, using the stars in Table III for orientation and scale value. Table V gives their co-ordinates relative to the *Nova* with the photometric magnitudes as far as measured; also three columns showing a comparison with the Harvard data. The co-ordinates are probably correct within a few tenths of a second. There is some uncertainty about the identifications with the stars in the Harvard *Circular*, as a comparison of the co-ordinates will show. Evidently the positions given in the circular are only approximate.

Table VI gives the observed magnitudes of the *Nova*, the second

TABLE VI

OBSERVED MAGNITUDES OF THE *Nova*

	C. S. T.	Mag.	Aperture
1905 September 2	8^h	10.3	12 v
5	8	10.4	12 v
21	10	11.0	24 v
27	7	11.32	6
28	7	10.82	12
29	8	11.1 ±	12 v
October 1	8	11.07	40

column giving Central Standard Time, six hours west of Greenwich. The last column gives the aperture in inches, with the letter v added for direct visual comparisons, not photometer measures. With the 40-inch the *Nova* appeared slightly nebulous, not focusing as sharply as the stars *k* and *l*, of about the same brightness. This appearance, with the slight color, will probably account for the inconsistency between the measures with the different apertures.

YERKES OBSERVATORY,
October 14, 1905.

NOTE.—In addition to the plates taken with the 24-inch reflector, parallax plates of *Nova Aquilae* were obtained with the 40-inch refractor by Mr. Frank Sullivan. These will be subsequently utilized for measurement, if the brightness of the *Nova* remains sufficient to permit other plates to be obtained at the complementary season for parallactic displacement.

The region of the *Nova* is included on at least ten plates obtained during this summer by Professor Barnard at Mount Wilson with the Bruce photographic telescope. Six of them were made in July and three in August. The images will probably not prove to be very good, as they will fall near the edges of the plates, in the field of poor definition. These negatives are now in transit by freight from California, with the Bruce telescope itself, and cannot be referred to more particularly at this time.

The faintness of the *Nova* prevented us from attempting to obtain the spectrum with the Bruce spectrograph. With the use of one prism (1 mm = 30 tenth-meters, at λ 4481) it is not feasible to photograph the spectra of stars fainter than the eighth magnitude.

EDWIN B. FROST.

Minor Contributions and Notes.

H 1175. *NOVA AQUILAE*, No. 2. 185604[1]

A second new star in the constellation *Aquila* has been found by Mrs. Fleming from an examination of the photographs of the Henry Draper Memorial. No trace of this star has been found on any photograph taken before August 18, 1905. It is important that this star should always be designated as No. 2, to distinguish it from *Nova Aquilae*, No. 1, which was first photographed on April 21, 1899, was discovered in the same way, and was described in the *Harvard Bulletin* of July 11, 1900, and in *Circular* 56.

The principal facts concerning a number of photographs of the region containing *Nova Aquilae*, No. 2, are given in Table I. The first column gives the letters designating the series, followed by the number, in that series, of each photograph. The letter A denotes that the photograph was taken with the 24-inch Bruce telescope, B with the 8-inch Bache telescope, C with the 11-inch Draper telescope, I with the 8-inch Draper telescope, and AC with the Cooke anastigmat. The year, month, and day are given in the second column, the Julian day and decimal in the third column, and the duration of the exposure in the fourth column. The photographic magnitude of the *Nova* is given in the fifth column for the later plates, and for the earlier plates the magnitude is that of the faintest star shown, preceded by the sign <, which denotes that the *Nova* was not seen and must have been fainter than the magnitude given. The scale is that defined by the photographic magnitudes of the comparison stars, given in the last column of Table III. There are probably two or three hundred

TABLE I

Description of Photographs

Plate	Date	Julian Day	Exp.	Mag.	Plate	Date	Julian Day	Exp	Mag.
	y m d		m			y m d		m	
B 2655	1888 5 22	0780.788	22	<12.1	AC 6590	1905 8 10	7068.678	85	<0.7
B 2656	1888 5 22	0780.806	16	<12.3	C 15907	1905 8 18	7076.635	75	0.07
B 3013	1888 10 18	0929.487	16	<12.3	AC 6614	1905 8 21	7079.602	63	0.27
I 1494	1890 7 20	1569.753	11	<12.0	AC 6632	1905 8 26	7084.540	67	10.04
I 11185	1894 6 13	2993.726	19	<12.6	AC 6643	1905 8 31	7089.547	60	10.28
A 1851	1896 6 17	3728.689	60	<13.6	I 33294	1905 8 31	7089.550	60	10.48
I 17844	1897 4 12	4027.857	10	<13.2	I 33295	1905 8 31	7089.608	60	10.43
A 6506	1903 8 15	6342.607	240	<15.7	C 15930	1905 8 31	7089.574	10	10.38
AC 6565	1905 8 2	7060.637	60	<11.5	C 15931	1905 8 31	7089.600	20	10.38
AC 6576	1905 8 4	7062.572	58	<10.8	C 15933	1905 8 31	7089.663	30	10.38

[1] *Harvard College Observatory Circular* 106.

REMARKS

B 2655. The earliest plate available for this examination. It extends the work over seventeen years.

A 6506. This plate shows stars much fainter than any other photograph of the region. The region of the *Nova* is near the edge of the plate; otherwise, still fainter stars would probably have been shown.

AC 6590. The last photograph before the appearance of the *Nova*. The region is on the edge of the plate, which, in the center, shows stars nearly as faint as AC 6565.

C 15907. A spectrum plate of $-3°.4460$, taken with one prism. The spectra of the stars $-4°.4663$, $-4°.4669$, $-5°.4876$, $-4°.4684$, and of the *Nova* are also shown. The *Nova* was discovered from this plate. Its spectrum, although faint, shows the lines $H\delta$, $H\gamma$, λ 4472, λ 4646, and $H\beta$, very broad and bright. The lines $H\gamma$ and $H\beta$ have accompanying dark lines on the edge of shorter wave-length. These dark lines are still better shown in a contact print from the original negative. λ 4646 is slightly stronger than the helium line λ 4472. The entire spectrum therefore closely resembles that of *Nova Persei*, No. 2, on March 30, 1901. See *Annals*, **56**, No. 3, Plate I.

AC 6614. Images near edge of plate, and much enlarged.

AC 6632. This and the two preceding plates were taken after the *Nova* appeared, and before it was discovered.

I 33294. A spectrum plate. The spectrum of the *Nova* is substantially the same as that shown on C 15907.

early photographs of the region, but only twenty-nine of them have been examined, as on none of them does the *Nova* appear. Only eleven of these plates have been included in Table I.

Since the discovery of the *Nova* on August 31, photographs of it and visual measurements of its brightness have been made on every clear night. The dates, photometric magnitudes, and photographic magnitudes are given in Table II.

TABLE II

MEASURES OF LIGHT

Date	Photometric	Photographic	Date	Photometric	Photographic
August 31	10.32	10.41	September 9	10.76	10.86
September 1	10.41	10.42	" 13	10.65	.....
" 5	10.60	10.58	" 14	10.94	10.77
" 6	10.57	10.63	" 21	11.14	11.16
" 8	10.65	10.86	" 22	11.55	11.23

A list of the comparison stars is contained in Table III. In the first portion of the table, which relates to the brighter stars, the first five columns give the designation, and the number, right ascension, declination, and magnitude according to the *Durchmusterung*, for the stars contained in that work. In the second part of the table, which contains the fainter stars, the second and fifth of these columns are omitted. The last two columns of the table give the photometric and photographic magnitudes.

The photometric magnitudes depend on measures made with the 12-inch meridian photometer for the stars *a* to *o*, and star *r*. Estimates were made, by Argelander's method, of the intervals between the stars and the magnitudes deduced as described in Volume **37**. The magnitudes of stars fainter than *r* were obtained by extrapolation and are somewhat uncertain. The stars *a* and *l* are much brighter visually than photographically. They have spectra of the second type. The other brighter comparison stars have spectra of the first type. The star *p* is fainter visually, but brighter photographically, than *q* or *r*. The positions of the stars not contained in the *Durchmusterung* were found by laying a plate, ruled in squares a millimeter on a side, on A 6506, and estimating the co-ordinates in twentieths of a millimeter.

TABLE III

COMPARISON STARS

Des	*DM.*	R. A. 1855	Dec. 1855	*DM*	Phtm.	Phtg.	Des.	R. A. 1855	Dec. 1855	Phtm.	Phtg.
		h m s	° ′					h m s	° ′		
N	....	18 54 25.1	−4 38.6	...	....	....	*o*	18 54 10.4	−4 38.0	12.10	11.90
a	−4° 4663	53 28.8	−4 38.4	6.8	6.98	8.67	*p*	54 41.0	−4 42.2	12.95	12.38
b	−4° 4668	54 11.9	−4 32.1	8.5	9.36	9.17	*q*	54 43.5	−4 42.6	12.38	12.69
c	−4° 4665	53 42.4	−4 52.5	8.8	9.42	9.57	*r*	54 34.3	−4 32.9	12.77	12.97
d	−5° 4844	54 57.2	−5 0.0	9.0	9.58	9.79	*s*	54 53.5	−4 40.3	13.53	13.27
e	−5° 4839	53 46.8	−5 10.0	9.1	9.96	9.80	*t*	54 33.5	−4 39.0	14.17	13.72
f	−4° 4675	55 21.2	−4 35.5	9.1	9.90	10.09	*u*	54 37.3	−4 40.5	13.98	13.97
g	−4° 4687	57 39.0	−4 44.7	0.2	10.02	10.17	*w*	54 30.3	−4 41.7	14.55	14.27
h	−4° 4656	52 5.1	−4 53.3	9.5	10.81	10.59	*x*	54 25.5	−4 39.8	14.98	14.62
k	....	54 18.3	−4 48.1	...	10.68	10.67	*y*	54 25.1	−4 38.3	....	14.82
l	−4° 4677	55 59.6	−4 43.8	9.2	10.26	10.95	*z*	54 29.0	−4 38.0	15.16	14.97
m	....	54 14.2	−4 37.7	...	11.28	11.30	*α*	54 24.8	−4 30.0	....	15.27
n	....	54 21.5	−4 32.5	...	11.54	11.45					

It was at first thought that the *Nova* might be identical with star *α*. As the plates available for comparison were on different scales, it was difficult to determine by inspection whether this was the case. Plates A 6506 and C 15938 were accordingly enlarged to the same scale, 10″=0.1 cm. Superposing them, it was obvious that no star appeared on the first plate in the position of the *Nova*.

TABLE IV

ADJACENT STARS

x	*y*	Mag.	*x*	*y*	Mag.	*x*	*y*	Mag.	*x*	*y*	Mag.	*x*	*y*	Mag.
″	″		″	″		″	″		″	″		″	″	
−111	+ 15	13.5	−75	+ 39	15.0	−15	+ 57	14.9	+47	−113	14.8	+107	+ 71	15.1
−107	− 25	15.2	−66	− 17	15.2	− 6	+135	14.8	+48	+105	14.9	+111	− 96	15.0
−105	+ 54	12.0	−66	− 30	15.2	− 5	− 18	15.3	+54	+ 96	15.0	+115	−102	15.2
−105	+ 43	14.2	−65	− 78	15.4	− 1	+ 18	14.9	+63	+ 36	15.0	+120	−107	14.9
−102	+ 45	14.3	−54	+ 84	15.0	0	0	*Nova*	+75	+ 67	15.0	+126	+ 93	14.4
− 99	+ 30	15.2	−45	+123	15.4	0	+147	15.4	+79	+ 77	15.0	+126	+ 67	15.4
− 95	+126	15.0	−41	+111	15.0	+ 6	− 72	14.6	+87	− 93	15.4	+126	−111	14.9
− 90	+ 12	14.8	−36	− 71	15.2	+13	+126	14.8	+95	+109	15.2	+127	− 33	13.7
− 84	+ 06	15.2	−30	+ 15	15.2	+18	+144	15.4	+06	+ 87	15.3	+132	+ 09	14.4
− 77	+ 75	14.9	−15	+ 78	14.8	+29	+ 18	15.2	+96	+ 57	15.5			

The positions and estimated photographic magnitudes of all the stars shown on A 6506, and not more than 2′ distant from the *Nova* in right ascension or declination, are given in Table IV. The stars for which the values of x are $-5''$, $-1''$, $0''$, $+6''$, $+63''$, and $+127''$ are a, y, *Nova*, x, z, and t, respectively.

On the photographs taken on August 18 and 21, the *Nova* was nearly equal in brightness to the star a, $-4^\circ.4663$, mag. 6.8. It was therefore at first supposed that the *Nova* was at least as bright as the seventh magnitude.

It will be seen that the time during which the *Nova* made its appearance was between August 10 and August 18. This interval may be greatly reduced when the plates taken in Arequipa are received. As, however, the Moon was full on August 14, the number of plates taken at that time would be small.

EDWARD C. PICKERING.

SEPTEMBER 23, 1905.

ON THE LIGHT- AND VELOCITY-CURVES OF *W SAGITTARII*

After the completion of my paper on the radial velocity and orbit of *W Sagittarii*,[1] there came to hand the excellent set of photometric observations of this star appearing in the *Annals of the Astronomical Observatory of Harvard College*, Vol. **46**, Part II. The deductions from these observations possess certain points of interest that may well be pointed out in connection with my own radial-velocity determinations of this variable.

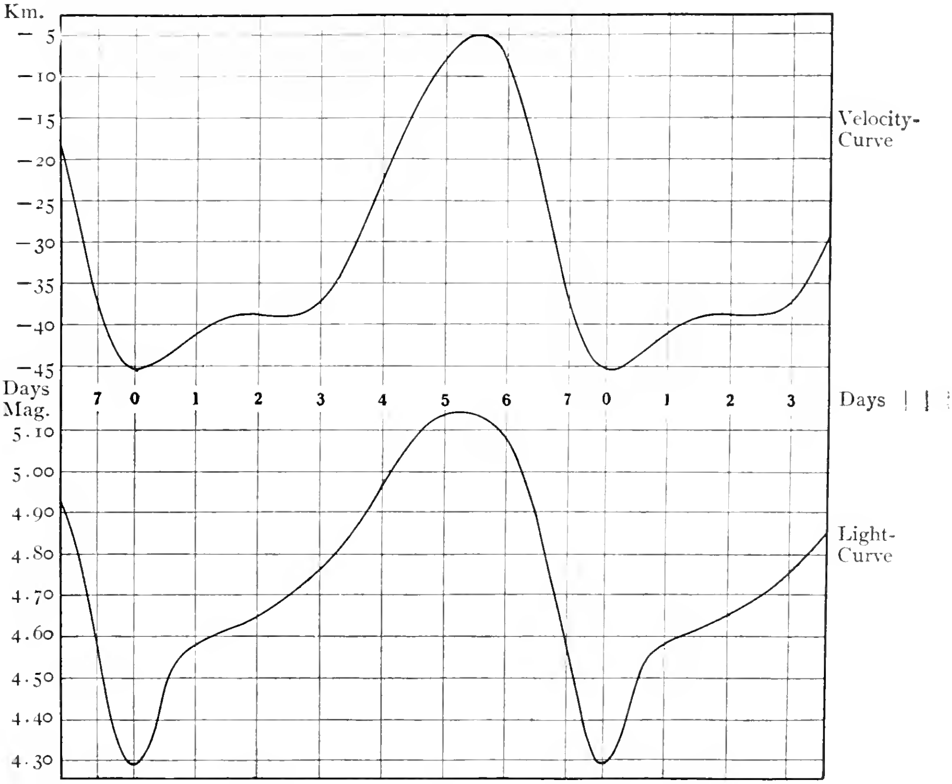
Forming the mean of ten determinations of light maxima of *W Sagittarii* from Table XVIII of the Harvard volume, and bringing this mean up to my epoch with Chandler's accurate period, I find for time of maximum, 1903, August 2.46 days. My value from Chandler's elements was, 1903, August 2.86 days. Adopting the new value, the only orbital element affected is T (time of periastron passage after light maximum), which becomes 6.69 days. Thus the light maximum occurs 0.99 day instead of 1.39 days after periastron time, and almost exactly at velocity minimum instead of 0.4 day after it.

The greatest interest lies in the comparison of the light- and velocity-curves. In the accompanying diagram the Harvard light-curve from Table XVII is drawn in parallel with the velocity-curve to exhibit the correspondence of detail both in form and position referred to maximum. The striking concordance observed is certainly more than a coincidence. Indeed, the strong general resemblance between the light- and velocity-

[1] *Astrophysical Journal*, **20**, 160, 1904.

curves of *W Sagittarii* is good evidence that the form of each is determined by the same underlying causes; but speculation on this point may properly wait until more data are available on similar stars.

The differences between the Harvard and the Schmidt light-curves seem hardly accountable on the ground of observational errors. It is not impossible that the true explanation is to be found in actual changes in the



VELOCITY- AND LIGHT-CURVES OF *W Sagittarii*

system of this star in the interval of thirty years between Schmidt's observations and those of Harvard College Observatory. This variation in the form of the light-curve, together with Schmidt's suspected perturbation in the light period, suggests that the light-changes of *W Sagittarii* are complicated by factors whose forms and magnitudes cannot be determined until the spectrographic and photometric observations cover many years.

R. H. CURTISS.

LICK OBSERVATORY,
May 20, 1905.

INTERNATIONAL UNION FOR CO-OPERATION IN SOLAR RESEARCH: MEETING AT OXFORD, SEPTEMBER 27–29, 1905[1]

By the kindness of the warden and fellows, the second meeting of the International Union for Co-operation in Solar Research was held at New College. The proceedings were opened on Wednesday, September 27, by Professor Turner, who briefly alluded to the St. Louis meeting, and expressed the hope that some good might be done by a few days' intercourse in the quiet of Oxford. In the absence of the warden, the subwarden, Mr. Matheson, welcomed the conference. He referred to the historic connection of New College with scientific research, particularly astronomy, and spoke of the general value of the meetings of men of different nationalities in promoting good feeling between the different countries in addition to their scientific importance.

Professor Schuster, on behalf of the conference, returned thanks to the warden and fellows. He went on to say that since the last meeting necessarily the business of the association had rested with the executive, and that body had acted on the principle that "it was better to do something irregularly than nothing regularly."

He proposed Sir William Christie as president for Wednesday, and M. Janssen as honorary president, and Professor Turner as president for Thursday. This was seconded by Sir Norman Lockyer.

At the meeting at 10:45 Sir William Christie presided, M. Janssen and Professor Schuster being on the platform with him. Professor Kayser proposed, and MM. Pérot and Fabry seconded, the following resolutions on new determinations of wave-lengths:

1. The wave-length of a suitable spectroscopic line shall be taken as the primary standard of wave-length. The wave-length of this line shall be fixed permanently and thereby define the unit in which all wave-lengths are to be measured. This unit shall differ as little as possible from 10^{-10} meters, and shall be called the Ångström.

2. Secondary standards are required at distances which should not be greater than 50 Angström units apart. These secondary standards should be referred to the primary standard by means of an interferometer method. The source of light should be obtained by means of an electric arc of from 6 to 10 amperes.

3. A committee shall be appointed to select the standards and to organize the determination of their wave-lengths in terms of the primary standard in at least two independent laboratories

[1] From the *Observatory* for October.

4. The same committee shall be charged with the selection of tertiary standards which shall be at distances of from 5 to 10 Å. The wave-lengths of these tertiary standards are to be obtained by interpolation with the help of gratings.

The resolutions were carried, and further details left to a committee.

The second resolution was moved by Professor Hale and seconded by M. Deslandres, "that a committee be appointed to draw up resolutions to be submitted to the meeting tomorrow."

Professor Hale said that a complete program for solar research would involve—

1. Solar photographs on, say, an 8-inch scale for the study of the general positions on the Sun's surface and for statistical inquiries.
2. Enlarged scale photographs like those of M. Janssen.
3. Photographs in calcium light with the spectroheliograph.
4. Photographs in the light from other metals.
5. Spectra of the K line in different parts of the Sun.
6. The spectra of sun-spots, the widening of the lines, radiation from sun-spots, etc.
7. Visual observations of chromosphere and spots.

He pointed out that there was urgent need for co-operation and an ample field for research in these many lines.

M. Deslandres, Mr. Newall, Professor Bélopolsky, and Sir Norman Lockyer were all agreed that co-operation was absolutely necessary, and need not be in the least degree detrimental to work requiring individual initiative.

Professor Fowler and Father Cortie spoke of the advantages to be obtained by a scheme of co-operation in work on the spectra of sun-spots.

At the meeting in the morning of Thursday, September 28, at which Professor Turner took the chair, the following resolutions dealing with solar radiation were submitted:

1. In order to secure uniformity, it is desirable that observations on the intensity of solar radiation in different localities shall be made as far as possible with the same type of instrument.
2. That for the present Ångström's pyrheliometer be adopted as the standard instrument.
3. That it is desirable to obtain accurate comparisons between the records of Ångström's pyrheliometer and other standard instruments; and that Mr. Abbot, Professor Callendar, Mr. W. A. Michelson, and Mr. W. E. Wilson be asked to assist the Union in this work.
4. That for the determination of the possible changes in the solar radiation power it is desirable to secure measurements of the intensity over limited ranges

of the spectrum, which are not affected by absorption due to ozone, aqueous vapor, and carbonic acid.

5. That a committee be appointed to draw up a scheme of co-operation, and proposals for the reduction of observations. That the committee be requested to communicate the scheme and proposals to the Executive Committee, with a view of initiating a system of observations according to the scheme.

6. That the union recognizes the great importance of measurements by direct photography, as well as by other methods, of the relative intensities of radiation emitted by different parts of the solar surface, and desires to include such measurements in the subjects to be dealt with by the Union.

The first five of these were brought forward by Professor Schuster, the sixth was added by Sir William Christie.

Professor Angström read a paper describing his pyrheliometer, and Professor Julius spoke in support of the resolutions generally.

Mr. W. N. Shaw explained that the International Meteorological Conference had adopted the resolutions Nos. 1 and 2, and urged that the same instruments and methods should be continued for a long time.

Sir W. Christie suggested that attempts should be made to photograph the umbræ of spots on a large scale and isolated from the surrounding parts of the Sun's disk.

Professor Hale explained Abbot's bolometric work on sun-spots, showing the difference in radiation when different lines were taken.

Professor Hull and Professor Bélopolsky also spoke on the resolutions.

Subsequently the following resolutions, which were the result of the deliberations of a committee consisting of Sir Norman Lockyer, Professor Hale, M. Deslandres, Professor Bélopolsky, and Professor Wolfer, after being amended at the meeting, were proposed by Sir Norman Lockyer and agreed to:

1. Co-operation is desirable in the various branches of solar research, such as visual and photographic observations of the solar surface, visual observations of prominences, and observations of the solar atmosphere with spectroheliographs of various types.

2. When an institution has collected and co-ordinated results from various sources, members of the Union shall be requested to place their observations at the disposal of the said institution.

3. In the case of investigations which have not yet been thus collected and co-ordinated, special committees nominated by the Union shall be charged with the work of preparation and organization. That these committees be requested to communicate the scheme and proposals to the Executive Committee, with a view of initiating a system of observations according to the scheme.

4. It is proposed forthwith to organize such co-operation in two branches of research:

a) The study of the spectra of sun-spots.

b) The study of the records, by means of the H and K light, of phenomena of the solar atmosphere.

5. The Union lays special stress on the fact that, notwithstanding the obvious utility of co-operation in certain cases, individual initiative is the chief factor in a very large number.

It is as much the duty of the Union to encourage original researches as to promote co-operation.

There was some discussion on these resolutions, leading to some amendment, in which most of those present took part. At the instigation of Sir William Christie, the words "special committees nominated" were substituted for "a committee specially nominated" in resolution 3, and the second part of this resolution was added in the course of the discussion. Professor Bélopolsky drew attention to differences of kinds of work—some in which co-operation would be helpful, others depending on individual initiative. Sir William Christie asked how far co-operation in work other than that with the spectroheliograph was desirable; and Father Cortie thought the danger of overlapping was avoided by co-operation.

Professor Fowler proposed that a committee should be formed to arrange for spectroscopic observations of sun-spots.

Professor W. W. Campbell instituted a discussion on the spectroscopic determination of solar rotation, and Dr. Halm described the instrument used by him at Edinburgh for this purpose.

The concluding meeting was held on the morning of Friday, September 29, when the proceedings were mainly of a formal character. It was resolved to accept the invitation of M. Janssen to Meudon in September 1907; that the Central Bureau should be at the University of Manchester, and the Computing Bureau at the University Observatory, Oxford, under the direction of Professor Turner; and committees were elected to deal with the four subjects: (1) standards of wave-length, (2) solar radiation, (3) co-operation in work with the spectroheliograph, (4) co-operation in the work on spectra of sun-spots. Professors Schuster and Hale were elected members of the Executive Committee, the first-named being chairman.

LIST OF DELEGATES AND VISITORS PRESENT AT THE MEETING

Name	Representing
Professor K. Ångström..........	Academy of Sciences, Stockholm
Professor A. Bélopolsky........	Academy of Sciences, St. Petersburg
Mr. F. A. Bellamy (Oxford)	
Professor W. W. Campbell......	National Academy of Sciences, Washing'n

Sir William Christie............	Royal Society, London
Father Cirera..................	Astronomical Society of France
Father Cortie...................	Royal Astronomical Society, London
Comte De la Baume Pluvinel...	Astronomical Society of France
M. H. Deslandres...............	Astronomical Society of France
Mr. H. Duffield (Manchester)	
Mr. F. W. Dyson (Greenwich)	
Professor W. S. Eichelberger (Washington)	
M. Fabry.......................	Physical Society of France
Professor A. Fowler............	Royal Astronomical Society, London
Professor G. E. Hale...........	National Academy of Sciences, Washing'n
Mr. J. Halm (Edinburgh)	
M. Hansky......................	Academy of Sciences, St. Petersburg
Major E. H. Hills..............	Royal Astronomical Society, London
Professor G. F. Hull (Hanover, N. H.)	
M. J. Janssen..................	Academy of Sciences, France
Professor W. H. Julius..........	Academy of Sciences, Amsterdam
Professor H. Kayser............	German Physical Society
Professor Littell (Washington)	
Sir Norman Lockyer............	Royal Society, London
Dr. W. J. Lockyer..............	Solar Commission of International Meteorological Committee
Mr. Georges Millochau (Meudon)	
Mr. H. F. Newall...............	Royal Society, London
M. Pérot.......................	Physical Society of France
Mr. H. C. Plummer (Oxford)	
Professor A. Schuster...........	Royal Society, London
Mr. W. N. Shaw................	International Meteorological Committee
M. Stefanik.....................	
Professor H. H. Turner.........	Royal Astronomical Society, London
Professor E. Weiss..............	International Association of Academies
Mr. W. E. Wilson...............	Royal Astronomical Society, London
Professor Wolfer (Zurich)	

Reviews

Handbuch der Spectroscopie. By H. Kayser. Band III. Pp. viii+604. Figs. 94 and three plates of spectra. Leipzig: S. Hirzel, 1905. 38 marks; bound, 42 marks.

This is the third volume of the set of five great treatises which were planned to record all that is of worth in the field of spectroscopy. The task is a formidable one, since, in addition to the references, there is a summary of the results of each research accompanied by critical remarks. The latter are given with great frankness; nevertheless, they are fair. In all cases completeness in record of literature is aimed at, and it would seem that little of value has been overlooked. In the preface of Volume I the author remarks that more than forty journals and treatises have been looked through from 1860 up to the present time—title and contents with notation of all spectroscopic citations. Not until then did he consult the *Fortschritte der Physik, Beiblätter zu Wiedemanns Annalen,* and the *Reports of the British Association.* The result is a review of more than 7,000 papers, the contents of which are now accessible to the reader. The work is bearing fruit. The frequent references to Kayser's *Spectroscopie* in recent papers on spectroscopic subjects show that it is filling a long-felt need.

The first volume contains the history of spectroscopy, and the description and theory of apparatus. The second volume deals with Kirchhoff's law, and its consequences, the emission of solids, the views concerning the incandescence of vapors and gases, the dependence of emission spectra upon pressure, temperature, magnetization, and finally the classification of the spectra. Originally it was intended to have the third volume deal with the phenomena of absorption, fluorescence, and phosphorescence; but the first part assumed such proportions that the author decided to divide it into two volumes. As a result, the third volume contains the description of apparatus and methods to investigate absorption spectra, an account of our knowledge of the dependence of absorption upon constitution, and finally a classification of the observational material for inorganic and artificial organic compounds. The next volume is to deal with the natural organic dye-stuffs from the plant and the animal kingdom, and also the phenomena which are associated with absorption: namely, dispersion, fluorescence and phosphorescence.

The author aptly remarks that the task in compiling these two volumes has not been a very agreeable one; that it consisted in collecting and sorting observational material of inferior value. Just why he should consider the data of an inferior order is not explained. Perhaps he did not wish to expose the failing of his fellow-workers, which is the proneness to overlook the necessity of dealing with pure materials. In addition to this fact, it must be remembered that but little has been done recently in absorption spectra; while, on the other hand, emission spectra are constantly being reinvestigated in the light of modern theories. Nevertheless, although the task has been burdensome, this volume fulfils the promise of the earlier ones in completeness and thoroughness of treatment. It deals exclusively with absorption spectra in the ultra-violet, visible and infra-red regions of the spectrum. Chapter I naturally deals with apparatus and methods used in investigating absorption spectra. Chapter II treats of the variation in absorption spectra, and the influence of the solvent, of the temperature, of the concentration, and of the thickness. Kundt's law of the shifting of the maximum of the absorption band of the solute toward the long wave-lengths, with increase in the refraction and dispersion of the solvent, is fully discussed, and the author shows that it is one of those laws which has many unexplained exceptions. Chapter III was written by Professor Hartley, of Dublin. This chapter contains a somewhat detailed account of what has been accomplished in the ultra-violet, visible and infra-red spectrum. The ultra-violet, being Hartley's own field, is most fully treated. His work dates back to the latter part of the seventies, when the constitution of the essential oils was practically unknown. It is interesting, therefore, to note that at this early date (1880) he showed from his absorption spectra that these oils contain benzene derivatives, which has since been proven by chemical analysis. The absorption curves found by Hartley for the ultra-violet, and the photographs of the infra-red, by Abney and Festing, are reproduced and form a valuable adjunct to the work as a whole. Chapter IV deals with the absorption of a list of compounds arbitrarily selected, many being of practical use. The chapter includes also the spectra of the rare earths, since it was found desirable to group them instead of giving a summary of each one. This is reserved for Chapter V, which contains an alphabetical list of all the known absorption spectra, except those which are to be included in the next volume.

Attention must be called to the matter of wave-lengths of absorption bands in the infra-red. In looking over Chapters IV and V, and noticing that one investigator found absorption bands of CS_2 at 4.62 μ and 8.72 μ, another found bands at 4.65 μ and 8.05 μ, and a third found bands at

$4.65\ \mu$ and $6.7\ \mu$, it would appear that the whole is in a state of chaos. As a matter of fact, the latter numbers refer to *one* band whose absolute value in wave-length is about $6.7\ \mu$. The older values, found by Ångström and by Julius, were obtained at a time when the dispersion of rock salt was unknown beyond $5\ \mu$, and the wave-lengths were obtained by extrapolation. Subsequent work by Rubens and others shows that these values are very much too large. Chwolson, in his *Lehrbuch*, has made the same slip, but it makes less difference than in the *Spectroscopy*, which is really a book for reference, to be quoted as an authority just as is true of the preceding volumes. In quoting these wave-lengths, beyond $5\ \mu$, the author should have called attention to this discrepancy. This is merely an oversight (see p. 384), and will not militate against the use of the volume as a whole, for a standard of reference.

WILLIAM W. COBLENTZ.

Astrometrie: oder, Die Lehre von der Ortsbestimmung im Himmelsraume. Erstes Heft: Die Sphärik und die Koordinatensysteme, sowie die Bezeichnungen und die sphärischen Koordinatenmessungen. Von WILHELM FOERSTER. Berlin: Georg Reimer, 1905. 4 marks.

All former students of Professor Foerster will recognize in the contents of this first *Heft*, which is soon to be followed by others, the main essence of his lectures on astrometry which he has been accustomed to deliver for a long series of years during the summer semesters at the University of Berlin. His successful attempt to build up from rather simple and very fundamental notions the entire structure of that part of the astronomical science which deals with the measurements of the macrocosmic configurations, the unique representation of the theory of the individual astronomical instruments as special types of one general form of instrument, brings a much-desired unification into that field of astronomy which is generally referred to as "spherical astronomy." In the small space allotted to this review it is impossible to do much more than simply quote the various heads of topics dealt with in the 160 pages which are before us: I, "Die Sphärik;" II, "Die Koordinatensysteme und Bezeichnungen;" III, "Die sphärischen Koordinatenmessungen." Section I starts out with general remarks on binocular and stereoscopic seeing, defines instrumentally the line of vision, and gives a general characterization of the measurement of angles. Emphasis is laid on the polar triangle. In Section II the orientation of a point S by the two co-ordinates σ and γ,

which are called *Poldistanz* and *Leitwinkel*, lays the basis to all later representations. Section III, which occupies pages 27 to 160, brings the theory of the equatorial, the transit instruments, and the horizontal universal instrument. The book will prove to be of great service to all who have become acquainted with astronomical instruments through actual observations. As a textbook for beginners it will likewise be very useful, provided that continued reference is taken to properly constructed models and to the instruments themselves. Otherwise the high view-point of regarding the individual phenomenon as a special case of a much more general type, which is the characteristic beauty of this book to the connoisseur, may prove to produce undesired effects with the uninitiated. For more than ten years my lecture notes on Professor Foerster's *Astrometry* have been used as a basis for an introduction to spherical astronomy. From the experience gained in this way I have become thoroughly convinced of the great pedagogical value inherent in this form of representation.

Kurt Laves.

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THE ARC IN HIGH VACUA

By R. E. LOVING

INTRODUCTION

If a discharge tube is made with the electrodes placed opposite and extending very close to each other, e. g., 1 mm apart, one may observe the following phenomena as the pressure is gradually reduced. With pressure of the order of 1 mm of mercury, the current passes in the form of the ordinary purple glow. As the pressure is lowered, the luminosity of the gas decreases, and there is a noted increase in the potential difference necessary to cause the discharge to pass. This last fact is strikingly illustrated in the classical experiment of Hittorf.[1] Professor Thomson, in his theory of the discharge through gases, has shown that the potential difference necessary to produce the discharge in a vacuum tube must become very much greater if the distance between the electrodes is made less than the length of the negative dark space at the existing pressure. If the potential difference between the electrodes is still further increased by putting a spark-gap in series with the tube in the circuit leading to the electrical machine, cathode rays are given off strongly even at a pressure of 1 mm. When a pressure of a few thousandths of a millimeter is reached, if the external spark-gap is increased to 2 or 2.5 cm, the profuse cathode discharge and the attendant phosphorescence over

[1] *Wied. Ann.*, **21**, 96, 1884.

the surrounding glass walls vanish, and the current passes in the form of a brilliant spark or arc between the electrodes.

Soon after the discovery of the Roentgen rays, Professor Rowland,[1] in the course of some experiments on the source of the radiation, noticed in one of his tubes, having aluminium electrodes about 1 mm apart, that when the pressure was extremely low, the discharge passed as a "spark or arc" between the electrodes. He observed that the spot of light on the anode was the seat of very strong Roentgen radiation. Professor R. W. Wood,[2] working independently, published about this time a paper on "A New Form of Cathode Discharge and the Production of X-Rays, together with Some Notes on Diffraction," in which he noted many of the properties of the discharge and mentioned some points deserving further study.

GENERAL STATEMENTS

Before taking up in order the several lines along which this investigation was directed, I shall give a brief description of the arc as I have produced it. Then I shall indicate the view which I was led to take concerning the nature of the discharge. Thus will be made apparent the points chosen for special study, the results of which have substantiated the view adopted. There will also appear the grounds on which I have chosen to speak of the discharge as an arc, although it is of course intermittent.

The electrode which I have found to give the most intense light are platinum beads about 1.5 mm in diameter, easily made by fusing the end of a platinum wire in the oxyhydrogen flame. If the vacuum is good, say a few thousandths of a millimeter, and the beads are placed 2 or 3 mm apart, the cathode rays go off in every direction from the negative electrode, but principally in the horizontal plane normal to the cathode wire and containing the negative bead. If now a spark-gap of 2 or 2.5 cm is introduced in the circuit leading to the machine, and the electrodes are brought nearer together, when they are within about 1 mm of each other, the phosphorescence on the walls of the vessel vanishes, and there appears the brilliant light on the anode bead.

But while under these circumstances there was no phosphores-

[1] *Physical Papers*, p. 574. [2] *Physical Review*, **5**, 1, 1897.

cence on the walls of the outer tube, there was a bright glow on the small capillary tube into which the anode wire was sealed (see Fig. 3), extending several centimeters beyond the seal. This phosphorescence on the anode tube did not appear at relatively high pressures, but only when the pressure and distance between the electrodes were such as to cause a very high potential difference—i. e., only when there was a strong electric field around the electrodes. It was also observed that when this glow first became distinct on the anode tube, there was often a faint glow on the cathode tube also, but this did not persist at very low pressures or when a wide external spark-gap was introduced. A tube having platinum-wire electrodes sealed into bulbs joined by a capillary showed that when the potential difference between the electrodes was not too high, cathode rays were given off with very rapid alternations by each wire, principally by the normal cathode, and less by the electrode joined to the positive pole of the machine. The discharge from the machine was therefore under these conditions oscillatory, as was shown also by a telephone placed in the circuit. Thus the occasional appearance on the cathode tube of the phosphorescent glow, such as persisted on the anode tube, but nowhere else when the potential difference between the electrodes became very high, was due to its being temporarily an anode. The persistent phosphorescence on the anode tube is due, I think, to two causes. Some cathode rays shot off from the cathode toward the anode do not strike the bead, but go just by its edge, and, moving parallel and very close to the anode wire and tube, are drawn in by the very strong field about the tube and so strike against it, causing the phosphorescence and a feeble emission of Roentgen rays. Moreover, there is a small quantity of gas present in the vessel, and gas is being given off continually by the beads, particularly by the anode, which suffers disintegration, so that the pressure about the electrodes is probably appreciably higher than the average pressure in the system which is registered by the gauge. Since this gas about the electrodes, and especially in the region around the positive wire and tube, is ionized by the cathode rays which go past the positive bead, we may expect that the electrons or negative particles thus set free will be drawn in toward the anode, and in moving through the high potential gradient near the anode tube will acquire sufficient energy to cause

phosphorescence. Such an accelerating action of the positive electrode on the cathode rays is illustrated in many Roentgen-ray tubes, where it is found that the emission by the anti-cathode is markedly increased if this is made the anode or is joined with the anode. In fact, instances may be cited in which the anti-cathode gave no evidence of even a feeble Roentgen radiation unless it was made anode, in which case it became a strong source. If the above explanation of the glow on the anode tube is correct, we should expect that any screen or obstacle, placed in close to any part of the anode wire or tube so as to obstruct a section of those cathode rays which pass by the anode beads, would produce at most only a shading in the glow on the tube, and not a complete shielding; for there would still remain the effect due to the electrons set free in the surrounding gas by the stray cathode particles.

After the arc has commenced to pass steadily, the distance between the electrodes may be increased to 1.5 mm, provided the vacuum is maintained by pumping (for the evolution of gas by the electrodes is considerable). Under these circumstances, which are the most favorable for observing the discharge, the source of light has not the form of a continuous line reaching from one bead to the other, as is the case with the ordinary spark, but rather looks like a crescent on the outer or near surface of the anode. The accompanying drawing will make my meaning plain. This brilliant light is usually surrounded by a sort of halo or corona, the color of which depends on the metal of the anode. This halo was most marked with a magnesium anode, and was of a yellowish-green color. If the discharge runs for some time, the end of the anode is markedly disintegrated. Red-hot particles are shot off from the end of the anode in every direction. This effect was most striking in the case of a titanium anode, when the luminous particles were shot off in great numbers, and often suffered several reflections from the walls of the vessels while still glowing. This disintegration of the anode is of course more marked for the volatile metals. There is also a deposit on the end of the cathode, and the amount of this deposit is apparently proportional to the loss of the anode. Two especially well-formed deposits and craters were noticed with a platinum cathode and magnesium and iron respectively as anode. I give in Fig. 2 a sectional view of the anode and cathode, before the discharge and after it had run

for about two hours. If the vacuum was about 0.001 mm and the external spark-gap was closed so that the discharge became less intermittent, the anode bead was intensely heated; the small platinum bead could thus be raised to a white heat in a minute or less. This heating of the anode is of course only an example of the well-known action of cathode rays on any obstacle against which they strike. If the discharge is intermittent, there is time between the showers for the heat to pass off by radiation and conduction along the wire. Again, if the pressure is not extremely low, the potential gradient is not great enough for the cathode particles to be given sufficient energy to cause visible heating by impact. If the anode, instead of being a bead on a small wire, was a small bar of metal 1.5 mm in diameter, no visible heating effect could be produced, conduction in this case being too rapid.

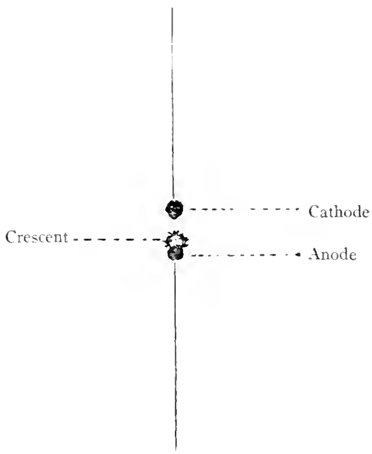


FIG. 1

Having observed that the anode wastes away as in the ordinary arc, that the light belongs to the anode rather than to the two elec-

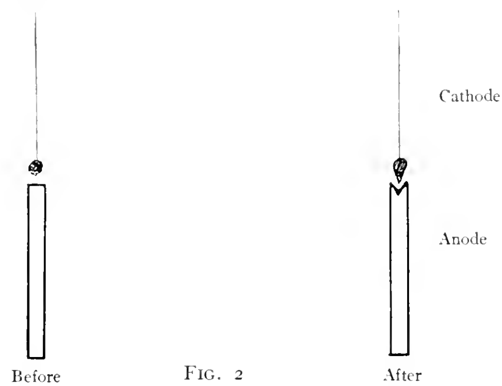


FIG. 2

trodes equally, that it is much more intense than the ordinary spark in air between the same electrodes with the same source of current, that the anode is (under certain conditions) visibly heated, I have

chosen to designate the discharge by the word "arc"; not meaning that I consider it similar to a uniform steady arc, but rather that it is in these respects analogous to the arc, and, when the anode is markedly heated, possesses the essential characteristic of the arc. The fact that the light is emitted by the bombarded anode, and that the luminosity as well as the heating of the anode seems to be due to the violent impact of the cathode particles, makes it plain that the discharge cannot be strictly an arc or a spark, for no such conditions exist in either as they are commonly produced. The phenomena might seem to approximate more nearly to a possible state of things in the chromosphere, where we may think of the matter as suffering severe impacts of the small parts against each other, due to the falling in, under the Sun's attraction, of material from the outer portions. Hence it was that I have studied the spectra of such substances as show strong chromospheric lines. Moreover, from the experiments of Hartmann[1] and others it appears that the conditions for the existence of the so-called characteristic spark lines are fully met in this case, so that we should expect in advance to find these, whether or not there are also present characteristic arc lines.

OBJECT OF INVESTIGATION

In the study of this discharge the points which seemed to merit special investigation were:

What becomes of the cathode rays when the profuse phosphorescence on the walls of the tube vanishes and the brilliant little arc appears?

What is the action of a magnetic field on the arc?

Is there any change in the current when the above-mentioned change in the discharge takes place?

What is the nature of the spectrum of the light emitted—does it correspond in general to the characteristic spark or arc spectrum, and is the character of the lines at all similar to the corresponding chromospheric lines?

APPARATUS

The vacuum was produced with a Geissler-Toepler mercury pump, and the pressures were read with a McLeod gauge. The source of current was always a six-plate Toepler-Holtz electrical machine.

[1] *Astrophysical Journal*, **17**, 270, 1903.

This was used rather than a coil, since such a machine gives a practically constant potential and unidirectional current. If working under favorable weather conditions, the current from the machine was about 0.15 milliampere on closed circuit and 0.1 milliampere with an external spark-gap of 2.5 cm in series with the tube. These readings were made on a Roentgen ammeter, kindly furnished by the Roentgen Manufacturing Co. of Philadelphia. It was in every case necessary to have one of the electrodes movable, hence the discharge apparatus was always mounted on a glass tube about 85 cm long, dipping into a mercury basin. The lower electrode was sealed into a glass tube bent into the form of a U, one arm extending up the long glass tube in which the mercury formed a barometer column, the other being held in a clamp which was movable by a slow motion screw. The particular form of tube and other apparatus depended on the experiment in hand, and will be described in connection therewith.

EXPERIMENTS ON CATHODE RAYS

The vacuum apparatus used in all experiments in this connection was a small bell-jar, 8 cm in diameter and 20 cm high, fastened to a ground aluminium plate with stopcock grease (prepared according to the formula of Travers:[1] 1 part vaseline, $\frac{1}{8}$ part paraffin, 2 parts rubber. The lower electrode was always connected to earth. The platinum wires *P* and *Q*, on which were fused the electrode beads, were sealed into capillary tubes, as thin-walled tubing was often punctured by the discharge, which seemed to prefer a path of 10 or 15 cm to the very short one between the beads. The tube carrying the electrode *P* was sealed into the mouth of the jar *J* with Khotinsky wax, and the aluminium plate *A* was similarly fastened on to the barometer tube *H*. Through *H* there passes, alongside the electrode tube, a glass rod *R* bent into a U at the bottom, and at the top into a shape indicated in the figure, so that any kind of phosphorescent screen or obstacle could be mounted on this rod by being fastened on to a small piece of glass tubing which fitted rather snugly over the end of the rod. This cap could be made secure and rigid by a little wax warmed and put on the end of the rod as the cap was pushed down over it. The rod *R* is capable of a rather

[1] *Study of Gases*, p. 24.

wide vertical motion, so that any part of any mounted screen could be brought opposite either of the electrodes P and Q. A limited freedom of rotation of R about itself as axis made it possible to bring the screen up to the electrodes or to hold it quite out of the line of the discharge.

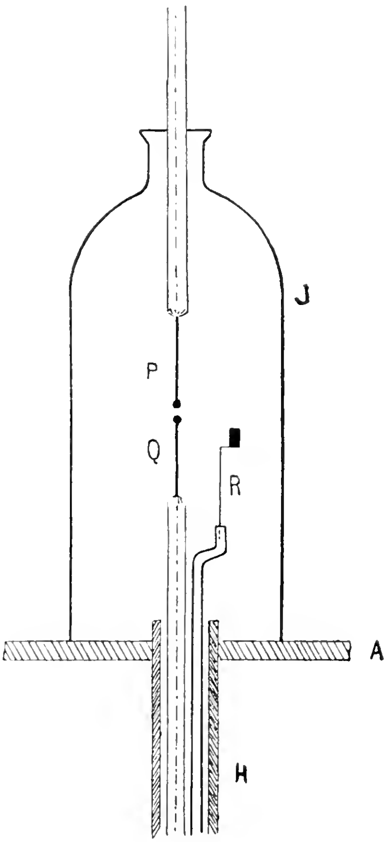


FIG. 3

Now, the very intense emission of Roentgen rays by the end of the anode very near to the cathode in a highly exhausted tube, as has been already alluded to, might seem to indicate a sort of focusing, so to speak, of the cathode rays on the anode surface just next to it. Moreover, some Roentgen-ray graphs taken by the writer with a pinhole camera seem to indicate that the surfaces most vigorously bombarded by the cathode rays during the arc discharge are the opposite surface of the anode, the wire, except just above the bead, and the more or less blunt end of the glass tube into which the anode wire is sealed. It may be remarked that on a plate obtained with a two-hour exposure at a distance of about 10 cm from the source there is no impression of the cathode, so that there seems to be no Roentgen radiation at the starting-point of the cathode particles. A distinct impression was made by the lower surface of the anode in three minutes.

Hence it was thought that, after the arc was formed, the distribution of the cathode rays was probably such as the following, viz.: the principal part of the discharge passes across directly from one electrode to the other, and the few cathode particles, shot out at a small angle with the vertical and going past the anode bead, are deflected by the electric field, which is very strong on account of the

extremely low pressure and the nearness of the electrodes. These are thus drawn in toward the anode, and so strike against the anode wire or the glass tube farther on. To test the views which I have indicated concerning the distribution of the cathode rays, and the cause of the persistent glow on the anode tube, the following experiments were made.

A small piece of copper foil about 5 mm square, and coated over with burnt ruby dust, which phosphoresces with a bright red color under the cathode rays, was mounted on the rod R (Fig. 3) with its plane vertical and perpendicular to a radius of the jar. By means of the rod the screen could be moved in a vertical or horizontal plane. Before the arc formed, if the screen was opposite the cathode bead or any part of that wire, there was a bright glow on the side of the screen next to the electrode and a weaker glow on the opposite side, the latter being evidently due to the diffuse cathode radiation from the walls of the jar. After the arc was formed there was no glow on the screen until it was moved up to or above the level of the upper bead, which was anode. In such a position, however, there was now a glow on the outside of the screen (though there was no radiation from the walls of the vessel) and none on the inside next to the electrodes. If the screen was moved up very close to the glass tube carrying the anode, there was a weakening of the phosphorescent glow on this tube, but not a complete shadow. A very strong phosphorescence was seen on the edge of the screen next to the cathode when the screen was so placed that its lower edge was opposite to or above the anode bead and was close in to the axis of the discharge. The fact that during the arc discharge there was, for the above-mentioned position of the screen, a glow only on the far side, shows that the cathode particles in the region are moving with appreciable velocity only toward the anode. This glow is thus due principally to the electrons generated in the region and drawn toward the anode by the electric field. The much stronger glow on the edge of the screen next to the cathode shows that, besides these secondary rays, there are some which come directly from the cathode against this edge.

Such a screen as was described above was then mounted with its plane horizontal. Before the arc passed, if the screen was on the

cathode side of the gap, there was a glow on the surface facing the gap. But if the arc was formed, there was no phosphorescence on the screen for such a position; but if it was moved opposite the anode wire or bead, and brought in very close to the wire, the glow appeared on the side facing the cathode; and this glow was much more intense on the parts of the screen nearest to the electrode wire. This was more marked if the screen was placed near the anode wire just beyond the bead. This screen, as the one above referred to, failed to cast any distinct or sharp shadow on the anode tube, which, as I have said, phosphoresced brightly for several centimeters beyond the seal of the platinum wire. This experiment also tends to confirm the opinion that some particles leave the cathode in a direction slightly inclined to the vertical and pass around the anode bead, but that most of them are confined to a region very close to the anode.

Besides the metallic ones, several mica screens were used, also screens or obstacles made of newly drawn small soda-glass wires. The phenomena were in each case similar to those of the experiments described above, and suggested the same explanation.

Now, with a fairly low pressure and small external spark-gap, it was often noticed that the phosphorescence on the bell-jar was confined to a narrow zone including the plane of the cathode bead, thus showing that the particles were shot out horizontally. As the necessary conditions for the arc were more nearly satisfied, the phosphorescent zone moved toward the anode end of the jar and became progressively less defined. The preceding experiments show that when the discharge passes as the arc, the cathode particles outside of the arc-gap are confined to the region close around the line of discharge and (excepting secondary radiation) are moving parallel to the discharge. It therefore seemed of interest to investigate whether there was a gradual change in the path of the rays from along a horizontal to a vertical direction.

A mica screen about 2 cm square, coated with ruby dust, was mounted with its plane vertical, and almost but not exactly containing the electrode wires. This gave sections of the cone of rays sent out by the cathode, and the shadow of a short glass wire which pierced the screen showed their path. Again, a mica disk mounted horizontally, and having a small hole in its center, could be placed

by means of the rod R in any desired position above or below the gap. As the cathode particles are at times shot out horizontally, and then, when the potential difference is increased, in a direction making an acute angle with the line joining the cathode to the anode, it was thought that the horizontal screen would indicate the angle at the vertex of the cone of rays from the cathode, and show whether this angle decreased continuously till the discharge passed as the arc; when, as already observed, no cathode particles make any large angle with the vertical. No result pointed to any other conclusion than that the change from the non-luminous to the so-called arc discharge was abrupt. There was no continuous and gradual concentrating of the rays along or nearly in the vertical, although the latter distribution always accompanied the arc. Furthermore, it was noticed that even when the mica disk was just opposite the end of the glass tube into which the anode was sealed, there still remained the phosphorescence for 3 or 4 cm along the anode tube.

I then determined to look more especially for the source of the persistent phosphorescence on the anode tube near to and just above the seal of the platinum wire into the glass. A small glass wire, about 1.5 mm in diameter, was bent into a ring large enough to go over the anode tube and leave a space of at least 1 mm all around. This was mounted on the rod R and, as the arc passed, was moved into various positions along the phosphorescent portion of the anode tube. Sometimes there seemed to be a faint shadow, but the cases were irregular and uncertain. Two little glass wires, about 3 mm long, stuck on to the anode tube radially, also failed to cast any distinct shadow along the tube. Bits of platinum wire gave similar results. The phosphorescence along the tube at points 2 or 3 cm from the end was then not caused principally by the glancing impact of rays shot out from the cathode and moving in paths only slightly inclined to the axis of the discharge.

Again, a thin glass disk, having in its center a hole just large enough for it to slip over the Pt beads, was mounted on the rod R. No position of this screen, whether just in the plane of the anode bead or quite up against the seal of this wire into the glass tube, gave any appreciable weakening of the glow on the anode tube. A small piece of glass tubing, 3 cm long and 2 or 3 mm larger in diameter

than the electrode tube, was mounted with the disk so that the two formed a sort of cap for the anode tube. This larger tube produced a very distinct shading, but there was still considerable phosphorescence on the protected or screened parts, although no primary cathode particles could strike the anode tube, and no secondary ones except such as were generated within the inclosing tube. The only explanation which occurs to me of the phenomena in question is that the glass surface of the disk and of the short outer piece of tubing, being struck by primary and also by secondary particles, emits into the inclosed space some radiations which, under the action of the strong field, bombard the anode tube and also act as ionizers. Since this space was very limited, the number of ions generated, and so the number striking against the anode tube, was smaller than if the tube had been removed, and so there was less intense phosphorescence than on portions of the anode tube that were not thus enclosed.

ACTION OF A MAGNETIC FIELD

In studying the action of a magnetic field on the discharge, the vacuum apparatus was a glass tube about 3 cm in diameter having a T-tube sealed into one side just opposite the electrodes and covered at its end with a glass plate sealed on with wax. In the first experiments to be described this side tube was normal to the plane of the axis of the magnet and the line of the discharge, and was 5 or 6 cm long, so that the glass plate would not blacken so rapidly with the platinum deposit; for the arc was observed through this side tube. The arrangement is shown in the drawing on the following page, except that the side tube is here a very short one along the axis of the magnet, which was a later arrangement. *N* and *S* are the conical pole-pieces of the electromagnet; these are bored through the center, so that when desired the arc can be viewed by looking along the axis *AB* of the magnet. A variable resistance in series with the exciting coils enabled the strength of the field to be altered at will. With low pressures, say 6 or 7 thousandths of a mm, if the external spark-gap was adjusted and the electrodes so placed that the arc was just on the point of forming—i. e., passed irregularly—the effect of the magnetic field was to cause the arc to pass regularly or steadily; the field seemed then to aid the formation of the arc discharge.

Again, the magnetic field often caused the anode bead to become visibly heated. Both of these actions may arise from the fact that a magnetic field in general hinders the discharge.[1] Here, then, its effect is to cause an increase in the potential difference between the electrodes, and is thus analogous to a lowering of the pressure.

As evidence of the fact that cathode rays have a very much higher velocity when the magnetic field is on, the following phenomenon may be mentioned. If the arc is not passing, but we have the phosphorescent glow over the tube, the magnetic field twists into spirals

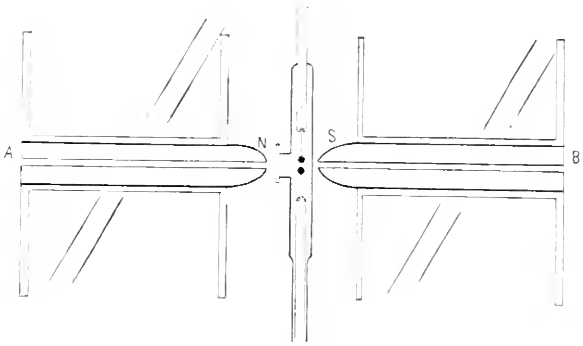


FIG. 4
(The oblique lines represent the Field Coils.)

the paths of all particles except such as are moving parallel to the field, and these therefore form on the tube a sort of image of the cathode. There thus remain two bright phosphorescent spots on either side of the tube, and the intensity of the glow is much increased in these images, as I call them. On cleaning the tube with aqua regia, it was found that in these places the glass appeared etched. It seems worth mentioning that this violent cathode discharge exerted some sort of deteriorating action on the walls of the tube around the discharge; for after a tube had been used for some days, it was found to crack very easily if put in the flame for any reason, and it was easy to break up the fragments of such a tube with one's fingers. I have never seen it stated that the glass of cathode tubes becomes specially fragile, and have no explanation to offer, but simply mention the fact.

As to a deviation along the normal to the plane of the electric and

[1] Thomson, *Conduction of Electricity through Gases*, p. 474.

magnetic force of the spot of light which marks the point of impact of the cathode particles, it need only be said that with such magnetic field-strengths as I could obtain, calculation showed that the possible deflection of the rays was less than could be observed, especially since the arc was not stationary, but wandered about over the anode slightly according as one part or another of the cathode was specially active.

It remains to be said that the magnetic field did not noticeably alter the distribution of the phosphorescent glow on the anode tube, though there was a slight weakening of the effect.

CURRENT MEASUREMENTS

Before I had the opportunity of using the Roentgen ammeter, I had made some readings with a small gas voltameter in order to see whether there was any appreciable variation of the current as the nature of the discharge changed. However, the current from the machine is very small and varies greatly with the weather conditions, so that I was not able to get any results which justified supposing that there was any material change in the current.

Measurements with the Roentgen ammeter did not show any change of current whether the discharge passed as the arc or whether there was the brilliant cathode glow over the tube, nor did the magnetic field have any readable effect. It is to be noted, however, that even a relatively large change in the resistance of the tube would make only a negligible change in the resistance of the whole circuit, including the machine itself, and that we do not have at hand a large source of current. Hence no large current changes should be expected for any of the variations which we have made in the conditions or nature of the discharge.

STUDY OF SPECTRUM

The spectrum of the discharge was produced with a Rowland concave grating of 60 cm radius. It was desirable to study specially the violet end of the spectrum, as most of the substances examined have the strongest lines in this region. Hence I used always a tube having a quartz window. The arrangement of the tube, slit, etc., is shown in the diagram. No lens was used, as the arrangement would have required a specially ground one of quartz, and very little would have been gained by it anyway. The time of exposure necessary

was about one or one and a quarter hours. The films used were Eastman Kodoid, orthochromatic. The form of the electrodes was for one a platinum bead, and for the other a small bar of the substance about 1.55 mm diameter, fastened in the end of a brass wire. This wire, about 3 cm long and having a screw socket joint in the middle was fastened rigidly to a heavy platinum wire sealed into the upper end of the U-tube. To change the electrode it was necessary only to lift off the bulb above the ground joint, unscrew the little brass end,

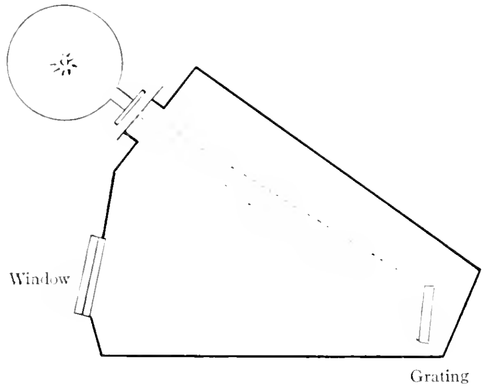


FIG. 5

and replace it by a similar one containing a different substance. For the calcium spectrum I used ordinary lime packed into a bored-out brass wire.

The first substances tried were *Mg* and *Pt* as anode and cathode respectively. There seemed to be no trace of *Pt* lines on the plate. It may be recalled here that I have said before that the light seemed to belong to the anode rather than to both electrodes; so I thought it worth while to test this point spectroscopically with two such metals as *Mg* and *Pt*, the one very easily volatilized and the other quite the opposite. I therefore carefully cleaned the *Pt* bead with acid and reduced the pressure to a few thousandths of a millimeter. If now the discharge was started with the *Pt* as anode, an exposure of as much as two hours failed to show any sign of the *Mg* lines. Of course, it was not possible to get the spectrum of the other electrode merely by reversing the current between exposures, on account of the previous deposit from the anode on the cathode. Nor was it possible to

reduce the pressure gradually and determine by eye observations at what pressure the anode only was active, for sufficient *Mg* got on the *Pt* anode to show in the spectrum even after the pressure was so low that the *Mg* cathode itself had ceased to play any part. In going from very low to higher pressures, the steps could not be made gradually, and so all that I can say is that so long as the pressure was low enough to cause the discharge to have the form of the little crescent of light on the anode, only the lines characteristic of the anode could be seen.

Having shown that the spectrum was characteristic of the anode only, if the discharge passed in the form which I have designated as the arc, I obtained the spectra of *Mg*, *Cu*, *Cr*, *Mn*, *Ti*, and *Fe*, respectively, by making anodes of these substances, the same *Pt* cathode serving throughout the series. Only the stronger lines made an impression on the plates in the time given an exposure; these, however, were amply sufficient for purposes of comparison such as I had in mind. The general character of the spectrum does not correspond to that of the arc or spark. (I could not take the ordinary arc or spark spectra on the same film with the spectrum of this anode light, and the intensities for these as given in the tables below are from Exner and Haschek.) Nearly all of the strong spark lines appear, and the spectrum is very much more like the spark than the arc. But the relative intensity is by no means the same for all the lines. Nor does there seem to be any close analogy between the intensities in this case and those of the chromosphere. There is observed here, however, an effect analogous to that brought out by Petaval and Hutton;[1] viz., certain lines have in this vacuum arc an intensity which is not at all in the same proportion to that of neighboring lines as in the ordinary arc or spark. It is doubtful if in this case the sole cause is diminution of pressure, but probably it is due both to the low pressure and to the fact that the luminosity is here excited under conditions materially different from those existing in the arc or spark in air.

I give below a list of the stronger lines showing on my plates, with their relative intensities in the ordinary arc and spark, chromosphere, and the arc as I have produced it in the vacuum tube. I shall designate these respectively by A, S, C, and Av.

[1] *Phil. Mag.*, **6**, 569, 1903.

MAGNESIUM

λ	Intensity A	S	C	Av	λ	Intensity A	S	C	Av
2777......	20	6		2	2929.......	2	200		6
2778......	20	5		1	2937.......	3	200		8
2780......	30	10		3	3829.......	30	200	5	5
2782......	20	5		1	3832.......	50	300	7	9
2783......	20	6		1	3838.......	100	500	10	15
2791......	5	100		12	4481.......	0	50	1	20
2796......	200	500		10	5073.......				?
2798......	2	100		8	5168.......			2	0
2803......	100	500		8	5173.......			4	1
2852......	500	100		8	5184.......			5	2

CALCIUM

λ	A	S	C	Av	λ	A	S	C	Av
3159......	10	50		1	4289.......	50	20	5	0
3179......	15	50		2	4299.......	30	20	1	00
3706......	10	50	10	3	4303.......	100	50	2	3
3737......	20	50	15	5	4308.......	30	20	5	0+
3934......	500	1000	75	25	4318.......	50	30	1	1+
3969......	300	500	60	20	4355.......	3	1		3
4227......	1000	100	8	5	4435.......	100	20		1
4283......	50	20	1	0	4455.......	200	30		2

CHROMIUM

λ	A	S	C	Av	λ	A	S	C	Av
2672......	3	8		1	2980.......	2	10		2
2677......	4	20		1	2985.......	2	10		1
2688......	2	10		2	2989.......	1	10		1
2704......	1	6		1	3016.......	2	3		1
2727......	1	5		0	3027.......		8		1
2749......	3	8		1	3041.......	4	10		2
2751......	3	10		1	3050.......	1	10		2
2752......	3	10		1	3119.......	3	10		1
2763......	3	10		1	3120.......	3	15		2
2767......	4	15		3	3125.......	3	20		3
2792		10		1	3132.......	4	20		4
2801......	1	10		1	3147.......	2	5		0
2812	1	10		1	3368.......	3	20		1+
2818......	1	8		0	3403.......	3	15		1−
2822......	1	10		2	3409.......	1	20		1
2831......	1	20		1	3579.......	30	20		2
2836......	4	30		3	3594.......	30	20		2
2843......	4	15		2	3605.......	30	20		2
2850......	4	10		2	3964.......	6	8		1
2851......	1	7		1	3970.......	5	8		1
2856......	3	10		1	3977.......	6	8		1
2863......	3	10		1	3984.......	4	5		1
2876......	2	5		3	3993.......	3	3		0
2899......	1	5		2	4254.......	50	50		4
2922......	1	3		1	4275.......	50	30	1	3
2927......		5		2	4290.......	30	30	1	3
2935	1	4		1	4337.......	10	8		1
2947......	1	3		0	4345.......	10	10		1
2954......	1	4		0	4352.......	15	10	1	2
2972......	2	10		2					

MANGANESE

λ	Intensity A	Intensity S	Intensity C	Intensity Av	λ	Intensity A	Intensity S	Intensity C	Intensity Av
2576......	4	30		5	2892.......	1	4		0
2594......	4	15		4	2898.......	1	3		0
2606......	4	10		4	2900.......	1	3		0
2610......	1	8		1	2933.......	3	15		3
2618......	2	8		2	2939.......	3	20		4
2626......	2	7		2	2949.......	3	30		5
2633......	1	7		2	3442.......	2	30		3
2638......	1	5		2	3460.......	2	20		2
2640......	1	5		1	3474.......	1	15		2
2656......	1	4		0	3483.......	2	12		1
2667......		4		1	3489.......	2	10		0
2702......	1	5		2	3807.......	1	8		1
2706......	1	8		2	3824.......	4	6		1
2709......		4		1	4030.......	100	20		1
2712......	1	5		2	4033.......	100	20		4
2719......		3		1	4034.......	50	10		3
2795......	50	4		2	4041.......	20	10		4
2805......	1	5		1	4048.......	8	7		1
2813......	1	3		0	4055.......	4	8		1
2815......	1	3		1	4064.......	5	6		1
2831......	2	3		0	4070.......	4	3		1+
2870......	1	4		1	4235.......	10	20		1+
2880......	1	5		1	4451.......	5	10		1+
2890......	1	10		2					

TITANIUM

λ	A	S	C	Av	λ	A	S	C	Av
3168......	5	15		0	3706.......	2	8		1
3191......	4	10		0	3742.......	3	10		3
3202......	3	10		0	3759.......	10	20		6
3234......	8	15		2	3761.......	10	10		5
3236......	5	6		2	3900.......	5	50		3
3239......	4	6		2	3914.......	5	20		3
3242......	4	10		1	4164.......	2	20		2
3249......	4	10		1	4172.......	1	15		2
3262......	4	15		1	4274.......	15	4		1
3323......	5	10		2	4290.......	10	10		2
3329......	6	10		1	4294.......	10	10		1
3332......	3	8		1	4300.......	15	8		1
3335......	5	10		1	4306.......	20	8		1
3342......	4	10		2	4308.......	4	8		1
3349......	8	10		4	4313.......	2	8	1	1
3362......	1	30		3	4315.......	5	5		1
3373......	4	20		3	4338.......	2	10		2
3384......	3	20		3	4358.......				3
3505......	3	30		3	4395.......	10	20		3
3511......	3	30		3	4418.......	2	6	1	1
3520......	2	8		1	4444.......	4	15	2	2
3536......	2	15		1	4468.......	4	15	5	2
3625......	2	8		1	4488.......	1	6		1
3641......	15	10		1	4501.......	4	15	6	2
3660......	3	10		1	4550.......	4	20	8	4
3662......	2	10		1	4564.......	3	10	5	2
3685......	8	100		5	4572.......	3	20	4	3

IRON

λ	Intensity				λ	Intensity			
	A	S	C	Av		A	S	C	Av
3735	50	10		1	4045	50	15	2	4
3749	30	10		3	4063	30	10		2
3758	30	8		1	4071	20	8		2
3763	20	6		1	4144	15	5		0
3816	20	10		0	4202	10	6		0
3820	50	10		1	4250	15	6		0
3826	30	8		1	4260	20	10	1	0
3841	15	5		1	4271	30	10		3
3860	30	6		1	4305	30	15		3
3878	15	5		0	4325	30	15		4
3886	20	5		1	4355		20		2
3928	15	4		0	4383	100	20	1	6
3969	15	5	1	0	4404	50	15	1	3
4005	15	6		0	4415	20	10	1	1

A comparison of these lists with those of Exner and Haschek will show that some strong spark lines are wanting on every plate, and many of the strong arc lines. With regard to the *Mg* group at λ 5183, it should be said that the small intensity given these is due to their being far in the green, where the plates were less sensitive. To the eye they appeared as strong as the line at λ 4481. It may be noted also that the *Mg* lines are much longer on the plate than those of any other substance. This is to be expected, as I have said before that the arc was surrounded by a much brighter halo with this metal than with any of the others. The strong *Hg* line at λ 4358 showed as a long line on every plate, the extensions above and below the arc being almost as bright as the central portion.

SUMMARY

The results of the foregoing experiments may be briefly summed up as follows:

If, at very low pressures, the discharge is caused to pass across a narrow gap, the cathode particles are shot off only by the near surface of the negative electrode, and almost all of them strike against the opposite face of the anode.

The principle action of a magnetic field on the discharge is to increase the potential difference in the gap, and consequently the kinetic energy of the cathode rays.

The luminosity of the anode is due to the violent impact of these cathode particles, and the spectrum of the light emitted is not analogous to that of either the spark or the arc. The fact that the spectrum is characteristic of the anode, and that the cathode makes no impression seems to merit special attention.

In conclusion, I wish to state that this research was carried on under the direction of Professor Ames. My best thanks are due to him for much valuable advice and criticism, and for the facilities placed at my disposal, and also to Professor Wood for many suggestions. The kindness and interest of student friends have been in many ways helpful.

Johns Hopkins University.

THE FIGURE OF THE SUN. II

By CHARLES LANE POOR

THE OBSERVATIONS OF SCHUR AND AMBRONN

Since my note, "The Figure of the Sun," was written, Ambronn has published, under the title of "Die Messungen des Sonnendurchmessers,"[1] an exhaustive research upon the shape and size of the Sun. This paper embodies the results of the solar investigations of Schur and Ambronn, made with the six-inch Repsold heliometer of the Göttingen Observatory, and extending over a period of nearly thirteen years, from 1890 to 1902. The conclusions drawn by Ambronn, from this great mass of observations, do not directly bear upon the theory advanced in my paper. A re-discussion of these observations was therefore undertaken, and some interesting facts were developed, tending to support and confirm the general results I had arrived at.

The idea that the diameter of the Sun may be variable is not new; a connection between the Sun's mean diameter and the sun-spot period has been suspected, and has been made the subject of several investigations in the past. When, therefore, the Repsold heliometer was mounted in Göttingen, Schur determined to investigate this subject thoroughly, and to make with that instrument a complete and uniform series of measures, which should extend over the whole of a sun-spot period. In carrying out this program, every conceivable precaution was taken to exclude systematic errors; in fact, two complete, parallel, and independent series of observations were made, one by Schur and one by Ambronn. Whenever possible, each observer obtained a series of four measures each week, two of the polar and two of the equatorial diameter. All the necessary instrumental constants for the reduction of these observations were obtained by each observer independently of the other. But the same methods and the same formulas of reduction were used in the two series; so that these series are directly comparable. The series of Schur extends from 1890 to the beginning of 1901; that of Ambronn,

[1] *Astronomische Mittheilungen der k. Sternwarte zu Göttingen*, Theil 7, 1905.

from 1890 to the end of 1902; both series thus covering an entire sun-spot period.

In reducing and discussing this great number of observations Ambronn investigates the questions of the figure and of the variability of the Sun separately. A brief résumé of his methods of investigating each of these points is given below, together with the conclusions he reaches in regard to these important questions.

1. *Variation of the Sun's diameter.*—Each series of observations is treated separately. Ambronn first finds the mean value of the Sun's diameter from all the observations of each series; then, subtracting this mean from the separate values, he finds the residual for each observation. From these residuals he finds the value of the mean residual for each year and tabulates these "yearly residuals," which thus show the yearly variation in the diameter.

In the first of these steps Ambronn was confronted with a difficulty: the series of observations were not strictly homogeneous. In October 1891 a prism was introduced into the instrument, in such a manner that the line joining the centers of the two images could always be brought into the same position relative to the eyes of the observer. This was to obviate any possible physiological influence which might cause the observer to measure the polar and equatorial diameters differently. An investigation, however, showed that the prism had a sensible effect upon the measures of all diameters, equatorial as well as polar. The diameters measured with the prism were all somewhat smaller than those measured without it. As the prism was used continuously after October 1891, the series of observations are divided into two periods by this date. The mean results from the observations in each period are given below, where the various values are expressed in scale-divisions, one division of the scale being approximately equal to 40″.

TABLE I

	Without Prism	Number	With Prism	Number
Schur	47.9919	25	47.9823	159
Ambronn	47.9819	27	47.9745	200

As a result of special measures made by Schur and Ambronn, both with and without the prism, Ambronn concludes that all obser-

vations made without the prism must be diminished by 0".4, or 0.01 scale-division, in order to make them comparable with those made with the prism.

Correcting all the observations made without the prism by this amount, taking the general means, and reducing scale to arc, Ambronn finally obtains for the definitive values of the Sun's diameter at distance unity:

Schur	1920".14	±0".040
Ambronn	1919".80	±0".036

From these means the yearly residuals were found, and, as given by Ambronn, are tabulated below.

TABLE II

Year	Schur	Ambronn	Mean
1890	−0".10	−0".08	−0".09
91	+0.03	−0.11	−0.04
92	+0.09	−0.08	0.00
93	+0.10	+0.06	+0.08
94	+0.10	+0.11	+0.10
95	−0.04	+0.25	+0.10
96	−0.10	+0.12	+0.01
97	−0.06	−0.12	−0.09
98	+0.01	−0.08	−0.04
99	+0.05	−0.06	0.00
1900	0.00	+0.02	+0.01
01	. . . . . .	+0.03	. . . . . .
02	. . . . . .	+0.09	. . . . . .

A simple inspection of these figures shows a certain periodicity. This is shown in the series of each observer and in the series of means. The periodic time of these variations is somewhere between six and eight years, and the amplitude about 0".1. The large residual (0".25) for the year 1895 is considered by Ambronn to be due to purely personal or accidental causes.

Ambronn further compares the curves which represent the above series of residuals with Wolfer's sun-spot curve for the corresponding years. The curve corresponding to the series of means is reproduced from Ambronn's paper and is given in Fig. 1, being the lowest curve of that figure. This curve, together with the above series of residuals, shows clearly, according to Ambronn, that there is no relation between the observed variations in the Sun's diameter and the relative frequency of sun-spots.

In considering these results of Ambronn, we note that he investigates the possible variation in the average or *mean* diameter of the Sun. The above residuals and the corresponding points on his curves are found by taking, in the series under consideration, the mean for each year of all the observations of both the polar and equatorial diameters. Thus his investigation would show whether there had been any change, periodic or secular, in the volume of the Sun, and not whether there had been any change in either the polar or the equatorial diameter. Changes in the relative sizes of the diameters of the Sun, or changes in its shape which do not alter its volume, could not be discovered by the methods used by Ambronn in this portion of his paper. His conclusions show that during the entire period of nearly thirteen years there was not present any periodic or secular variation in the Sun's volume, larger than that represented by a change of 0."1 in the mean diameter of that body. This is not inconsistent with the views advanced in my paper. Ambronn merely shows that if the Sun be a vibrating body, it must so vibrate as to retain a constant volume, or a volume sensibly constant.

2. *The figure of the Sun.*—On each day of observation the polar and equatorial diameters were both measured twice, so that the research furnishes a great mass of data regarding the shape of the Sun. The values of the differences between the diameters, in the sense polar minus equatorial, are tabulated and given by Ambronn. From these are formed the mean values of this difference for each year; and from these yearly means, the mean value for the entire series of observations.

Unfortunately the tables of yearly means, as given by Ambronn, in Appendix 4, and also on page 44 of his memoir, contain errors, which mask the periodic character of this quantity. These yearly means were, therefore, all recomputed from the tabulated values of the daily observations, and the following corrections to Ambronn's computations were noted:

Page	108,	yearly mean	1891	for	+0."02	read	+0."06
"	110,	"	1896	"	+0.05	"	−0.05
"	110,	"	1898	"	−0.11	"	+0.11
"	111,	"	1900	"	+0.04	"	+0.02

These errors are also found in the table on page 44, with the exception of that for 1896, where the correct sign is given.

As we have already seen, the observations during the first two years, 1890–1891, were made under instrumental conditions different from those during the rest of the interval. Ambronn, therefore forms the means of all the observations, and also means excluding these two years, to find definitive results. But as these results were obtained from erroneous yearly means, the final conclusions are also in error. I give below the final values as given by Ambronn on page 45 of his memoir, together with the corrected values:

TABLE III

MEAN VALUE OF THE DIFFERENCE (P.—E.)

AMBRONN'S RESULTS

	Schur	Ambronn	Mean
Mean of all observations..................	+0″.008	+0″.022	+0″.015
Mean excepting those of 1890 and 1891...	−0.007	+0.002	−0.003

CORRECTED RESULTS

	Schur	Ambronn	Mean
Mean of all observations..................	+0″.030	+0″.022	+0″.028
Mean excepting those of 1890 and 1891...	+0.014	+0.002	+0.008

These corrected results show the two series to be much more consistent than do the results derived by Ambronn. The final mean shows that the polar diameter exceeds the equatorial by +0″.028, and this value agrees closely with that, +0″.038, obtained by Auwers in "Die Venus-Durchgänge, 1874 und 1882."

The mean errors of the above results are given by Ambronn as

for Schur, ±0″.015;
for Ambronn, ±0″.009.

Comparing these with the values of the quantity (P.-E.) which he found, Ambronn concludes that the deviations are accidental, and that the Sun is sensibly a sphere. If, however, we compare these with the corrected values, we find that the values of Schur and Ambronn are each more than twice the size of their respective mean errors. The results can hardly, therefore, be considered as accidental.

In testing this result, Ambronn investigates the effect of the inclination of the measured diameter on the result to determine whether there was any tendency on the part of the observer to measure vertical diameters differently from horizontal. He could find no such

effect, but he calls special attention to the observations made during the two years, 1890–1891, which show the polar diameter to be decidedly the greater; and points out the fact that these results may be due to physiological causes, for during this interval no precautions were taken to obviate this difficulty. As has been noted, a prism was attached to the heliometer, in October 1891, in such a manner that all the diameters of the Sun were measured in the same relative position as regards the vertical, and from that date on the observations are perfectly homogeneous.

Ambronn also investigates the possibility of errors in the constants of refraction which were used in reducing the observations. In the winter months the Sun was at an average lower altitude at the time of observation than in the summer months. Hence, if there were any systematic errors in computing the differential refraction, such errors would be apparent when the observations are grouped according to the months in which they were made. When the observations are so grouped, no periodic variation is shown, and Ambronn concludes, therefore, that the constants and the methods used in computing the differential refraction are sensibly correct.

RE-DISCUSSION OF AMBRONN'S RESULTS

Making the corrections, already noted, to Ambronn's series of yearly means, we have the following series of values:

TABLE IV

MEAN OF YEARS (P.–E.)

Year	Schur		Ambronn		Mean	Weighted Mean	Wt.
	P.–E.	No. of Obs.	P.–E.	No. of Obs.	P.–E.	P.–E.	
1890	+0.13	10	+0.12	14	+0.12	+0.13	6
1891	+0.06	17	+0.14	14	+0.10	+0.10	6
1892	–0.06	12	+0.07	16	+0.005	+0.02	7
1893	–0.09	10	–0.01	14	–0.05	–0.07	6
1894	+0.10	11	–0.07	11	+0.015	–0.04	10
1895	+0.04	13	+0.03	15	+0.035	+0.03	9
1896	–0.05	19	–0.01	18	–0.03	–0.03	8
1897	+0.02	26	–0.04	21	–0.01	–0.01	13
1898	+0.11	21	+0.07	21	+0.09	+0.08	11
1899	+0.05	21	–0.03	24	+0.01	–0.01	12
1900	+0.02	24	+0.02	21	+0.02	+0.02	20
1901	+0.43	1	+0.06	27	+0.245	+0.07	10
1902	......	..	–0.06	15	–0.06	–0.06	5

In forming the weighted mean for the different years, weights were assigned to the observations of Schur and Ambronn in conformity with the values of the mean error, for each year, of a single observation, as given by Ambronn. Upon the assumption that the shape and size of the Sun are constant for each year, Ambronn found, from the separate observations made during that year, the value of the mean error of a single observation, and these mean errors are tabulated in Appendix 4. From these and the number of observations were found by the ordinary formulas the weights assigned to the yearly means.

While the determinations vary, a simple inspection of the above table shows that during the period from 1890 to 1902 there was a periodic change in the difference between the polar and equatorial diameters. This is clearly indicated in the series of observations of Schur, in that of Ambronn, in the series of unweighted means, and more clearly yet in the series of weighted means. In the earlier measures the polar diameter was decidedly the larger; in the years 1892, 1893, and 1894 the equatorial diameter was the larger; in the later years the polar diameter was again the larger.

These changes in the relative sizes of the polar and equatorial diameters are shown in the diagrams in Fig. 1. In this, No. 1 represents the relative frequency of sun-spots; the heavy curve being taken from the Greenwich observations and showing the proportionate area of the Sun's surface covered by spots; the lighter smooth curve being that of Wolfer's "sun-spot relative numbers." No. 2 shows the variation in the figure of the Sun, as represented by the yearly means of the observations of Schur and Ambronn; the unweighted and weighted means for each year being shown on the diagram. In this figure the dotted curve represents Wolfer's curve of sun-spot frequency, and this curve is identical with that in No. 1. In No. 3 are shown the observations of Schur and Ambronn; the heavy curve representing Ambronn's observations, the dotted curve those of Schur. No. 4 is Ambronn's curve, and this shows the variation in the mean diameter of the Sun as deduced from all the observations of both Schur and Ambronn.

Nos. 2 and 3 show clearly the changes in the shape of the Sun. The individual curves of Schur and Ambronn are similar; the posi-

tions of maxima and minima are nearly the same in both. The mean curve shows a general resemblance to Wolfer's sun-spot curve; both curves rise rapidly to a maximum in 1893, and then gradually

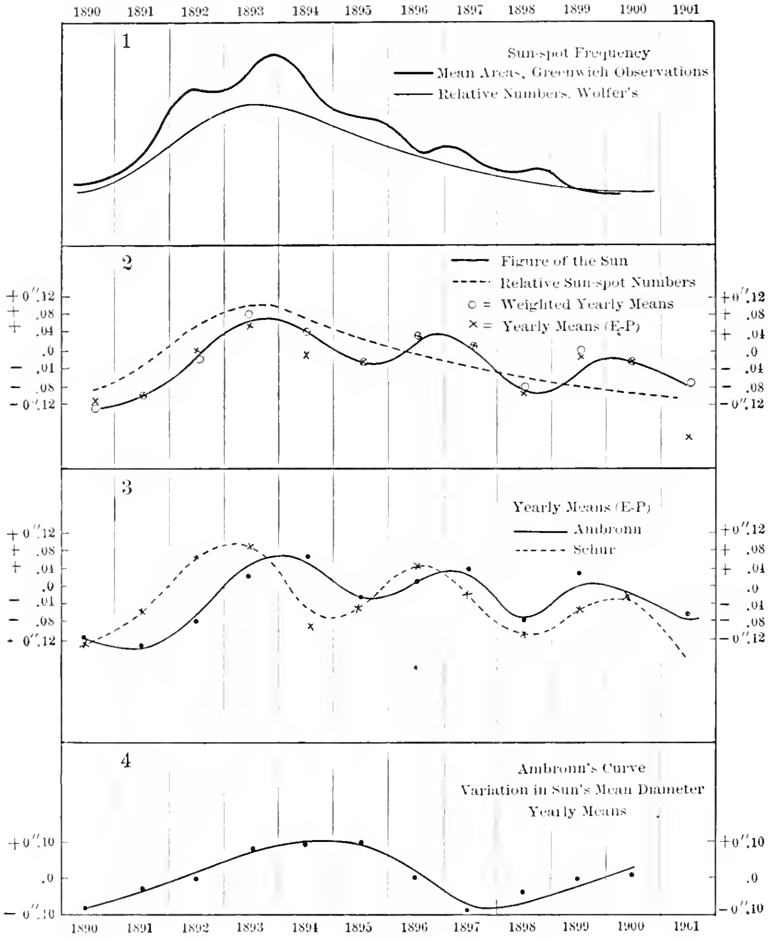


FIG. 1

fall off to a minimum in 1901. The figure curve shows, however, two subsidiary maxima in the years 1896–1897, and 1899. These subsidiary maxima do not appear in Wolfer's curve, but the first one is clearly indicated in the Greenwich sun-spot curve, which shows a decided maximum at the beginning of 1897.

From the curves in No. 2 may be found the residuals upon the supposition that the Sun is a sphere, and also upon the hypothesis that its figure varies with the number of sun-spots. Forming these residuals, we shall have:

TABLE V

RESIDUALS

Date	Sphere	Variable Figure	Date	Sphere	Variable Figure
1890........	−0.13	−0.04	1896.......	+0.03	+0.04
1891........	−0.10	−0.05	1897.......	+0.01	+0.05
1892........	−0.02	−0.07	1898.......	−0.08	−0.02
1893........	+0.07	−0.02	1899.......	+0.01	+0.08
1894........	+0.04	−0.03	1900.......	−0.02	+0.07
1895........	−0.03	−0.05	1901.......	−0.07	+0.03

From these we find for the sum of the squares of the residuals, on the two hypotheses:

	[vv]
Hypothesis of sphere,	$0''.0475$
Hypothesis of variable figure,	0.0295

This shows that the hypothesis that the figure varies proportionately with Wolfer's sun-spot numbers represents these observations of Schur and Ambronn much better than does the hypothesis that the Sun is a sphere.

Thus these observations indicate clearly that the Sun's figure is subject to periodic changes, and they point toward the conclusion that the period of these fluctuations is the same as that of the sun-spot frequency. The amplitude of these variations, as shown by these observations, is extremely small, being not more than $0''.2$.

To test this question of the variability of the Sun's figure still further, I formed the means of the observed values of (P.−E.) for every three months, making the dates symmetrical with the position of the Sun's axis. On June 5 and December 6 the axis of rotation of the Sun is perpendicular to our line of sight, and on these dates the measures will give directly the polar diameter. The periods, therefore, in which the observations were grouped are as follows:

January	15—April	15
April	15—July	15
July	15—October	15
October	15—January	15

These means are tabulated below, being arranged according to the mean date of observation; the weights being simply the number of observations from which the mean is formed in each interval.

TABLE VI

MEAN OF EVERY THREE MONTHS (P.–E.)

Schur			Ambronn			Mean		
Date	P. − E.	Wt.	Date	P. − E.	Wt.	Date	P. − E.	Wt.
1890, May 31	+0.11	5	1890, June 14	+0.05	5	1890, June 7	+0.08	10
Oct. 1	+0.16	2	Aug. 22	+0.22	5	Sept. 2	+0.20	7
Nov. 17	+0.14	3	Nov. 17	+0.09	4	Nov. 17	+0.11	7
1891, Mch. 17	−0.08	4	1891, Feb. 28	+0.09	4	1891, Mch. 8	0.00	8
June 3	+0.30	7	June 2	+0.19	5	June 3	+0.25	12
Sept. 23	+0.09	3	Aug. 10	+0.18	3	Sept. 1	+0.14	6
Dec. 6	−0.37	4	Nov. 25	+0.04	2	Dec. 2	−0.23	6
1892, Apr. 1	−0.04	4	1892, Mch. 2	+0.09	6	1892, Mch. 14	+0.04	10
June 4	+0.05	4	June 4	+0.11	5	June 6	+0.08	9
Sept. 24	−0.27	2	Aug. 22	+0.10	3	Sept. 4	−0.05	5
Nov. 24	+0.38	1	Nov. 14	−0.10	2	Nov. 17	+0.06	3
1893, Mch. 24	−0.20	4	1893, Mch. 25	−0.14	4	1893, Mch. 27	−0.17	8
May 29	−0.05	5	May 29	+0.08	4	May 29	+0.01	9
Aug. 4	+0.09	1	Aug. 13	+0.04	2	Aug. 10	+0.06	3
1894, Mch. 24	+0.16	3	1894, Mch. 16	−0.09	3	1894, Mch. 20	+0.04	6
May 30	+0.14	6	June 30	−0.04	3	June 9	+0.08	9
July 24	−0.25	1	Aug. 19	−0.17	3	Aug. 13	−0.19	4
Dec. 10	+0.03	1	Nov. 23	+0.08	2	Nov. 29	+0.06	3
1895, Mch. 18	+0.08	3	1895, Mch. 6	−0.52	1	1895, Mch. 15	−0.07	4
May 31	0.00	8	May 30	+0.07	5	May 31	+0.03	13
July 16	0.00	1	Aug. 25	+0.05	7	Aug. 20	+0.04	8
Oct. 18	+0.27	1	Nov. 24	+0.16	2	Nov. 11	+0.20	3
1896, Feb. 10	−0.11	4	1896, Feb. 21	+0.14	4	1896, Feb. 16	+0.02	8
June 8	−0.27	7	June 8	−0.09	7	June 8	−0.18	14
Aug. 25	+0.09	3	Aug. 31	−0.05	5	Aug. 29	0.00	8
Nov. 22	+0.23	5	Dec. 19	+0.03	3	Dec. 3	+0.16	8
1897, Mch. 13	+0.06	4	1897, Mch. 14	+0.18	2	1897, Mch. 13	+0.10	6
June 6	0.00	10	June 1	−0.07	9	June 4	−0.03	19
Sept. 4	−0.03	8	Aug. 29	+0.05	4	Sept. 2	0.00	12
Nov. 8	+0.14	4	Dec. 12	−0.09	7	Nov. 30	−0.01	11
1898, Mch. 17	+0.07	4	1898, Feb. 28	+0.21	4	1898, Mch. 8	+0.14	8
May 29	+0.19	7	June 9	−0.08	6	June 3	+0.07	13
July 30	+0.12	5	Aug. 20	+0.07	6	Aug. 10	+0.09	11
Nov. 19	+0.04	5	Dec. 9	+0.06	4	Nov. 29	+0.05	9
1899, Feb. 23	−0.02	8	1899, Mch. 6	−0.06	7	1899, Feb. 28	−0.04	15
May 29	+0.07	7	May 31	−0.10	6	May 30	−0.01	13
Aug. 2	+0.14	5	Aug. 21	+0.09	7	Aug. 13	+0.11	12
Nov. 4	−0.04	1	Dec. 3	+0.02	3	Nov. 26	0.00	4
1900, Feb. 22	+0.12	7	1900, Mch. 8	+0.10	6	1900, Feb. 28	+0.11	13
June 1	+0.07	8	June 2	−0.01	8	June 2	+0.03	16
Aug. 28	+0.01	6	Aug. 23	−0.02	7	Aug. 25	−0.01	13
Dec. 8	−0.17	3	1901, Jan. 8	+0.45	3	Dec. 24	+0.14	6

An inspection of these means shows that the value of (P. – E.) varies very irregularly, jumping from large negative to large positive values. There is a break in the series during eight months during the latter part of 1893 and the early part of 1894. Thus the series falls into two parts. During the first part, from 1890 to the middle of 1893, the value of (P. – E.) was on the whole decreasing; the equatorial diameter increasing with respect to the polar diameter. This is shown by the observations of each observer and by the mean. In the second part, from 1894 to 1901, the value of (P. – E.) on the whole shows a tendency to increase; during this interval the equatorial diameter was shrinking relatively to the polar. The break in the observations is extremely unfortunate, for the sun-spot maximum, according to Newcomb, occurred in the latter part of 1893.

These results are exhibited in Fig. 2. The upper curve shows the variation in the spotted area of the Sun, as shown by the Greenwich observations; the second curve, the variation in the magnetic declination in minutes of arc; and the fourth curve, the variation in the vertical force of the Earth's magnetism. The curves are taken from *Monthly Notices, R. A. S.*, Volume **63**. The third curve shows the variations in the Sun's figure as plotted from the above table. This curve of the Sun's figure shows a general resemblance to all the other three curves. The resemblance to the sun-spot curve is as striking in case of the curve of figure as in that of the vertical magnetic force. Not only do the curves agree in their general characteristics, but in many cases the curve of figure shows subsidiary maxima and minima agreeing with those in the other curves. The figure curve shows a high maximum in the latter part of 1891. Similar maxima are found in the sun-spot and in the declination curves; similar coincidences in the maxima are found in the middle of 1894, the early part of 1895, the early part of 1896, and the latter part of 1897.

After 1898 the figure curve departs from the other three. During the years 1899 and 1900 the curve of figure is too high, it does not fall to so low a minimum as do the others, and the minimum appears to be somewhat earlier in this curve than in the other three.

These observations of Schur and Ambronn thus tend to confirm the general result given in my former paper. They seem to show that the ratio between the polar and equatorial radii of the Sun is

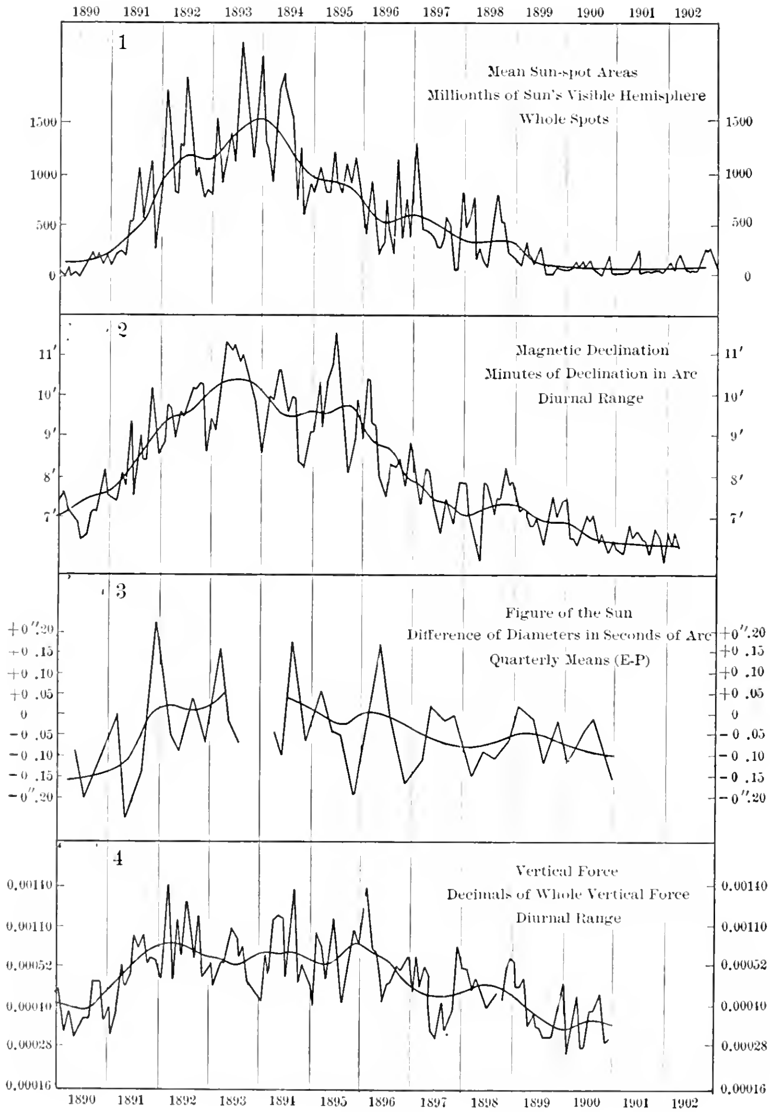

1890 1891 1892 1893 1894 1895 1896 1897 1898 1899 1900 1901 1902
1
Mean Sun-spot Areas
Millionths of Sun's Visible Hemisphere
Whole Spots
1500
1000
500
0
2
Magnetic Declination
Minutes of Declination in Arc
Diurnal Range
11′
10′
9′
8′
7′
3
Figure of the Sun
Difference of Diameters in Seconds of Arc
Quarterly Means (E-P)
+0″.20
+0 .15
+0 .10
+0 .05
0
−0 .05
−0 .10
−0 .15
−0″.20
4
Vertical Force
Decimals of Whole Vertical Force
Diurnal Range
0.00140
0.00110
0.00052
0.00040
0.00028
0.00016

FIG. 2

variable, and that this variability is periodic. The exact length of this period is uncertain, but it appears to be nearly the same as the sun-spot period. The amplitude of this variation is about $0''.2$; the difference between the largest positive and negative values being about $0''.5$.

These heliometer measures thus tend to supplement and confirm the conclusions originally drawn from the solar photographs of Lewis M. Rutherfurd. These photographs clearly show the figure of the Sun to be variable; but unfortunately they do not extend over a sufficient number of years to determine the period of this variability. On the other hand, the amplitude of this variation, as shown by the photographs, is considerably greater than that shown by the heliometer measures.

COLUMBIA UNIVERSITY,
October 1905.

OBSERVATIONS OF STANDARD VELOCITY STARS WITH THE LOWELL SPECTROGRAPH (1905)

By V. M. SLIPHER

In the present paper are given the results of my observations of the list of "Standard Velocity Stars,"[1] made with the Lowell Spectrograph during the summer and autumn of the present year. Owing to the circumstance that the time that the spectrograph is available for stellar radial velocity work is limited, I have not been able to follow closely the recommendation[1] that the three observations of each star be made at the beginning, middle, and end of the two months symmetrical about the date of the star's opposition with the Sun. Inasmuch as *α Crateris*, the faintest star of the regular list, has been, and will be for some time yet, too near the Sun for observation, I have substituted for it *γ Cephei*, the faintest star of the supplementary list, in order to bring these observations to an early conclusion. The ten stars that I have observed are, then, the following:

α *Arietis*	β *Ophiuchi*
α *Persei*	γ *Aquilae*
β *Leporis*	ε *Pegasi*
β *Geminorum*	γ *Piscium*
α *Boötis*	γ *Cephei*

I have secured, as was suggested, extra spectrograms of *α Persei* and *α Boötis;* and, in order to check the performance of the spectrograph, I have measured at frequent intervals the spectrographic velocities of *Venus*, *Mars*, and the Moon.

The spectrograph,[2] as employed in these observations, consists essentially of a collimator of 30 mm aperture and 490 mm focus, a train of three 63° dense flint prisms and a camera of 35 mm aperture and 471 mm focus, the whole inclosed in a box supplied with

[1] See Frost on "Coöperation in Observing Radial Velocities of Selected Stars," *Astrophysical Journal*, **16**, 169, 1902.

[2] A detailed description of this instrument was published in the *Astrophysical Journal* for July, 1904 (**20**, 1–20).

electrical heating. The construction of this instrument partakes of the universal type, having a device for automatically keeping the prisms in the position of minimum deviation, a feature almost indispensable in our varied program of spectroscopic work. But there is an insufficient number of clamp screws to hold the prisms rigidly without causing injurious pressure on the glass of the prisms, each prism being clamped by only one screw, which presses centrally upon the top plate of its mounting. When this screw is clamped too tightly, unequal pressure is transmitted to the prism, destroying its homogeneity. Although realizing that by so doing I was impairing the definition of the spectrograms, I have nevertheless turned down very tightly the clamp screws and thus insured the rigidity of the prisms. I have in this way obtained entirely trustworthy spectrograms, but, as might be supposed, the definition in the spectrum is rather inferior, being no better on Seed 23 plates than it should be on the coarser 27 emulsion. The full power of the spectrograph has therefore not been realized, and the agreement of the velocities from different lines of the same plate is not so close as it should be with a spectrograph of this size.

In these observations, the prisms have been used set at minimum deviation for wave-length 4415. The linear dispersion at different points through the part of the spectrum covered by my measures is as follows:

Wave-Length	Tenth-Meters per mm
4250	9.9
4300	10.6
4350	11.4
4400	12.3
4450	13.2
4500	14.1
4550	15.0

The star spectrum usually has a width on the plates of one-third of a millimeter, and is separated from the two parts of the comparison spectrum by about a tenth of a millimeter.

All the details relative to the making of the spectrograms are given in the accompanying table, which will be readily understood. The date of the observation is given in Greenwich Mean Time. Except in the case of a few of the short exposures, the comparison

JOURNAL OF OBSERVATIONS

Object	Plate Number	Date, 1905	Length of Exposure	Slit-Width	Comparison Spectrum	Temperature Inside Prism-Box	Seeing Sky	Seeing Image	Seed's No. of Plates	Remarks
β *Geminorum*	L 1833	April 7^d 17^h 0^m	57^m	$0^{mm}.019$	*Ti, Cr*	12°.47–12°.56			23	
α *Boötis*......	L 1850	14 20 15	37	0.020	*Ti, Cr*	8.13– 8.18	Poor		23	
Mars.........	L 1868	28 18 42	70	0.020	*Fe, Ti, Cr*	11.43–11.43	3	3	23	
Mars.........	L 1881	May 18 16 30	72	0.022	*Mo, Fe*	15.00–15.02	3	3	23	
γ *Aquilae*.....	L 1921	July 5 21 5	120	0.025	*Mo, Ti, Fe, Cr*	24.73–24.76	4	2	27	Spectrograph readjusted
γ *Aquilae*.....	L 1926	7 20 25	128	0.028	*Mo, Fe*	24.72–24.70	4	2–1	27	
Venus.........	L 1937	11 0 21	8	0.018	*Mo, Fe*	22.38–22.45	3	3	23	
Moon.........	L 1944	13 18 35	30	0.020	*Mo, Fe*	19.26–19.15			23	
β *Ophiuchi*...	L 1947	14 19 10	110	0.028	*Mo, Fe*	18.20–18.21			27 N. H.	
ε *Pegasi*......	L 1948	14 21 18	90	0.028	*Mo, Fe*	18.20–18.23			27 N. H.	
γ *Aquilae*.....	L 1952	15 19 50	120	0.028	*Mo, Fe*	19.90–19.98	4	3	27 N. H.	
ε *Pegasi*......	L 2007	Aug. 10 20 42	120	0.028	*Fe, V*	17.90–18.02	4	2	27 N. H.	Spectrograph readjusted
α *Boötis*......	L 2011	12 16 6	40	0.020	*Fe, Cr, V*	19.40–19.55	5	2	23	
Moon.........	L 2013	12 19 28	30	0.020	*Fe, Cr, V*	19.50–19.40	5	4	23	
α *Boötis*......	L 2016	15 16 8	40	0.020	*Fe, V*	21.20–21.18			23	
β *Ophiuchi*...	L 2017	15 17 50	120	0.027	*Fe, V*	21.14–21.15			27 N. H.	
α *Boötis*......	L 2043	29 15 34	40	0.024	*Fe, V*	21.37–21.55	4	1–2	23	Spectrograph readjusted
α *Persei*......	L 2049	30 23 6	58	0.025	*Fe, V*	20.65–20.70	3–4	3–4	23	
α *Bootis*......	L 2053	31 15 31	40	0.022	*Fe, V*	19.66–19.70	3	1–2	23	
ε *Pegasi*......	L 2054	Sept. 6 18 48	120	0.028	*Fe, V*	15.70–15.74	4	3	27 N. H.	
β *Ophiuchi*...	L 2058	8 16 32	120	0.028	*Fe, V*	17.56–17.56	3–4	1–2	27 N. H.	
α *Arietis*.....	L 2067	12 20 48	60	0.022	*Fe, V*	20.16–20.10	3–4	3–4	27 N.H.	
α *Persei*......	L 2068	12 21 59	40	0.024	*Fe, V*	20.10–20.13	3	3–4	27 N. H.	

JOURNAL OF OBSERVATIONS—*Continued*

Object	Plate Number	Date, 1905	Length of Exposure	Slit Width	Comparison Spectrum	Temperature Inside Prism-Box	Seeing Sky	Seeing Image	Seed's No. of Plates	Remarks
				mm						
α *Persei*......	L 2079	Sept. 25d 20h 40m	30m	0.025	*Fe*, *V*	13°.10–13°.10	4	3	27 N. H.	Spectrograph readjusted
ε *Pegasi*......	L 2080	27 17 28	60	0.027	*Fe*, *V*	16.66–16.66	4	4	27 N. H.	
γ *Piscium*....	L 2081	27 19 15	120 ±	0.029	*Fe*, *V*	16.67–16.63	3–0	4	27 N. H.	Clouds
α *Arietis*.....	L 2085	Oct. 2 20 8	50	0.020	*Fe*, *V*	13.62–13.68	4	3	27 N. H.	
β *Leporis*.....	L 2087	2 23 57	83	0.026	*Fe*, *V*	13.56–			27 N. H.	
Moon........	L 2091	5 15 40	50 ±	0.020	*Fe*, *V*	18.17–18.11			23	Hazy
α *Arietis*.....	L 2094	5 18 54	50	0.024	*Fe*, *V*	18.10–18.14	3	3	27 N. H.	
Mars........	L 2096	6 13 52	55	0.024	*Fe*, *V*	18.80–18.76	3	2	27 N. H.	
α *Persei*......	L 2100	7 22 53	20	0.024	*Fe*, *V*	15.96–15.96	3–4	2	27 N. H.	
β *Geminorum*.	L 2101	7 23 39	28	0.022	*Fe*, *V*	15.96–16.00	3–4	1–2	27 N. H.	Guiding interrupted
γ *Cephei*.....	L 2109	12 20 42	105 ±	0.027	*Fe*, *V*	13.98–13.95	4	3–4	27 N. H.	
β *Leporis*.....	L 2111	12 23 21	88	0.026	*Fe*, *V*	13.90–13.87			27 N. H.	
Venus........	L 2113	13 1 15	12 ±	0.020	*Fe*, *V*	13.96–14.00			23	Clouds
β *Geminorum*.	L 2117	15 1 5	20	0.023	*Fe*, *V*	9.60– 9.64	4	3	27 N. H.	
										Spectrograph readjusted
γ *Piscium*....	L 2122	27 18 7	150	0.029	*Fe*, *V*	11.00–11.07	4	3–4	27 N. H.	
γ *Cephei*.....	L 2123	27 20 45	120	0.029	*Fe*, *V*	11.00–11.08	4	3	27 N. H.	
α *Persei*......	L 2124	27 22 7	15	0.029	*Fe*, *V*	11.06–11.04	4	3	27 N. H.	
β *Leporis*.....	L 2125	27 23 17	84	0.029	*Fe*, *V*	11.01–10.94	4	3	27 N. H.	
γ *Piscium*....	L 2129	Nov. 2 17 3	125 ±	0.029	*Fe*, *V*	8.90– 9.00	3–0–2	2	27 N. H.	Clouds
γ *Cephei*.....	L 2130	2 19 50	50 ±	0.028	*Fe*, *V*	9.12– 9.14	3–0	2	27 N. H.	Clouds

has been photographed at the beginning and end of the star exposure. The table gives in one column two readings of a large-scale thermometer whose bulb is inside the prism-box near the base of the middle prism. For the most part, the two readings are those made at the beginning and end of the exposure, but for the later plates they are the highest and lowest readings of the thermometer. The temperature control has worked well, and the range in the readings of the prism thermometer for the longest exposures ordinarily does not exceed 0°.1 C. and frequently is less than 0°.05. The double column headed "Seeing" gives the condition of the sky as regards transparency and the character of the stellar image, both on a scale increasing from 0 to 5, where 5 means perfection. The remark "Spectrograph readjusted" means that the spectrograph has been used for other lines of work requiring different adjustments, during the interval against which that note is placed. I have endeavored to keep all adjustments the same throughout this series of observations.

The electric spark has furnished the comparison spectrum. The induction coil supplying the high potential current receives its power from a 104-volt alternating current. A condenser is inserted in the secondary from the coil. To insure the complete illumination of the collimator lens with the light from the spark, a ground glass has been interposed between the electrodes and the slit.

Except for a few of the earlier plates, I have employed for comparison the spectrum of an alloy containing 10 per cent. of vanadium and 90 per cent. of iron. By occulting the twelve bright lines from λ 4379 to λ 4415 during the greater part of the exposure to the spark with a projection on the slide working in the end of the camera tube, an excellent series of uniformly spaced comparison lines is obtained. With only fairly well-timed exposures, there are many more good lines than are needed, so that it is always possible to choose for measures those lying nearest the best star lines.

I have employed throughout Rowland's wave-lengths for the comparison lines; for the vanadium lines, the arc values;[1] and for the iron lines, the arc values for those lines whose arc wave-lengths

[1] Published by Rowland and Harrison in *Astrophysical Journal*, **7**, 273, April 1898.

he has published,[1] for the others, the values given in his table of solar wave-lengths.

Rowland's solar wave-lengths have been used for the wave-lengths of the stellar lines. I have, as far as possible, measured single lines, but have also employed a number of composite lines which appear single, and well suited to measurement, on my spectrograms. For the wave-lengths of these composite, or blended, lines, I have, as is customary, used the values resulting from giving to the wave-length of each component of the blend the weight of its intensity given in Rowland's table, and taking the weighted mean. The weakest line ordinarily taken into account is that of "o" intensity, which has generally been given a weight of one-half.

On some of the last Moon and planet plates, I have measured a rather large number of lines, both single and blended, for the purpose of seeing how the velocities from the blended lines compare with those from single lines. To the same end, I have also measured the strong solar lines at $\lambda\lambda$ 4326, 4384, 4405, and 4415. A comparison of the results from single and from blended and from the very strong lines shows that measures on the single lines are not noticeably more accurate than on the blends and heavy lines, and also that the values for the wave-lengths of the blended lines are reliable. Of course, with stars of the advanced solar type of spectrum, the class to which most of the "Standard Velocity Stars" belong, the relative intensities of lines must frequently be different from what they are in the Sun, and therefore the wave-lengths of the blends in such cases must be inaccurate. I have observed, for instance, that the blend λ 4352.935 $\begin{cases} 4352.908\ (4)\ Fe \\ 4352.044\ (1)^{2}\ V \end{cases}$ in certain stars gives a larger positive velocity than the mean value of the other lines. However, similar uncertainties must attach to some of the lines which are single in the Sun. As an example of this kind may be mentioned the line at λ 4468.663, an excellent single in the Sun, of intensity 5, due to

[1] "A New Table of Standard Wave-lengths," *Astronomy and Astrophysics*, **12**, April 1903; and Frost's Scheiner's *Astronomical Spectroscopy*, p. 363.

[2] The vanadium lines are generally stronger in these stars than in the Sun, and in this blend I have given the V component weight 1, although its intensity is given as o by Rowland. I have used this wave-length for the blend, with Moon and planets, as well as with the stars.

titanium, which appears as a single on the star plates but which, in *a Boötis* for example, gives a too large positive velocity.[1]

I continued to measure certain stellar lines after I knew solar wave-lengths were not entirely applicable and that they were giving spurious velocities. The employment of such lines, however, has only slightly affected the velocity of a plate and they can at any time be excluded or their velocities corrected when the wave-lengths have been more accurately determined. The inclusion of such lines by the different co-operators in their first year's observations would give provisional corrections to their wave-lengths, thus making the lines useful for velocity observations of these and other stars of the same spectral type. I am of the opinion that, after all, one of the most important results of this co-operation in radial velocity observations will be the knowledge gained of the wave-lengths of the star lines.

The plates have been measured with a screw microscope[2] designed especially for measurement of spectrum plates. The screw, which has a pitch of half a millimeter, was examined for errors. Periodic errors were not revealed by the tests, although errors of run were quite apparent, and were of such a character as would be explained by a tapering of the screw from the middle toward the ends. I have not attempted to apply corrections to the measures to take up this error (which accumulates at a rate of about 0.3 μ per revolution), for the reason that its gradual change would practically affect equally the star and near-by comparison line. I do not consider that the measures are appreciably affected by this imperfection of the screw. I have always measured the plate in both positions, violet-right and violet-left, under the microscope, making generally four settings on the star line and two each on the upper and lower part of the comparison line. The best star lines have been measured, regardless of whether or not they existed in the comparison spectrum. The comparison lines lying nearest the measured star lines have been selected, the distance between the star and the comparison line amounting only in exceptional cases to as much as 3 tenth-meters. This close proximity of the spark and the star line practically renders inoperative the errors in run of the micrometer screw.

[1] Frost's and Adams's velocities verify my own as regards the wave-length of this line.

[2] This instrument was made by Gaertner & Co., of Chicago, and is a duplicate of those used by Frost and Adams.

A magnification of 21 diameters has been used in the measurements.

The measures in the two positions of the plate have not been reduced separately, but have been combined and the mean taken before the reduction was begun.

I have adopted the method of reducing each plate independently of every other, by computing for each plate a new Hartmann formula in the simple form

$$\lambda-\lambda_{\circ}=\frac{C}{R-R_{\circ}},$$

where R denotes the screw reading. The constants $R_{\circ}$, C, and $\lambda_{\circ}$ of the formula are computed (in the order given) from the observed screw-readings and known wave-lengths of three comparison lines so selected that there is one near each end and the third near the middle of the portion of spectrum measured. By casting away a factor to make the reading on one of the lines zero, and by the use of logarithms, the constants are derived in about eight minutes. The wave-lengths of all star and comparison lines are then computed. The differences between the computed and normal wave-lengths of the numerous comparison lines furnish the necessary corrections for reducing the star wave-lengths to the true dispersion-curve of the plate. I have applied these corrections to the star lines without the use of a curve, making linear interpolations where needed; the mean of the errors of two neighboring comparison lines frequently being employed for the correction to the intervening star line. The differences between these corrected stellar wave-lengths and their normal values are then taken as the velocity displacements for the star lines. These displacements are speedily converted into velocity in the line of sight by a Crelle's table suitably supplied with notes.

The theoretical velocities of the planets and the Moon have been computed from data given in the *American Ephemeris*, by the aid of Professor Campbell's convenient formulæ. In the reduction of the star velocities to the Sun, Schlesinger's line-of-sight constants have been employed for computing the factor V_a due to the Earth's orbital velocity. The other factor, V_d, due to the Earth's diurnal rotation, is read from a table. In the case of the earlier plates the correction for prismatic curvature has been applied to the mean velocity, and appears at the foot of the reduction table. In the other cases it has

been introduced earlier in the reductions and affects the velocities of the individual lines.

Object	Number of Plate	Greenwich M. T.	Velocity			No. of Lines	Quality of Plate
			Obs.	Comp.	Residu'ls O.—C.		
			km	km	km		
Mars.........	L 1868	1905 April 28d 18h 42m	− 8.39	− 7.02	−0.5	21	Good
Mars......	L 1881	May 18 15 30	− 1.10	− 1.62	+0.5	17	Good
Venus.......	L 1937	July 11 0 21	+13.72	+13.42	+0.3	26	Overexposed
Moon........	L 1944	13 18 35	+ 1.30	+ 0.52	+0.8	27	Underexposed
Moon.....	L 2013	Aug. 12 19 28	+ 0.53	+ 0.25	+0.3	26	Good
Moon........	L 2001	Oct. 5 15 40	+ 0.55	+ 0.65	−0.1	24	Excellent
Mars........	L 2096	6 13 52	+ 0.02	+ 0.16	−0.1	35	Good
Venus.......	L 2113	13 1 15	+ 8.70	+ 8.64	+0.1	36	Excellent

The results from all the planet and Moon plates, made at intervals to test the performance of the spectrograph, are here summarized in a brief table. These check plates cover the whole period during which the "Standard Velocity Stars" have been under observation. The last two of these plates are also printed in detail to show the lines measured and to illustrate the character of the results from the individual lines. The mean value of O.—C. = +0.15 km is doubtless only accidental as it is due to the rather large positive value of one of the less reliable plates. (I consider plates having *V* comparison lines much more reliable than those having the *Mo* lines.) It seems safe to conclude from these tests that the spectrograph has not been affected by appreciable systematic errors during the period covered by this series of velocity observations.

In the following pages are given in tabulated form the detailed reductions of all the plates of the "Standard Velocity Stars." The date of the plate is given in Greenwich Mean Time, above the table. The hour angle is also added. Just over the head of the table is a note descriptive of the quality of the plate. The first column of the table contains the wave-length of the star line and the second column, the velocity deduced for the line, given to the tenth of a kilometer per second. At the foot of these columns is given the mean of the velocities from the several lines, followed by V_a and V_d, the reductions to the Sun; and next the value of the star's radial velocity. Below these will be found the mean error $\epsilon = \pm\sqrt{\frac{\Sigma v^2}{n-1}}$ of the determination of the velocity from a single line, and the mean

error $\epsilon_0 = \pm\sqrt{\frac{\Sigma v'^2}{m(n-1)}}$ of the final velocity of the star deduced from the plate.

The stars are arranged in the order of their right ascensions and the plates of each are given in chronological order.

MARS—L 2096

1905 Oct. $6^d\ 13^h\ 52^m$
Hour angle W $1^h\ 27^m$
Planet spectrum good; comparison lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4274.911	+11.7km
93.241	11.7
94.273	8.7
4307.938	6.9
14.321	8.2
15.178	9.1
18.817	9.7
25.951	9.4
37.216	6.6
40.634	10.7
52.006	9.3
52.935	8.7
59.784	9.1
76.107	5.8
78.419	11.9
79.396	5.5
80.883	12.3
83.720	8.5
95.286	7.4
4404.951	7.1
06.810	6.7
07.851	7.2
08.549	7.1
15.244	10.2
27.420	9.1
35.184	7.5
42.510	10.3
43.976	13.0
47.892	10.0
56.030	10.7
59.304	11.7
68.663	8.3
76.214	8.5
82.376	11.0
94.738	6.4
Mean	+9.02km
Computed vel.	+9.16
O.—C.	−0.1km

No. of Martian lines 35
No. of comp. lines 32

VENUS—L 2113

1905 Oct. $13^d\ 1^h\ 15^m$
Hour angle E $4^h\ 15^m$
Planet spectrum excellent; comparison lines (*V*, *Fe*) excellent.

Line λ (Solar)	Velocity
4238.970	+9.1km
39.975	6.7
45.455	6.3
47.580	11.2
50.287	8.2
50.959	7.2
54.505	5.5
71.934	7.6
74.911	8.4
93.241	7.7
94.273	7.7
4306.938	6.8
14.321	10.2
15.178	7.4
18.817	9.6
25.951	7.2
40.634	8.4
52.006	9.1
52.935	8.9
59.784	7.0
83.720	7.2
95.286	8.3
4404.951	7.0
07.851	6.4
08.549	10.8
27.420	9.8
35.184	9.9
42.510	8.0
43.976	10.8
47.892	10.3
56.030	11.2
59.304	9.8
68.663	9.9
76.214	11.1
82.376	11.0
4528.798	11.3
Mean	+8.70km
Computed vel.	+8.64
O.—C.=	+0.1km

No. of *Venus* lines 36
No. of comp. lines 30

α *ARIETIS*—L 2067

1905 Sept. 12^d 20^h 48^m
Hour angle E 1^h 20^m
Star spectrum fair; comparison lines (*V*, *Fe*) excellent.

Line λ (Solar)	Velocity
4315.178	−35.3^{km}
18.817	33.5
28.080	35.0
37.216	35.2
40.634	33.7
52.006	36.3
52.935	33.6
59.784	37.7
76.107	38.4
95.286	37.7
4407.851	39.4
08.549	33.7
27.420	33.5
28.711	38.7
42.510	35.9
47.892	35.0
59.304	32.3
68.663	34.0
76.214	35.7
91.620	34.6
4505.003	31.0
28.798	35.5

Mean	−35.26^{km}
V_a +20.80	
V_d + 0.12	
Red. to Sun	+20.92
Rad. vel.	−14.3^{km}

No. of star lines. 22
No. of comp. lines 25
ϵ ±2.05
ϵ_0 ±0.43

α *ARIETIS*—L 2085

1905 Oct. 2^d 20^h 8^m
Hour angle E 0^h 30^m
Star spectrum good; comparison lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4315.178	−29.7^{km}
18.817	24.9
28.080	25.9
40.634	27.3
41.530	26.9
52.006	30.6
52.935	27.2
59.784	30.9
76.107	28.5
95.286	28.0
4406.810	30.0
4407.851	−27.3^{km}
08.549	24.1
27.420	25.5
28.711	26.4
41.881	29.8
42.510	23.6
47.892	24.8
56.030	26.5
60.460	23.5
66.701	25.3
68.663	25.8
76.214	25.6
82.376	25.0
94.738	27.5
97.046	26.3
4501.422	30.9
28.798	24.8

Mean	−26.88^{km}
V_a +12.81	
V_d + 0.05	
Red. to Sun	+12.86
Rad. vel.	−14.0^{km}

No. of star lines 28
No. of comp. lines 27
ϵ ±2.21
ϵ_0 ±0.42

α *ARIETIS*—L 2094

1905 Oct. 5^d 18^h 54^m
Hour angle E 1^h 40^m
Star spectrum good; comparision lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4318.817	−25.2^{km}
28.080	27.7
39.731	28.0
40.634	27.1
52.006	26.4
52.935	24.0
59.784	28.7
76.107	28.2
95.286	28.2
99.903	28.4
4406.810	27.2
07.851	28.9
27.420	23.4
28.711	28.2
35.851	28.2
41.881	26.8
43.976	22.0
47.892	24.9
56.030	22.6
68.663	25.4
76.214	25.8
82.376	24.5
82.904	22.3
4490.950	−27.6^{km}
97.046	24.9
97.842	23.5

Mean	−26.08^{km}
V_a +11.47	
V_d + 0.15	
Red. to Sun	+11.62
Rad. vel.	−14.5^{km}

No. of star lines 26
No. of comp. lines 27
ϵ ±2.17
ϵ_0 ±0.43

α *PERSEI*—L 2049

1905 August 30^d 23^h 6^m
Hour angle E 1^h 0^m
Star spectrum good; comparison lines (*Fe*, *V*) excellent

Line λ (Solar)	Velocity
4308.023	−26.0^{km}
13.034	29.0
37.216	25.4
59.784	22.3
76.107	27.4
83.720	30.8
94.225	27.5
95.201	26.1
4404.927	28.0
16.985	29.9
27.420	23.6
43.976	26.0
47.892	25.2
59.301	25.5
66.727	28.0
68.663	28.9
76.214	29.9
82.376	24.3
94.738	30.7
4501.448	26.2
08.455	23.9
15.508	26.4
28.798	27.3

Mean	−26.89^{km}
Curve corr.	− 0.50
V_a +25.29	
V_d + 0.06	
Red. to Sun	+25.35
Rad. vel.	− 2.0^{km}

No. of star lines 23
No. of comp. lines 19
ϵ ±2.10
ϵ_0 ±0.40

α *PERSEI*—L 2068

1905 Sept. 12^d 21^h 50^m
Hour angle E 1^h 20^m
Star spectrum over-exposed; comparison lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4294.273	−23.1 km
4300.211	21.6
03.419	26.0
05.871	28.9
08.023	24.0
13.034	27.4
15.178	23.0
25.939	23.3
40.634	24.0
52.006	28.1
83.720	28.9
91.146	24.8
95.201	26.1
4404.927	28.8
16.985	27.3
27.420	22.9
59.301	26.3
76.214	25.8
81.400	31.3
91.570	29.6
4508.455	26.9
15.508	26.2
28.798	27.8

Mean −26.18 km
V_a +24.04
V_d + 0.08
Red. to Sun +24.12

Rad. vel. − 2.1 km

No. of stars line 23
No. of comp. lines 20
ϵ ±2.53
ϵ_0 ±0.53

α *PERSEI*—L 2079

1905 Sept. 25^d 20^h 40^m
Hour angle E 1^h 35^m
Star spectrum good: comparison lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4294.273	−25.2 km
4300.211	23.0
08.023	26.3
13.034	28.7
15.178	23.2
25.939	24.8
40.634	24.0
52.006	29.2
52.908	22.9
83.720	25.9
94.225	23.1
4395.201	−24.6 km
96.008	22.9
4404.927	24.3
16.985	24.0
43.976	22.8
50.654	23.4
59.301	21.4
66.727	23.8
68.663	24.8
91.570	25.2
94.738	27.2
4501.448	21.4
08.455	23.0
15.508	24.5
20.397	21.5
28.798	24.1
34.139	18.7
49.767	23.8
54.211	25.3

Mean −24.10 km
V_a +21.59
V_d + 0.10
Red. to Sun +21.69

Rad. vel. − 2.4 km

No. of star lines 30
No. of comp. lines 28
ϵ ±2.12
ϵ_0 ±0.39

α *PERSEI*—L 2100

1905 Oct. 7^d 22^h 53^m
Hour angle W 1^h 10^m
Star spectrum very good: comparison lines (*V*, *Fe*) good

Line λ (Solar)	Velocity
4294.273	−21.1 km
4308.023	17.9
13.034	21.4
14.321	20.0
15.178	20.6
25.183	21.4
25.939	20.1
38.084	25.3
40.634	22.9
52.006	24.1
52.908	19.9
59.784	15.2
76.107	23.0
83.720	23.1
94.225	20.5
95.201	20.5
4404.927	22.0
16.985	21.3
17.884	19.8
43.976	23.2
50.654	21.2
59.301	20.3
4468.663	−22.6 km
69.545	23.4
76.214	21.9
81.400	20.1
82.376	19.4
89.351	20.9
91.570	19.5
94.738	23.6
97.023	18.0
4501.448	21.9
08.455	19.4
15.508	20.9
20.397	22.4
28.798	22.4

Mean −21.14 km
V_a +18.32
V_d − 0.07
Red. to Sun +18.25

Rad. vel. − 2.9 km

No. of star lines 36
No. of comp. lines 28
ϵ ±1.94
ϵ_0 ±0.32

α *PERSEI*—L 2124

1905 Oct. 27^d 22^h 7^m
Hour angle W 1^h 47^m
Star spectrum rather strong; comparison lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4308.023	−12.0 km
13.034	12.5
14.321	11.5
25.939	13.7
40.634	14.9
52.908	11.6
76.107	13.1
83.720	15.4
95.201	15.0
4404.927	15.8
16.985	16.4
43.976	13.9
47.892	15.4
50.654	13.6
59.301	15.5
76.214	17.4
4501.448	15.7
28.798	15.9

Mean −14.40 km
V_a +11.18
V_d − 0.11
Red. to Sun +11.07

Rad. vel. − 3.3 km

No. of star lines 18
No. of comp. lines 18
ϵ ±1.74
ϵ_0 ±0.41

β LEPORIS—L 2087

1905 Oct. 2d 23h 56m
Hour angle E 0h 8m
Star spectrum good; comparison lines (V, Fe) good.

Line λ (Solar)	Velocity
4315.178	−33.6km
25.951	29.7
28.080	30.4
37.216	32.3
40.634	30.8
41.530	33.9
52.006	32.4
52.935	32.5
59.784	34.7
76.107	32.6
83.720	35.0
91.146	35.0
4404.951	30.8
06.810	32.4
07.851	32.8
15.244	33.5
27.420	34.3
42.510	35.2
47.892	30.0
56.030	34.7
59.304	32.5
60.460	31.3
66.701	30.3
68.663	33.7
76.214	33.6
82.376	33.0
85.846	32.9
94.738	34.0
4500.480	32.7
01.422	32.5
15.475	30.5
28.708	31.8

Mean	−32.67km
V_a	+19.84
V_d	+ 0.01
Red. to Sun	+19.85
Rad. vel.	−12.8km
No. of star lines	32
No. of comp. lines	30
ε	±1.57
ϵ_0	±0.28

β LEPORIS—L 2111

1905 Oct. 12d 23h 20m
Hour angle E 0h 6m
Star spectrum somewhat weak; comparison lines (Fe, V) good.

Line λ (Solar)	Velocity
4314.321	−30.3km
25.951	32.9
4331.762	−30.7km
40.634	31.0
41.530	32.1
52.935	32.2
59.784	33.0
79.396	33.9
83.720	33.9
4404.951	32.6
06.810	31.7
07.851	31.4
08.549	33.3
27.420	32.0
35.184	28.4
43.076	29.4
47.892	30.1
59.304	32.8
60.460	33.2
68.663	31.5
76.214	30.0
94.738	30.0
4501.422	30.2
08.455	29.8
15.475	30.7
28.708	28.8

Mean	−31.38km
V_a	+18.22
V_d	+ 0.01
Red. to Sun	+18.23
Rad. vel.	−13.2km
No. of star lines	26
No. of comp. lines	25
ε	±1.56
ϵ_0	±0.30

β LEPORIS—L 2125

1905 Oct. 27d 23h 17m
Hour angle W 0h 47m
Star spectrum fair; comparison lines (Fe, V) good.

Line λ (Solar)	Velocity
4315.178	−27.1km
25.951	26.7
28.080	29.1
40.634	24.9
52.006	29.6
52.935	29.9
59.784	28.4
79.396	31.4
83.720	31.3
95.286	28.7
4404.951	28.7
06.810	32.9
08.549	28.8
25.608	24.6
27.420	26.6
42.510	27.0
47.892	29.0
4459.304	−25.2km
68.663	28.9
76.214	23.6
4501.422	24.9
08.455	28.1
22.853	23.6
28.798	24.3

Mean	−27.64km
V_a	+14.75
V_d	− 0.07
Red. to Sun	+14.68
Rad. vel.	−13.0km
No. of star lines	24
No. of comp. lines	24
ε	±2.56
ϵ_0	±0.52

β GEMINORUM—L 1833

1905 April 7d 17h 0m
Hour angle W 2h 58m
Star spectrum fair; comparison lines (Ti, Cr) weak.

Line λ (Solar)	Velocity
4274.911	+36.9km
93.241	35.5
94.273	30.5
4306.938	32.6
14.321	37.5
15.178	30.9
18.817	35.9
28.080	33.3
39.731	32.7
40.634	35.7
49.107	37.7
52.006	34.5
52.935	35.1
59.784	31.9
99.903	31.0
4406.810	31.0
07.851	34.6
08.549	32.3
27.420	30.8
42.510	33.0
57.656	33.7
59.304	32.2

Mean	+33.60km
Curv. cor.	− 0.60
V_a	−29.40
V_d	− 0.23
Red. to Sun	−29.63
Rad. vel.	+3.4km
No. of star lines	22
No. of comp. lines	13
ε	±2.13
ϵ_0	±0.45

β GEMINORUM—L 2101

1905 Oct. 7[d] 23[h] 39[m]
Hour angle E 2[h] 22[m]
Star spectrum fair only: comparison lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4293.241	−27.8km
4314.321	21.6
15.178	28.2
18.817	23.1
25.951	26.6
28.080	27.7
37.216	27.7
40.634	29.6
52.006	28.4
52.935	26.8
59.784	27.4
83.720	27.7
95.286	27.4
99.903	29.9
4406.810	25.5
08.549	25.3
15.244	26.0
27.420	28.0
42.510	25.0
47.892	26.7
59.304	23.6
68.663	25.2
76.214	27.6
82.376	25.6
85.846	23.8
4528.708	25.7

Mean	−26.46km
V_a +29.41	
V_d + 0.20	
Red. to Sun	+29.61
Rad. vel.	− 3.2km
No. of star lines	26
No. of comp. lines	22
ε ±1.98	
ϵ_0 ±0.39	

β GEMINORUM—L 2117

1905 Oct. 15[d] 1[h] 5[m]
Hour angle E 0[h] 20[m]
Star spectrum fair only; comparison lines (*V*, *Fe*) a trifle weak.

Line λ (Solar)	Velocity
4314.321	−27.2km
15.178	29.8
18.817	27.5
4328.080	−28.9km
52.935	29.0
59.784	29.9
95.286	25.9
4407.851	26.2
08.549	23.0
27.420	25.5
28.711	27.8
47.892	23.3
56.030	23.2
57.656	26.4
68.663	24.7
76.214	26.5
82.376	26.2
94.738	24.5
4501.422	25.6
28.708	25.0

Mean	−26.30km
V_a +29.66	
V_d + 0.03	
Red. to Sun	+29.69
Rad. vel.	− 3.4km
No. of star lines	20
No. of comp. lines	20
ε ±2.08	
ϵ_0 ±0.46	

α BOÖTIS—L 1850

1905 April 14[d] 20[h] 15[m]
Hour angle 0[h] 0[m]
Star spectrum fair only: comparison lines (*Ti*, *Cr*) weak.

Line λ (Solar)	Velocity
4293.241	−5.5km
4318.817	3.3
52.006	5.2
52.935	1.2
59.784	4.4
76.107	5.6
79.396	5.5
94.161	6.0
95.286	5.5
99.903	4.8
4400.615	3.5
06.810	6.5
08.549	2.5
27.420	4.3
42.510	4.4
45.641	2.0
47.892	3.4
57.656	7.4
68.663	6.2

Mean	−4.59km
Curv. corr.	−0.50
V_a −0.37	
V_d ±0.00	
Red. to Sun	−0.37
Rad. vel.	−5.5km
No. of star lines	19
No. of comp. lines	15
ε ±1.69	
ϵ_0 ±0.37	

α BOÖTIS—L 2011

1905 Aug. 12[d] 16[h] 6[m]
Hour angle W 3[h] 50[m]
Star spectrum excellent: comparison lines (*Fe*, *V*) good.

Line λ (Solar)	Velocity
4344.507	+19.6km
52.935	19.3
59.784	17.2
69.033	20.0
79.396	17.5
90.149	16.1
91.146	17.9
4406.810	19.3
07.851	17.5
18.499	21.0
27.420	20.2
28.711	17.5
35.851	17.6
41.881	16.9
42.510	19.6
47.892	21.0
56.030	21.2
57.656	20.0
59.304	20.0
60.460	18.3
68.663	19.9
76.214	19.5
82.376	21.1
97.046	20.6
4501.422	20.8
28.798	20.7
29.774	19.7
34.953	20.6

Mean	+19.30km
Curv. corr.	− 0.55
V_a −22.40	
V_d − 0.30	
Red. to Sun	−22.70
Rad. vel.	− 4.0km
No. of star lines	28
No. of comp. lines	29
ε ±1.45	
ϵ_0 ±0.28	

α BOÖTIS—L 2016

1905 Aug. 15^d 16^h 8^m
Hour angle W 4^h 10^m
Star spectrum good; comparision lines (*V*, *Fe*) good.

Line λ (Solar)	Velocity
4352.935	+18.8^km
59.784	15.0
79.396	15.6
89.413	15.7
4406.810	16.3
07.851	15.1
18.499	19.2
27.420	19.9
28.711	17.9
35.851	14.0
41.881	14.9
42.510	18.1
47.892	19.9
57.656	18.1
60.460	19.1
61.818	19.6
66.701	20.4
68.663	19.7
76.214	18.6
82.904	19.7
94.738	19.9
97.046	21.4
4501.422	18.7
28.798	20.7
34.953	18.6

Mean		+18.20^km
Curv. corr.		−0.45
V_a	−21.79	
V_d	−0.32	
Red. to Sun		−22.11
Rad. vel.		−4.4^km

No. of star lines	25
No. of comp. lines	22
ε	±2.08
ϵ_0	±0.42

α BOÖTIS—L 2043

1905 Aug. 20^d 15^h 34^m
Hour angle W 4^h 30^m
Star spectrum excellent; comparison spectrum lines (*V*, *Fe*) excellent

Line λ (Solar)	Velocity
4352.006	+13.6^km
52.935	16.9
59.784	14.2
79.396	12.1
89.413	15.0
4390.149	+14.0^km
99.903	13.8
4406.810	12.5
07.851	12.3
15.722	12.9
27.420	16.0
28.711	13.3
41.881	12.3
42.510	13.4
47.892	15.2
57.656	13.9
59.304	14.7
60.460	15.5
68.663	16.0
76.214	14.3
82.376	15.9
94.738	12.3
97.046	14.6

Mean		+14.12^km
Curv. corr.		−0.55
V_a	−18.22	
V_d	−0.33	
Red. to Sun		−18.55
Rad. vel.		−5.0^km

No. of star lines	23
No. of comp. lines	23
ε	±1.26
ϵ_0	±0.28

α BOÖTIS—L 2053

1905 Aug. 31^d 15^h 31^m
Hour angle W 4^h 35^m
Star spectrum excellent; comparison lines (*Fe*, *V*) good.

Line λ (Solar)	Velocity
4337.216	+16.1^km
48.045	13.8
52.935	16.3
59.784	13.3
69.933	14.2
79.396	11.1
89.413	14.8
91.146	11.3
4406.810	12.9
07.851	12.6
27.420	15.0
28.711	11.4
42.510	13.5
47.892	15.4
56.030	15.0
59.304	15.5
60.460	11.4
68.663	15.3
76.214	15.6
82.376	14.0

Mean		+14.07^km
Curv. corr.		−0.60
V_a	−17.62	
V_d	−0.33	
Red. to Sun		−17.95
Rad. vel.		−4.5^km

No. of star lines	20
No. of comp. lines	20
ε	±1.60
ϵ_0	±0.36

β OPHIUCHI—L 1947

1905 July 14^d 10^h 10^m
Hour angle W 1^h 45^m
Star spectrum fair; comparison lines (*Mo*, *Fe*) fair.

Line λ (Solar)	Velocity
4352.006	+0.3^km
52.935	4.0
59.784	2.3
79.396	−1.4
4406.810	1.5
07.851	+1.7
08.549	1.8
27.420	2.1
38.510	−1.1
42.510	+1.8
47.892	2.4
57.656	4.6
59.304	2.6
60.460	3.4
68.663	−0.5
76.214	0.5
90.950	1.3
97.046	+1.3
4528.798	0.5
34.953	−1.9

Mean		+1.03^km
Curv. corr.		−0.55
V_a	−12.25	
V_d	−0.16	
Red. to Sun		−12.41
Rad. vel.		−11.9^km

No. of star lines	20
No. of comp. lines	21
ε	±1.95
ϵ_0	±0.44

β OPHIUCHI—L 2017

1905 Aug. 15ᵈ 17ʰ 50ᵐ
Hour angle W 2ʰ 30ᵐ
Star spectrum good; comparison lines (*Fe, V*) strong.

Line λ (Solar)	Velocity
4328.080	+15.0km
39.731	10.8
49.107	14.2
52.006	10.5
52.935	13.1
59.784	11.9
79.396	10.1
89.413	12.6
99.903	9.4
4406.810	11.4
07.851	13.0
08.549	13.2
27.420	13.5
42.510	10.5
47.892	8.1
57.656	13.9
59.304	14.0
60.460	11.3
68.663	14.8
76.214	12.7
90.950	11.0

Mean	+12.14km
Curv. corr.	− 0.42
V_a −22.33	
V_d − 0.23	
Red. to Sun	−22.56
Rad. vel.	−10.8km

No. of star lines	21
No. of comp. lines	23
ε ±1.71	
ϵ_0 ±0.37	

β OPHIUCHI—L 2058

1905 Sept. 8ᵈ 16ʰ 32ᵐ
Hour angle W 2ʰ 35ᵐ
Star spectrum fair; comparison lines (*Fe, V*) good.

Line λ (Solar)	Velocity
4328.080	+14.8km
31.762	12.7
52.006	12.8
52.935	15.4
59.784	13.8
69.868	18.4
79.396	12.4
4395.286	−13.9km
4406.810	16.9
08.549	13.6
15.244	18.1
27.420	16.0
28.711	11.9
42.510	16.5
47.892	14.3
59.304	18.4
60.460	18.0
69.549	19.4
76.214	14.6
82.376	18.1
94.738	12.7
4522.853	15.3
28.708	14.4

Mean	+15.31km
Curv. corr.	− 0.45
V_a −25.89	
V_d − 0.22	
Red. to Sun	−26.11
Rad. vel.	−11.3km

No. of star lines	23
No. of comp. lines	23
ε ±2.32	
ϵ_0 ±0.48	

γ AQUILAE—L 1921

1905 July 5ᵈ 21ʰ 5ᵐ
Hour angle W 0ʰ 50ᵐ
Star spectrum good; comparison lines (*Ti, Mo, Cr, Fe*) overexposed.

Line λ (Solar)	Velocity
4321.931	−13.4km
28.080	4.5
31.762	7.7
34.967	6.2
39.731	11.0
52.006	9.6
52.935	6.3
59.784	8.4
64.273	9.8
76.107	13.0
79.396	11.2
95.286	9.0
4400.615	7.7
27.420	7.3
42.510	9.8
47.892	10.1
59.304	9.3
68.663	8.1
75.026	7.1
76.214	10.6

Mean	−9.00km
Curv. corr.	−0.50
V_a +6.89	
V_d −0.08	
Red. to Sun	+6.81
Rad. vel.	−2.7km

No. of star lines	20
No. of comp. lines	14
ε ±2.30	
ϵ_0 ±0.51	

γ AQUILAE—L 1926

1905 July 7ᵈ 20ʰ 25ᵐ
Hour angle W 0ʰ 20ᵐ
Star spectrum good; comparison lines (*Mo, Fe*) weak.

Line λ (Solar)	Velocity
4328.080	−4.9km
31.762	6.9
39.731	7.1
52.935	6.6
59.784	7.4
69.933	7.2
79.396	9.7
95.286	9.1
4407.851	12.4
27.420	3.9
42.510	9.9
47.892	8.8
57.656	10.5
59.304	7.1
68.663	5.9

Mean	−7.83km
Curv. corr.	−0.50
V_a +6.08	
V_d −0.03	
Red. to Sun	+6.05
Rad. vel.	−2.3km

No. of star lines	15
No. of comp. lines	18
ε ±2.23	
ϵ_0 ±0.57	

γ *AQUILAE*—L 1952

1905 July 15^d 19^h 50^m
Hour angle W 0^h 15^m
Star spectrum fair; comparison lines (*Mo. Fe*) fair.

Line λ (Solar)	Velocity
4328.080	-2.4^{km}
49.107	1.8
52.935	2.2
59.784	2.4
62.262	6.9
76.107	2.7
79.396	3.7
4400.615	2.7
06.810	4.6
07.851	5.2
27.420	2.2
42.510	4.9
47.892	6.5
56.030	6.7
57.656	6.6
60.460	7.1
68.663	0.7
72.956	4.4

Mean -4.10^{km}
V_a $+2.77$
V_d -0.02
Red. to Sun $+2.75$

Rad. vel. -1.3^{km}

No. of star lines 18
No. of comp. lines 13
ϵ ± 2.06
ϵ_0 ± 0.49

ε *PEGASI*—L 1948

1905 July 14^d 21^h 18^m
Hour angle E 0^h 24^m
Star spectrum good; comparison lines (*Fe, Mo*) good

Line λ (Solar)	Velocity
4331.762	-12.6^{km}
52.935	7.9
59.784	6.1
76.107	11.4
79.396	10.9
4407.851	9.7
33.390	9.6
41.881	15.0
42.510	11.5
45.641	10.6
56.030	14.0
57.656	10.2
59.304	8.3
4468.663	-6.7^{km}
76.214	11.3
82.376	9.1
82.904	10.3
4501.422	9.5
05.003	8.7
28.798	10.7
29.774	10.0

Mean -10.20^{km}
Curv. corr. -0.50
V_a $+16.77$
V_d $+0.04$
Red. to Sun $+16.81$

Rad. vel. $+6.1^{km}$

No. of star lines 21
No. of comp. lines 17
ϵ ± 2.18
ϵ_0 ± 0.47

ε *PEGASI*—L 2007

1905 Aug. 10^d 20^h 42^m
Hour angle W 0^h 50^m
Star spectrum very good; comparison lines (*Ti, Fe*) good.

Line λ (Solar)	Velocity
4318.817	$+4.6^{km}$
28.080	1.2
31.762	4.2
47.403	1.9
49.107	-0.6
52.935	$+2.1$
59.784	2.5
76.107	-2.9
79.396	1.9
89.413	1.2
91.146	2.9
94.161	$+1.1$
95.286	0.4
4406.810	-1.4
07.851	1.4
27.420	$+0.4$
41.881	-1.5
42.510	2.0
45.641	$+2.4$
47.892	0.9
57.656	-2.8
59.304	$+0.9$
60.460	-1.7
68.663	$+1.4$
76.214	0.4
85.846	2.5
97.046	-1.6
4500.480	$+3.4$
05.003	1.8
12.063	0.1
4512.906	-0.8^{km}
14.513	0.9
15.475	3.0
28.798	-1.7

Mean $+0.39^{km}$
Curv. corr. -0.45
V_a $+5.65$
V_d -0.08
Red. to Sun $+5.57$

Rad. vel. $+5.5^{km}$

No. of star lines 34
No. of comp. lines 25
ϵ ± 2.01
ϵ_0 ± 0.34

ε *PEGASI*—L 2054

1905 Sept. 6^d 18^h 48^m
Hour angle W 0^h 45^m
Star spectrum good; comparison lines (*V, Fe*) fair.

Line λ (Solar)	Velocity
4331.762	$+15.5^{km}$
49.107	12.5
52.935	16.1
59.784	14.2
76.107	11.2
79.396	11.9
89.413	14.0
91.146	11.6
95.286	14.7
98.272	14.1
4427.420	16.1
42.510	15.8
47.892	11.2
56.030	15.8
59.304	16.8
60.460	11.7
68.663	16.7
76.214	12.2
82.376	13.9
94.738	12.5
4515.475	16.6
28.798	14.9

Mean $+14.10^{km}$
V_a -6.74
V_d -0.08
Red. to Sun -6.82

Rad. vel. $+7.3^{km}$

No. of star lines 22
No. of comp. lines 23
ϵ ± 1.79
ϵ_0 ± 0.39

ε *PEGASI*—L 2080

1905 Sept. 27^d 17^h 28^m
Hour angle W 0^h 48^m
Star spectrum good; comparison lines (*Fe, V*) somewhat strong.

Line λ (Solar)	Velocity
4328.080	+18.7^km
49.107	19.3
52.935	23.0
56.110	19.9
59.784	18.4
76.107	19.0
89.413	21.2
90.149	19.8
95.286	18.0
4406.810	19.8
07.851	22.1
27.420	24.9
28.711	19.3
35.851	20.8
41.881	20.0
42.510	23.4
45.641	23.3
47.892	22.0
57.656	23.7
59.304	24.2
68.663	26.2
76.214	21.7
82.376	25.0
94.738	16.4
97.046	23.6

Mean +21.35^km
V_a −15.71
V_d − 0.08
Red. to Sun −15.79

Rad. vel. + 5.6^km

No. of star lines 25
No. of comp. lines 25
ϵ ±2.21
ϵ_0 ±0.44

γ *PISCIUM*—L 2081

1905 Sept. 27^d 19^h 15^m
Hour angle W 1^h 25^m
Star spectrum fair; comparison lines (*V, Fe*) good.

Line λ (Solar)	Velocity
4314.321	− 4.0^km
15.178	8.4
25.951	3.7
28.080	3.0
37.216	5.4
40.634	5.7
41.530	0.7
52.006	7.5
52.935	6.0
4359.784	−7.8^km
76.107	6.8
79.396	5.3
83.720	5.5
95.286	8.1
4406.810	2.3
08.549	3.1
15.244	2.5
27.420	4.3
41.881	1.5
42.510	1.0
45.641	0.2
47.892	4.0
57.656	3.1
59.304	1.7
68.663	5.0
76.214	3.0
82.376	1.4
88.363	0.0
94.738	4.9
97.046	0.0

Mean −3.86^km
V_a −7.79
V_d −0.13
Red. to Sun − 7.92

Rad. vel. −11.8^km

No. of star lines 30
No. of comp. lines 23
ϵ ±2.56
ϵ_0 ±0.45

γ *PISCIUM*—L 2122

1905 Oct. 27^d 18^h 22^m
Hour angle W 2^h 10^m
Star spectrum fair; comparison lines (*V, Fe*) good.

Line λ (Solar)	Velocity
4294.273	+15.0^km
4315.178	12.0
28.080	13.4
40.634	11.0
52.006	13.9
52.935	14.4
59.784	9.3
78.419	9.2
79.396	7.3
83.720	9.3
95.286	7.0
4404.951	6.3
06.810	12.0
07.851	8.0
08.549	9.0
15.244	13.9
27.420	10.9
42.510	10.0
68.663	8.4
76.214	7.2
94.738	5.4
4501.422	−9.4^km
08.455	10.1
28.798	11.9

Mean +10.19^km
V_a −20.99
V_d − 0.20
Red. to Sun −21.19

Rad. vel. −1.0^km

No. of star lines 24
No. of comp. lines 24
ϵ ±2.60
ϵ_0 ±0.53

γ *PISCIUM*—L 2129

1905 Nov. 2^d 17^h 0^m
Hour angle W 1^h 20^m
Star spectrum fair; comparison lines (*V, Fe*) good.

Line λ (Solar)	Velocity
4293.241	+11.6^km
94.273	10.6
4306.938	11.4
15.178	11.6
25.951	12.3
31.762	10.0
37.216	13.8
40.634	12.3
52.006	9.0
52.935	14.3
59.784	8.3
77.407	12.3
95.286	10.0
4404.951	9.8
08.549	12.3
27.420	11.2
41.881	11.6
42.510	12.9
47.892	13.7
57.656	12.4
59.304	12.0
60.460	12.9
76.214	15.4
82.376	15.4
4501.422	12.6
28.798	15.5

Mean +12.12^km
V_a −23.08
V_d − 0.12
Red. to Sun −23.20

Rad. vel. −11.1^km

No. of star lines 26
No. of comp. lines 24
ϵ ±1.88
ϵ_0 ±0.37

γ CEPHEI—L 2109

1905 Oct. 12ᵈ 20ʰ 45ᵐ
Hour angle W 3ʰ 5ᵐ
Star spectrum fair; comparison lines (V, Fe) good.

Line λ (Solar)	Velocity
4293.241	-50.8^{km}
94.273	49.4
4315.178	52.4
18.817	49.8
28.080	48.2
37.216	50.9
39.731	53.0
52.006	49.1
52.935	47.8
59.784	51.5
77.407	44.8
95.286	50.2
4406.810	48.3
07.851	51.8
08.549	49.0
27.420	46.4
28.711	49.8
42.510	47.7
43.976	46.5
47.892	46.3
57.656	43.3
59.304	48.0
68.663	51.6
76.214	45.9
82.376	44.9
97.046	44.5
4528.798	47.7

Mean	-48.50^{km}
V_a +7.96	
V_d −0.06	
Red. to Sun	+ 7.90
Rad. vel.	-40.6^{km}

No. of star lines	27
No. of comp. lines	28
ϵ	±2.60
ϵ_0	±0.50

γ CEPHEI—L 2123

1905 Oct. 27ᵈ 20ʰ 45ᵐ
Hour angle W 4ʰ 5ᵐ
Star spectrum fair; comparison lines (V, Fe) good.

Line λ (Solar)	Velocity
4315.178	-45.9^{km}
28.080	44.5
40.634	44.4
52.935	46.0
59.784	47.4
79.396	49.5
95.286	46.1
4408.549	49.2
27.420	46.7
28.711	49.3
47.892	47.1
59.304	45.1
68.663	47.2
76.214	47.7
94.738	47.6
96.046	49.8
4501.422	49.9
28.798	47.9

Mean	-47.30^{km}
V_a +5.15	
V_d −0.08	
Red. to Sun	+5.07
Rad. vel.	-42.2^{km}

No. of star lines	18
No. of comp. lines	19
ϵ	±2.00
ϵ_0	±0.47

γ CEPHEI—L 2130

1905 Nov. 2ᵈ 19ʰ 50ᵐ
Hour angle W 3ʰ 35ᵐ
Star spectrum weak and unsymmetrical; comparison lines (V, Fe) good.

Line λ (Solar)	Velocity
4315.178	-47.9^{km}
28.080	48.2
52.006	49.8
52.935	46.9
59.784	48.3
4408.549	49.6
09.328	44.6
15.244	43.2
27.420	46.3
28.711	48.5
57.656	49.6
59.304	47.8
66.701	42.9
75.026	47.2
76.214	46.6
82.376	42.0
4528.798	47.6

Mean	-46.88^{km}
V_a +3.91	
V_d −0.07	
Red. to Sun	+ 3.84
Rad. vel.	-43.0^{km}

No. of star lines	17
No. of comp. lines	18
ϵ	±2.39
ϵ_0	±0.58

The resulting velocities for the different plates tabulated above are here collected into a table. The first part of this table contains the values of the velocity deduced from each star plate, followed by their unweighted mean, which is given as the velocity of the star. In the second part of the table are given for comparison the results by other observers of the same star.

It will be noticed that I have in general measured many more lines than is common in such observations. This has increased the accuracy of my velocities by decreasing the effect of accidental errors of measurement, which arise from the somewhat inferior definition

in the spectrograms. Although fairly accurate results are obtained in this way, the extra labor in the measurement and reduction is quite considerable. The spectrograph is soon to be remodeled so as to improve the definition and to render accurate velocity observations possible with less labor.

α *ARIETIS*

Slipher		Other Observers			
Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
Sept. 12^{d} 21^{h}...	−14.3 km	Frost[1]...........	−13.5 km	3	0.8 km
Oct. 2 20 ...	−14.0	Adams[1]..........	−13.9		0.7
Oct. 5 19 ...	−14.5	Adams[2]..........	−13.7	1	...
		Campbell[3].......	−14.1	4	0.6
Mean......	−14.3	Newall[4]..........	−14.3	3	2.8
		Lord and Maag[5]..	−12.4[7]	5	1.8
		Lord[3]...........	−14.0	2	2.7
		Newall[6]	−16.4	8	6.3

α *PERSEI*

Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
Aug. 30^{d} 23^{h}...	− 2.0 km	Frost...........	− 2.3 km	3	1.6 km
Sept. 12 22 ...	− 2.1	Adams..........	− 2.0		1.3
Sept. 25 21 ...	− 2.4	Campbell[8].......	− 2.2	10	2.0
Oct. 7 23 ...	− 2.9	Bélopolsky[9]......	− 2.9	8	3.7
Oct. 27 22 ...	− 3.3	Lord and Maag...	+ 0.6	5	3.7
		Newall..........	− 2.6	14	5.7
Mean......	− 2.5	Vogel[10]..........	− 3.2	13	3.3
		Newall..........	− 4.6	5	5.5

β *LEPORIS*

Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
Oct. 3^{d} 0^{h}...	−12.8 km	Frost...........	−12.2 km	1	... km
Oct. 12 23 ...	−13.2	Adams..........	−12.6		
Oct. 27 23 ...	−13.0				
Mean......	−13.0				

[1] "Spectrographic Observations of Standard Velocity Stars (1902–1903)," *Astrophysical Journal*, **18**, 273, 1903.

[2] *Ibid.*, **15**, 24, 1902.

[3] *Ibid.*, **8**, 150, 1898.

[4] *Monthly Notices*, **63**, 298, 1903.

[5] *Astrophysical Journal*, **21**, 297, 1905.

[6] *Monthly Notices*, **65**, 651, 1905.

[7] See footnote 2, page 339.

[8] *Lick Bulletin*, No. 4, p. 24.

[9] *Astrophysical Journal*, **19**, 85, 1904.

[10] *Ibid.*, **13**, 322, 1901.

β GEMINORUM

Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
April 7d 17h...	+ 3.4km	Frost............	+ 3.2km	3	0.6km
Oct. 8 0 ...	+ 3.2	Adams..........	+ 3.7		0.2
Oct. 15 1 ...	+ 3.4	Lord and Maag..	+ 5.3	5	5.4
		Bélopolsky.......	+ 3.4	9	1.4
Mean......	+ 3.3	Newall..........	+ 2.0	6	3.0

α BOÖTIS

Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
April 14d 19h...	− 5.5km	Frost............	− 4.7km	5	1.3km
Aug. 12 16 ...	− 4.0	Adams..........	− 4.9		0.9
Aug. 15 16 ...	− 4.4	Bélopolsky.......	− 6.1	9	3.3
Aug. 29 16 ...	− 5.0	Lord and Maag...	− 3.2	7	3.2
Aug. 31 16 ...	− 4.5	Frost and Adams[1]	− 4.3	8	1.8
		Newall..........	− 5.8	5	2.7
Mean......	− 4.7	Newall..........	− 6.6	19	4.5

β OPHIUCHI

Slipher		Other Observers			
Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
July 15d 18h...	−11.9km	Frost............	−11.3km	3	0.8km
Aug. 15 18 ...	−10.8	Adams..........	−10.9		0.7
Sept. 8 17 ...	−11.3	Newall..........	−15.9	2	1.9
Mean......	−11.3				

γ AQUILAE

Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
July 5d 21h...	− 2.7km	Frost............	− 1.4km	3	0.7km
July 7 20 ...	− 2.3	Adams..........	− 2.2		1.0
July 15 20 ...	− 1.3	Bélopolsky.......	− 2.0	10	3.8
		Newall..........	− 1.9	4	4.2
Mean......	− 2.1				

ε PEGASI

Date, 1905	Velocity	Observer	Velocity	No. of Plates	Range
July 14d 21h...	+ 6.1km	Frost............	+ 6.2km	3	0.5km
Aug. 10 21 ...	+ 5.5	Adams..........	+ 6.2		0.4
Sept. 6 19 ...	+ 7.3	Campbell[2].......	+ 5.7	4	1.2
Sept. 27 17 ...	+ 5.6	Bélopolsky.......	+ 6.0	7	1.4
		Lord and Maag...	+ 6.1	5	5.8
Mean......	+ 6.1	Newall..........	+ 3.3	3	2.6

[1] *Publications of the Yerkes Observatory*, Vol. II, Part 4, p. 35, 1903.

[2] *Loc. cit.*

γ *PISCIUM*

Sept. 27ᵈ 20ʰ...	−11.8km	Frost...........	−10.7km	3	0.4km
Oct. 27 18 ...	−11.0	Adams..........	−11.1		1.1
Nov. 2 17 ...	−11.1				
Mean......	−11.3				

γ *CEPHEI*

Oct. 12ᵈ 21ʰ...	−40.6km	Frost...........	−41.0km	3	1.0km
Oct. 27 21 ...	−42.2	Adams..........	−41.4		0.2
Nov. 2 20 ...	−43.0	Bélopolsky.......	−39.9	4	2.7
Mean......	−41.9				

As regards the quality of the plates, the velocity of γ *Cephei* is subject to the greatest inaccuracy, due to the weak character of the last plate. The velocity of γ *Aquilae* is also somewhat uncertain, owing to lack of knowledge of the wave-lengths of the *Mo* lines, there being apparent disagreement between the arc[1] and spark values.

Comparison of my results with those of other observers seems to point toward slightly greater negative values for my velocities.[2] But as this depends largely upon the value got for γ *Cephei*, the most discordantly observed star of the ten, I consider it only apparent and due to accidental causes. It might, however, be interesting in this connection to point out that there is a slight difference between the arc wave-lengths[3] of the *V* lines (λ 4300–4500) and Rowland's solar wave-lengths of the lines assigned to *V*, the latter being about 0.0025 tenth-meters greater than the former.

The performance of the 24-inch glass has been, in these observations, entirely satisfactory, as may be judged from a comparison of the exposure times with those of the same stars by Frost and Adams with the great Yerkes refractor. The altitude of this observatory and the transparency of the sky must contribute very greatly

[1] The wave-lengths of the *Mo* lines in the arc were published by Hasselberg in the *Astrophysical Journal*, **17**, 20, January 1903.

[2] Mention should be made here that Professor Lord has called attention to the fact that his and Mr. Maag's velocities are systematically too large positive by about two kilometers.

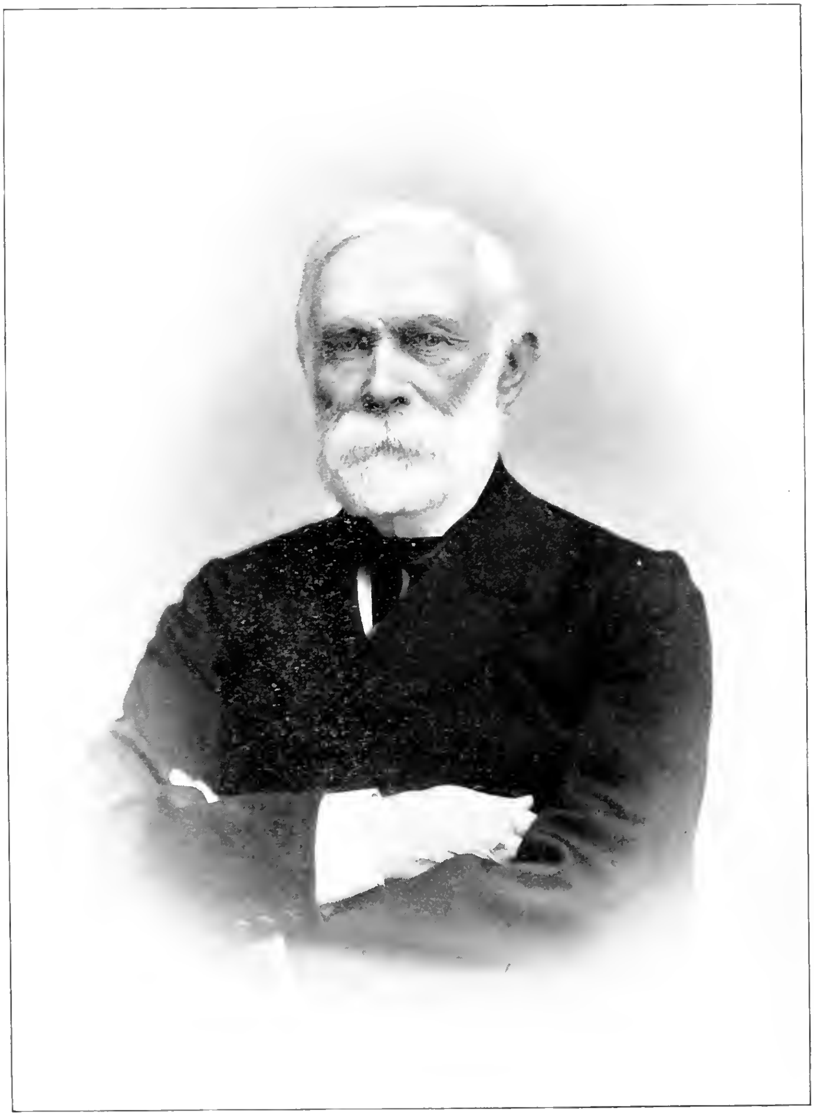
[3] Rowland and Harrison, *Astrophysical Journal*, **7**, 273, 1898.

to the light-power of the equipment. Under fair conditions, with good guiding, satisfactory spectrograms of *a Persei*, for example, would be made (through a 0.025 mm slit) with a 15-minute exposure. My last plate of this star was given that length of exposure and was amply timed, whereas the shortest exposure given this star with the Yerkes equipment was 30 minutes. My earlier plates of this series were, in general, rather over-timed.

In conclusion, I wish to acknowledge my indebtedness to Professor Lowell for encouragement in carrying on these observations, and to Mr. J. C. Duncan, fellow in this observatory, for checking the reductions to the Sun and assisting in preparing the tables for the press.

LOWELL OBSERVATORY,
FLAGSTAFF, ARIZ.,
November 7, 1905.

PLATE XI



T. R. Thalén

Minor Contributions and Notes

TOBIAS ROBERT THALÉN

Tobias Robert Thalén was born on December 28, 1827, in Köping, Sweden. His parents were Jacob Thalén, principal of the school in that place, later pastor in Fläckebo, and his wife, Maria Elizabeth Weijel. After concluding his studies at the school and gymnasium in Westerås, he entered the university at Upsala in 1849, where in 1854 he became a candidate in philosophy, and later in the same year received the degree of doctor of philosophy. Thalén's first scientific work was in the field of mathematics and astronomy, and he became in 1856 docent in astronomy. From 1856 to 1859 he carried on his physical and mathematical studies in England, France, and Germany, and after his return he became docent in physics, and in 1861 adjunct in physics and mechanics at the University of Upsala. For the year 1869–1870 he held the professorship of general and applied physics at the technical school at Stockholm. We find Thalén in the following year again at Upsala, where he was professor of physics and mechanics. On December 19, 1873, Thalén was appointed as the first occupant of the especially established chair of mechanics at the University of Upsala. But in the following year, on August 6, after the sudden death of Ångström, he became professor of physics, and retained this office until he retired on September 1, 1896.

Thalén was one of the most distinguished professors of his time at the University of Upsala. His lectures were characterized by clearness and elegance, and several of his students have become prominent in physics. Thalén also took part in the conduct of the University, in addition to being a member of the Consistorium. From 1883 to 1896 he was a member, and from 1890 to 1896 chairman, of the select committee of the Board of Finances of the University, and from 1889 to 1891 he was *Prorector*. He was also a leading spirit in the Royal Society of Sciences at Upsala, of which he was the librarian from 1860 to 1902, and permanent secretary from 1880 to 1901, thereafter an honorary member. From 1885 to 1899 he was a member of the international metrological committee at Paris, and he took part in numerous meetings of the committee at Breteuil; he was also Sweden's delegate at the metrological conferences at Paris in 1889 and 1895, and he officially transported from Paris to Stockholm the pro-

totypes of weights and measures prepared for Sweden. On October 18, 1901, he was elected an honorary member of the metrological committee. From 1900 to 1903 he was one of the five members of the Nobel committee for physics, which had the duty of dealing with all questions concerning the assignment of the Nobel prize for physics, and of submitting to the Academy of Sciences at Stockholm nominations of those to receive the prize.

Thalén's scientific activity in the domain of physics concerned itself in part with the study of electricity and magnetism and in part with optical observations, but particularly with spectroscopy. His methods of finding deposits of iron ore by means of magnetic measurements have been recognized as of practical significance, and were awarded the medal of the first class of the geographical society by the International Congress of the Geographical Sciences at Paris in 1875. His most important contributions were in the field of spectroscopy. While Ångström was occupying himself with the solar spectrum in general, and in particular with the determination of the wave-lengths in this spectrum, Thalén made measurements of the wave-lengths of the lines of several metals. In this way the value of Ångström's *Spectre normal du Soleil* was still further increased, as the origin of a large part of the Fraunhofer lines could be established with certainty. In a yet greater measure is Thalén's memoir *Sur le spectre du fer obtenu à l'aide de l'arc électrique* (Upsala, 1885) of value as a supplement to the *Spectre normal du Soleil.* It is well known that certain deviations were soon found between the wave-lengths determined by Ångström, and by other physicists, which were of such an order of magnitude that Ångström had himself taken steps to find their cause, but his untimely death prevented him from obtaining better values for his wave-lengths. In the memoir just mentioned, however, Thalén established the fact that these deviations for the most part depend upon the inadequate accuracy of the comparison made in 1866 at Paris between the *mètre à traits* of the Physical Laboratory at Upsala and the *étalon prototype du Conservatoire des arts et métiers.* In his later memoir, *Sur la détermination absolue des longueurs d'onde de quelque raies du Spectre Solaire* (Upsala, 1898), Thalén investigated this question still more thoroughly and found a more precise correction for Ångström's wave-lengths, which brings them into almost exact agreement with the wave-lengths determined by Michelson and Benoit by interference methods. It is true that doubts have been raised as to this method, but it nevertheless appears as though it had been brought into the forefront at the meeting in Oxford of the International Union for Co-operation in Solar Research. In any case, these two investigations

are witnesses to the exemplary accuracy of Thalén's measurements. Finally, Thalén extended his spectroscopic investigations to several newly discovered or rare substances, and, in conjunction with Ångström, to the spectra of the metalloids.

Thalén's services to science were rewarded by membership in numerous learned societies, and the Royal Society of London awarded him the Rumford gold medal; he frequently received awards from Swedish learned societies. It is hardly necessary to mention that numerous orders were bestowed upon him.

Thalén married in 1862 Tonny Carolina Kraak, and one daughter was born from this marriage, who is now the wife of the secretary of the University of Upsala, J. von Bahr.

As appears from what has been said, Thalén was to be called a fortunate man in many respects, and when he requested to be allowed to retire, there seemed to be much to promise him a quiet and peaceful old age. But all this changed in 1901, for in this year his wife was taken from him after a brief but severe sickness and a consequent operation. In the winter of 1903 he himself suffered a fracture of the hip bone from a fall on an icy street. From that time on he was not only a deeply bowed, but even a broken man. On July 27, 1905, he was released by death from his cares and sorrows.

N. C. Dunér.

Upsala,
October 18, 1905.

DE WITT BRISTOL BRACE

On October 2 occurred the death of DeWitt Bristol Brace, professor of physics in the University of Nebraska. By his valuable published researches and by the administration of his professorial office he had won for himself, before his premature death, a prominent position among the physicists of this country.

Professor Brace was born in Wilson, N. Y., on January 5, 1859. He was prepared for college in Lockport, N. Y., and was graduated at Boston University in 1881. After graduation he spent two years at Johns Hopkins University under Rowland, and two years at the University of Berlin under Helmholtz and Kirchhoff. In Berlin he began the series of optical researches, to which his life was devoted, by a study of the magnetic rotation of the plane of polarization, and published the results of his work in his inaugural dissertation, when he received the doctor's degree in 1885. This first paper was an earnest of the work which he was to do in the

future. It is replete with discoveries and suggestions for new work, and was highly commended by Helmholtz for its originality.

After his return to this country he was for a year an assistant professor at the University of Michigan, and soon afterward took up the work at the University of Nebraska, as professor of physics, which for seventeen years, until his death, he conducted with such conspicuous success. From the beginning he felt that his duty as professor was not limited by his courses of instruction, but that he was also bound to promote original research. He had the highest university ideals. To his enthusiasm and to his stimulating influence is largely due the great number of physical investigations which have been carried on in the University of Nebraska in recent years. The new physical laboratory of that university was planned by him especially for research, and he looked forward to years of happy labor in it with his colleagues and his friends. Most appropriately, this laboratory, which was so much the product of his mind and heart, will be named after him, and will no doubt illustrate for many years to come, by the work which proceeds from it, the inspiring example of its designer.

Brace's own contributions to physical science were almost exclusively in the domain of optics. By the invention of his sensitive-strip polarizer, and his half-shade elliptic polarizer, he extended the range of observation far beyond that previously attained, and he devised and partly executed many experiments in which this increased sensitiveness could be used in the study of fundamental optical problems. Returning to the question which he dealt with in his first published paper, he succeeded in showing that the beam of polarized light which undergoes rotation in a magnetic field is susceptible of resolution into two circularly polarized beams. He showed that, to a very high order of sensitiveness, no effect is impressed upon a ray of light by a magnetic field, if its lines of force are at right angles to the ray. He showed that, up to the third order of the ratio of the velocities, no double refraction could be observed in a medium due to its motion through the ether. He planned and tested a method for determining the velocity of light, from which he expected still greater accuracy than that attained in the classical researches of Michelson and Newcomb. He executed several repetitions, with greatly improved instrumental appliances, of classical experiments bearing on the fundamental question of the relative motion of matter and the ether. It is sad to relate that much of the work which he laid out for himself remains incomplete. He had planned more extensive investigations of the ether drift, and was only waiting for the completion of his new laboratory to undertake this impor-

tant task. No nobler memorial could be raised to him, nor one more after his own heart, than the execution of these long-meditated plans by those who will take up his labors in the place which he had designed for them.

W. F. MAGIE.

PRINCETON UNIVERSITY,
November 15, 1905.

WALTER F. WISLICENUS

It is with great regret that we record the untimely death, at the age of forty-six, of Walter Friedrich Wislicenus, *Ausserordentlicher* Professor of Astronomy at the University of Strassburg. His observational activity began in 1882, before the completion of his university studies, when he took part in the observations of the transit of *Venus* with the third German expedition. For more than six years thereafter he was assistant in the Strassburg Observatory and largely concerned with meridian-circle observations. He also regularly observed the Sun with the heliometer. His doctor's thesis, *Beitrag zur Bestimmung der Rotationszeit des Planeten Mars*, was published in 1886. He became *Privatdozent* in Strassburg in 1888, qualifying with his paper on *Untersuchungen über den absoluten persönlichen Fehler bei Durchgangsbeobachtungen*. He was appointed professor in 1894. He was a successful teacher, and his public lectures were characterized by their clearness.

It was a matter of regret to his friends that circumstances did not put him in a position where he could become a practical observer in the field of astrophysics, which held for him a very great interest. As a result of this, his scientific activity, aside from teaching, was more directed toward literary and bibliographical lines. His *Tafeln zur Bestimmung der jährlichen Auf- und Untergänge der Gestirne* constituted the twentieth volume of the publications of the *Astronomische Gesellschaft*. His *Astronomische Chronologie* (1895) is a book of value to historians and archæologists, as well as to astronomers. He also contributed important articles to Valentiner's *Handwörterbuch der Astronomie*, and he wrote several booklets in popular scientific series.

During recent years he had devoted himself with great diligence to the editorship of the *Astronomisches Jahresbericht*, established by himself, and now in its seventh volume. His service in founding this valuable bibliography of current astronomy was a great one, and will be increasingly appreciated as time goes on.

He was of charming personality, and in his quiet dignity a fine illustration of the gentleman and scholar.

F.

SOME REMARKS ON DR. O. C. LESTER'S CONTRIBUTION "ON THE OXYGEN ABSORPTION BANDS OF THE SOLAR SPECTRUM"[1]

Owing to some delay in delivery, this paper did not come under my notice until six months after date of publication; consequently a number of inaccuracies contained therein have, in the meantime, remained unchallenged.

I refer to some comments on my paper on the same subject in the *Proceedings of the Royal Society of 1893.*

This latter, which is mostly of a tabular nature, describes the analytical process which led to a formula expressing the relation between the lines composing the absorption bands A, B, and α; its application to the resolution of the congested groups forming the heads of these bands into pairs; and the general subdivision of all the bands into series.

The apparent complexity of the head portions is shown to be due to the overlapping and interlacing of several pairs near the edges of the bands.

The resolution is illustrated by means of a graphical construction in which the axis of y at the origin is a tangent at the vertex of a parabolic curve, the axis of x coinciding with the scale.

The extensions of the lines of a series are shown to intersect the curve at uniformly increasing distances from the axis of x, and the formula referred to is derived from the known properties of the curve.

The two head series are shown to be distinct and independent of the two series forming the trains, the entire band being composed of at least four series.

The greater intensity of certain head lines, the gradual variation in the separation of the components of pairs, and other characteristics which are the natural consequence of such independence, are also mentioned.[2]

All this, which, from a spectroscopic point of view, has become a matter of ancient history, can hardly be presented to the readers of the *Astrophysical Journal* as a new discovery. I feel that I ought not to overlook any such implication.

On p. 92 the following statement is made: "In his study of the single band by means of the parabola, Higgs shows a smooth curve connecting the lines of the head and tail series as if they were parts of the same band," etc.

I reply simply, but emphatically, that I do not show any such smooth

[1] *Astrophysical Journal,* **20,** 81, September 1904.

[2] A limited number of copies are still available for distribution.

curve connecting the lines of the head and train, but, on the contrary, show plainly that the head and train series are in every case independent of each other.

In the text of the original memoir it is distinctly stated that each band is divided into four series. Correspondingly in the tables the parameters of four different curves are given for each band, the four vertices of which occupy as many different positions on the wave-length scale.

On page 82 appears this remark: "Higgs confines himself to the study of the relations of the lines of a single band, taking B as an example."

It is not clear what can be the object of this statement, for not only were the α and B bands fully discussed, as far as the lines could be measured with any degree of accuracy, but several series of the A band which were not previously known to exist were examined and tabulated.

The faint band in the green beyond D, which is invisible except by the aid of the most powerful instruments, was unknown at that time; but the following extract from a description of my *Photographic Studies* (1893) is evidence that the principal line was found to possess all the characteristics of absorption by the oxygen of our atmosphere:

> No. 9.—N.E. wind, dry. Below freezing. Air lines extremely faint. One line, λ 5789.4, is unaffected by the low temperature and by comparison with other low Sun sections, such as 10 and 86, evidently maintains an intensity proportional to the Sun's altitude. The position is in close agreement with that of an absorption band for liquid oxygen as observed by Egoroff, Liveing and Dewar, and also with the hypothetical position for a fourth group in sequence with A, B, and α.

The paragraph on page 82 concludes as follows: "There are two parallel parabolas corresponding to the two series in each band, the vertices coinciding with the beginning of each series."

As a matter of fact, the elements of four different parabolas are given, corresponding to the four series in each band. The last part of the sentence is misleading, if it is meant to convey that the vertices coincide with the first lines of the series instead of the origins. The foregoing distinction, as I will endeavor to show, has an important bearing on the construction of a general formula, not only for the oxygen absorption bands of the solar spectrum, but for any other spectrum series whose second differences expressed in wave-lengths or wave-frequencies are likewise practically constant.[1]

[1] The extremely minute deviations referred to in the original memoir cannot be taken into account until the measurements can be relied on to within a few thousandths of a tenth-meter.

First, let it be assumed that the initial line coincides with the origin at the vertex; then, from the above-named condition and the nature of the curve, we know that

$$\lambda = v + \frac{1}{p} n^2 \; . \tag{1}$$

But from the condition alone we also know that the wave-length of *nth* line is, in part, the product of the arithmetical progression, $1 + 3 + 5 + \ldots\ldots + r$, that is, n^2 into the semi-constant second difference denoted by b, so that

$$\lambda = A + bn^2 \; . \qquad \text{(2) Deslandres' formula}$$

In applying the above to wave-frequencies a change of signs becomes necessary. No. 2 is applicable to bands of many gaseous spectra whose lines at the edges of bands are in close formation. The solar group commencing at λ 3883.5 on Ångström's scale is probably an example.

For bands whose edges are in more open formation there is but one possible case where it can be applied, and that is where all of the first differences happen to be odd multiples of the semi-constant difference b.

The second of the "Secondary Series" in the train of A which I discovered in 1890 furnishes a remarkable and, as far as I know, unique example of its application; here the first differences are odd multiples of the semi-constant difference.

It may here be remarked that missing lines of a series, if any, are necessarily included in the reckoning from the origin.

In general, the first differences are in excess of the odd multiple, as shown in the appended column, from which it will be seen that the excess is also a constant:

$$\begin{array}{l} \text{Origin} \\ \text{—} \end{array}$$

$$\left.\begin{array}{l} \quad b+k \\ \text{—} \\ \quad 3b+k \\ \text{—} \\ \quad 5b+k \\ \text{—} \\ \quad \cdot \quad \cdot \quad \cdot \\ \quad \cdot \quad \cdot \quad \cdot \\ \quad \cdot \quad \cdot \quad \cdot \\ \quad rb+k \\ N \end{array}\right\} \text{sum} = kn + bn^2 \; . \tag{3}$$

In this case the first line of a series cannot coincide with the vertex or

origin, and the expression for the wave-length is a modification of (1), or

$$\lambda = v + \frac{1}{p}(n \pm c)^2 , \tag{4}$$

which is applicable to every possible case.

This is my parabolic formula, which, reduced to the straight line as in (2), becomes

$$\lambda = O + b(n \pm c)^2 , \tag{5}$$

where O, the origin, has the same value as V, but does not coincide with A, denoted as the first line of the series.

In practice this difference between V and A requires to be known; denote it by x, which we know is $=\frac{y^2}{p} = b\, y^2$ from the nature of the curve and the equality of b and $\frac{1}{p}$, and as $k = 2b\, y$ and $y = \frac{k}{2b}$, then $x = \frac{k^2}{4b} = \frac{(j-b)^2}{4b}$, where j denotes the first difference.

Formula (4) was used simply because of its direct application to the graphical construction given in the memoir; but the mechanical and physical sciences supply us with numerous other instances in which one quantity varies as the square of another, several of which might serve as illustrative principles.

As an example, the reader may conceive a point to move along the scale with a uniformly increasing speed, and the spaces between the lines of any series will be described in equal intervals of time; but whatever form the expression assumes, the coefficient of n^2 has one signification and one only: it is the inseparable accompaniment of the hypothesis with which we set out.

In dealing with the discrepancies between the measurements and calculations on page 95, Dr. Lester states that the character of the variations plainly indicates that Deslandres' constant b is not really a constant, and in the concluding pages proposes to amend Deslandres' law by ascribing to that factor as many values as there are lines in a series; and finally on page 98, after discarding it altogether, he adopts two other constants which have no meaning whatever except that they combine in producing an approximate agreement between the measures and calculations in the formula $N = a + kn + c^{-1}n^2$.

It must be obvious that this or any other formula which does not involve the semi-constant second difference as a coefficient of n^2 has no *raison d'être;* the amendment then resolves itself into a *reductio ad absurdum.*

If it is considered desirable to involve both first and second powers of n, why not at once apply the ready-made textbook formula for uniformly accelerated motion? In which case we have

$$N = a + kn + bn^2 \;, \tag{6}$$

where b retains its constancy and k the signification already assigned to it in (3). In applying (6) to the wave-length scale the terms are positive, and the calculations would agree precisely with those of my tables, but the numbering of the lines of a series from 0 to n would not, of course, include the missing steps from 0, the initial line, to the origin.

The lines of Dr. Lester's continuation table on page 92 evidently constitute the first three pairs of my "Secondary group" which is independent of both head and tail of the main band.

GEORGE HIGGS.

LIVERPOOL,
September 15, 1905.

SECOND NOTE ON "ORTHOCHROMATIC" PLATES

The "sensitiveness-curves" in Fig. 3 accompanying the present paper were plotted from negatives obtained under precisely similar conditions to those described in my "preliminary note."[1]

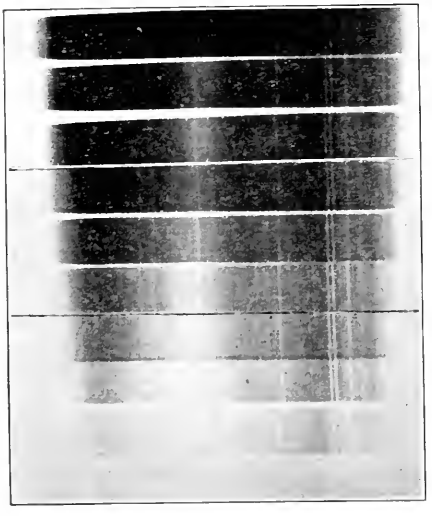


FIG. 1

In considering the series recording the selective sensitiveness of the Cramer "Slow Isochromatic" plate an interesting condition was observed which, in brief, amounts to a reversal of curve according to whether "under" or "normal" exposure be considered.

In the record of this plate (Fig. 1) it will be noted that beginning with the 5 seconds exposure, up to and including that of 30 seconds, the maximum sensitiveness lies decidedly in the violet about λ 3900–4100. With the 1 minute exposure, however, the yellow-green sensitiveness is slightly in excess of the former, and gains in density very rapidly with increasing exposure, until at 8 minutes it is far ahead and has then reached

[1] See p. 153.

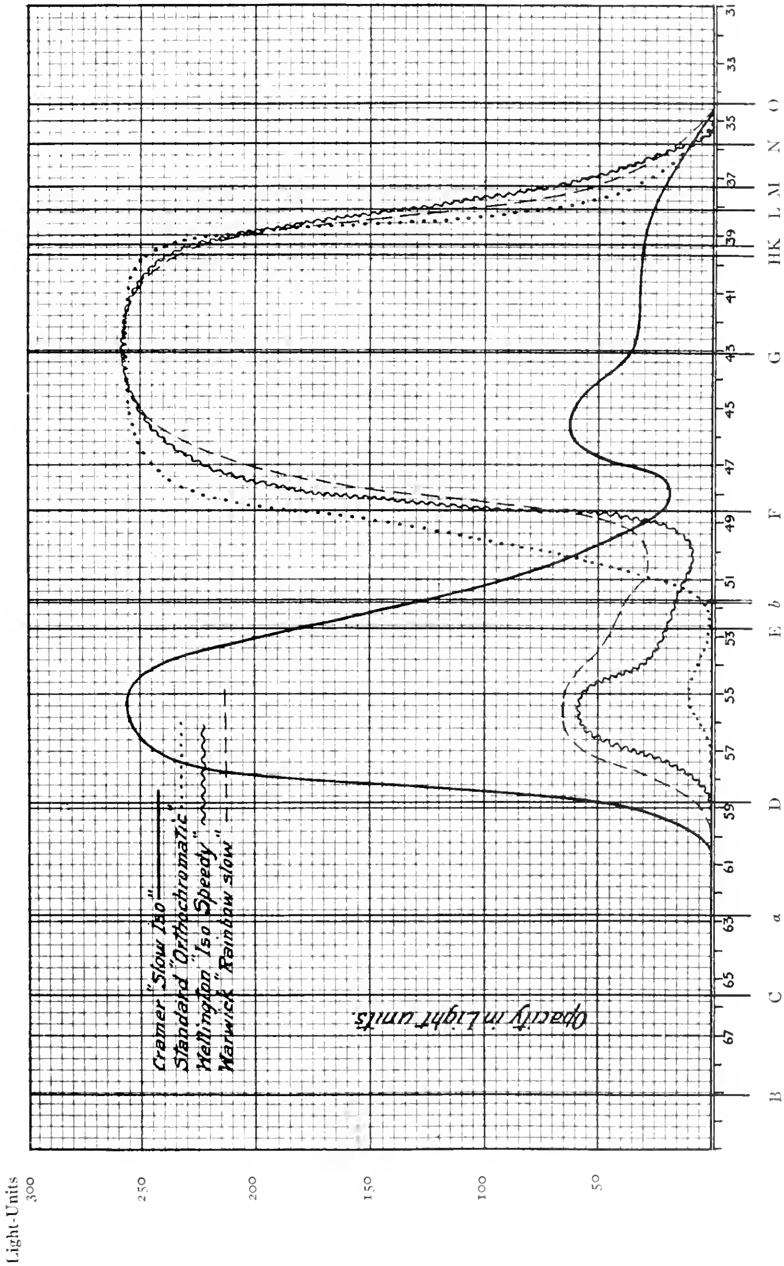


FIG. 2.—Sensitiveness-Curves in Light-Units.

the point of greatest allowable opacity. In the meantime the blue-violet has but slightly increased.

The dye incorporated in the emulsion during the preparation of the plate stains it with a heavy greenish-orange hue, which shows a definite absorption band in the yellow-green from λ 5400–5800; while in the violet the absorption is very strongly marked, shading off gradually in the blue. The sensitiveness-curve for normal exposure is therefore resultant from a combination of emulsion and "light-filter."

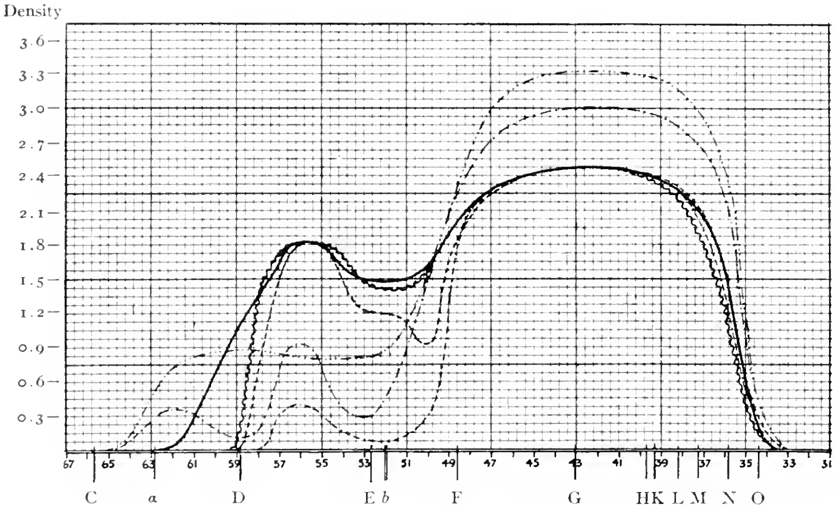


FIG. 3.—Density-Curves Corresponding to Plates Plotted in Fig. 2 (p. 155).

The light which falls upon the surface of the film ("underexposure") affects first the blue-violet—the region of maximum sensitiveness; but, as it penetrates (by lengthened exposure) farther into the film, the violet and blue light is more and more absorbed, while the yellow and green is transmitted with but slight loss.

It will be noted that these curves, Fig. 2 (p. 155) and Fig. 2 (herewith), have been plotted with "opacity in light-units." This differs from the method of Hurter and Driffield, who measure opacity, but plot *density*. The investigations of these workers[1] have proven that in a theoretically perfect negative the quantities of silver reduced at different points are proportional to the logarithm of the light producing them; the deposit of silver (density) representing the amount of chemical work accomplished by the light. By

[1] *Journ. of Soc. of Chem. Industry*, May 1890; also *Photo-Miniature*, **5**, No. 56, Nov. 1903.

plotting these spectrum negatives as "opacity in light-units" the curves serve as an indication of the relative exposure for pure color. At the same time they should also be plotted as densities, for, the transparency of the light being reduced by the density, such a curve is the measure of the printing value (Figs. 3 and 4).

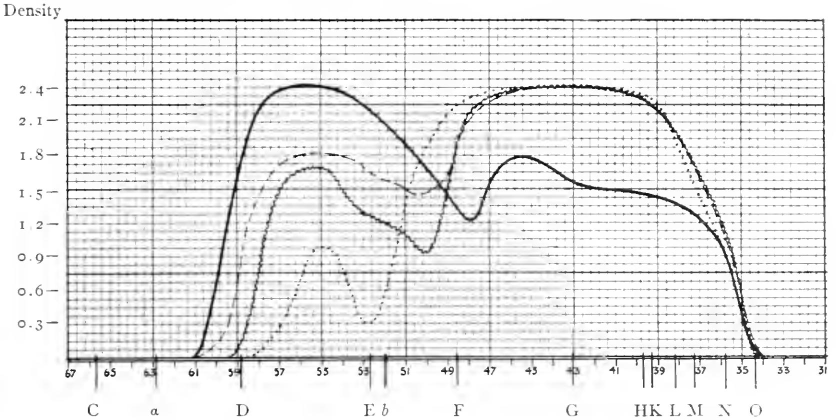


FIG. 4.—Density-Curves Corresponding to Plates Plotted in Fig. 2.

As a check upon the opacity estimation of these curves it was thought advisable to adopt some method of "proving" them. A density-curve was therefore selected (Cramer "Instantaneous Isochromatic") which had been plotted from the opacity-curve already published,[1] and which should

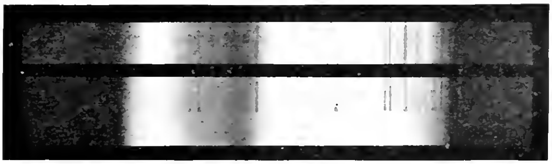


FIG. 5

be, theoretically, exactly the inverse of the original. This was mounted upon a sheet of opaque paper and its area carefully cut out, then placed in a camera, and an image projected of a size comparable with the original negative. This last adjustment was readily effected by making pinholes through the paper mask at the positions of the Fraunhofer lines (abscissæ) and focusing the bright images to size. By means of a plate swinging vertically the image of the curve was caused to impress itself with varying

[1] See p. 155.

density, the pinholes being distinctly shown as slightly darker lines crossing the negative of the artificial spectrum thus obtained.

This negative was found to be closely comparable with the original spectrum negative, when due allowance was made for the effect of the Fraunhofer lines in the latter. Comparison prints are shown in Fig. 5, in which one-half of the height of the artificial spectrum has had the lines drawn in by hand, while the remaining half is untouched.

The result of the speed tests for the plates represented in this second note is (while still taking the Cramer "Instantaneous Isochromatic" as 1.0) as follows:

Standard "orthochromatic"	= 0.75
Cramer "slow isochromatic"	= 9.00
Warwick "Rainbow" (slow)	= 2.00
Wellington "Iso speedy"	= 1.17

Thus it would appear that the "slow isochromatic" has a speed of one-ninth that of the "instantaneous isochromatic."

ROBERT JAMES WALLACE.

YERKES OBSERVATORY,
October 27, 1905.

NOTE ON THE EVOLUTION OF THE SOLAR SYSTEM

In the October number of the *Astrophysical Journal*, Professor Moulton, in the leading article, attacks the idea that the retrograde revolution of *Phoebe* may be explained by the hypothesis that *Saturn* itself formerly revolved in a retrograde direction; that the solar tides reversed this rotation, so that the planet presented for a time only one face to the Sun; and that subsequent condensation accelerated its speed to the velocity which it now possesses.

In all that he says of this supposition I quite agree with him, although I am surprised that he should have thought it worth while to devote so much space to disproving an explanation so obviously improbable. What I fail to understand, however, is why he should associate my name in any way with this ancient theory. Perhaps when it was propounded by Kirkwood in 1864[1] it did not seem so improbable as it does today, but that was a good while ago.

How Professor Moulton should have so completely missed the thread of my explanation I cannot understand, but would suggest that he should read some of my papers on the subject. The theory of planetary inversion has been before the public now for the last twelve years. It was propounded in order to explain some peculiarities of *Jupiter's* satellites, and the anomalous revolution of the satellites of *Uranus*, a revolution which

[1] *Am. Jour. Sci.*, 38, 1.

no other theory has ever even attempted to explain, before or since, so far as I am aware.

When the retrograde revolution of *Phoebe* was discovered, it was found that the inversion theory would fully explain it as it stood, without modification. Indeed, *Phoebe* furnished a very pretty illustration of it. Incidentally it also explains the inclined orbits of the sixth and seventh satellites of *Jupiter*.

I would suggest to Professor Moulton that when he has read one of my articles, he should procure a gyroscope and watch the wheel gradually shift its plane—it is a rather interesting sight.[1] At first it will, for instance, be parallel to the plane of the orbit of the satellite of *Neptune*, then of *Uranus*, then of *Saturn*, and finally, when the planes of revolution and rotation again coincide, to that of *Jupiter*.

WILLIAM H. PICKERING.

October 18, 1905.

AN APOLOGY AND AN EXPLANATION

1. I desire to apologize to Professor W. H. Pickering for having misinterpreted his views in my paper on the evolution of the solar system. But there was no hint in the paper that the theory of tidal retardation was original with him. It has been well known since the time of Delaunay; and so well known since the researches of Darwin in 1878 that it was not deemed necessary to refer to its origin. The statements were intended to mean simply that Professor Pickering applied this idea to the evolution of the Saturnian system. Since he denies having had any reference to it, I must express my deep regret that I ascribed to him such views.

2. I wish to make a few comments on the dynamics of the question, and to show that, under the hypotheses adopted by Professor Pickering, the only effect on the rotation of the planet would be precisely that which I represented him as having had in mind. Since in all his papers he used general language with vague references to the gyroscope, rather than the precise terminology of technical dynamics, his ideas were not perfectly clear to me. My excuse for ascribing to him the views which I did, is that, while I was not absolutely sure from his language what he had in mind, I assumed that his conclusions followed from the hypotheses which he adopted.

The problem in question is to determine the effects on the rotation of a planet of the attraction of the Sun for the tides which it has raised upon the planet. The first principle of the dynamics of rigid bodies is that all the forces which act upon a body may be resolved into three rectangular components applied to its center of gravity, and three couples about three

[1] *Nature*, **71**, 608.

rectangular axes. Since the rectangular components do not affect the rotation, they may be omitted from this discussion.

Let us take the x and y-axes in the plane of the orbit of the planet, and the z-axis perpendicular to this plane. The couple considered by Professor Pickering is the one around the z-axis. This follows from the fact that he says the force which he is considering is perpendicular to that which produces the precession.

Now consider the rotation. Just as any translation may be resolved into three rectangular components, so any rotation may be resolved into rotations around three axes. The rate of rotation around the x-axis may be represented by a vector from the origin along the x-axis, which may be called Oa. The rotations around the other axes may be represented similarly by Ob and Oc. The instantaneous axis of rotation has the direction of the resultant of these three vectors, and the rate of rotation is proportional to the length of the resultant.

The second principle of the dynamics of a rigid body is that the rate of change of rotation around an axis is proportional to the couple around that axis. This means that the couple around the z-axis changes the rate of rotation around the z-axis, and does not affect the rotations around the other axes.

Consider the rotation of the planet. Suppose it is rotating around an axis perpendicular to the plane of its orbit. Then the vectors Oa and Ob are zero, while Oc extends in the negative direction from the origin. A slightly lagging tide will give a positive couple around the z-axis. This will increase Oc algebraically. If it continues long enough Oc will become zero, when the planet will have no rotation. After that Oc will become positive, and the rotation will be positive. This is the ordinary statement of the case.

Suppose now that the rotation is around any axis, but that the component of rotation around the z-axis is negative. In this case Oa and Ob are not zero. The couple around the z-axis will act precisely as before, for its effects are independent of the rotations around the x and y-axes. That is, it will change the rotation around the z-axis just as it would if there were no rotations around the other axes. Also, it will not change the other rotations. Hence it can never bring the planet's equator into the plane of its orbit. Since the question is respecting the change of moment of momentum around the z-axis from negative to positive, it follows that there is no reason whatever for introducing the rotations around the x and y-axes. The only result of doing it is to lead to a confusion of ideas when the problem is not treated by the methods employed in dynamics. It is easy to point out the source of the confusion. When the rotation around the z axis had become zero, there still remained a rotation, for

the rotations *Oa* and *Ob* retained their original values. Hence the body never stopped rotating, while its axis of rotation turned over. But this does not mean that the body turned over, or that its rate of rotation remained unchanged. On the contrary, since the rate of rotation of a body is equal to the resultant of its three components, it decreased until the z-component became zero, and then increased again. The whole thing corresponds to the fact that if a body is projected upward not vertically, it will fall back to the Earth without its velocity ever having become zero. Just as the influence of the Earth's gravitation on the vertical component is independent of the other components, so the influence of the couple around the z-axis is independent of the other components of rotation. Therefore their introduction has added nothing to the problem except possibly a little misunderstanding of it.

F. R. MOULTON.

THE UNIVERSITY OF CHICAGO,
November 8, 1905.

REPLY TO PROFESSOR F. R. MOULTON

I have carefully read over Professor Moulton's reply to my letter, and it still appears to me that the effect of the annual tides on a planet having a retrograde rotation will be not simply to stop and then reverse this rotation, leaving its plane unchanged, as Professor Moulton claims, but rather to cause it to turn over, so that the rate of rotation shall be unchanged, while its plane is by this means rendered parallel to the orbit of the planet.

I am sorry to differ from Professor Moulton on a point in such elementary mechanics. The simplest case to consider, it seems to me, is where the rotations about the y and z axes are reduced to zero. We have now only a rotation about the x axis. This is the case, very nearly, of the planet *Uranus* at the present time. Let us now introduce a minute couple tending to cause a rotation about the z axis, due to the annual tide raised by the Sun. The result is to shift the direction of the axis of rotation of the body so that instead of being parallel to x, as before, it is now inclined slightly toward that of z. This may be represented by the component of the vectors in these two axes. The tidal force still acting about the z axis, the axis of the body inclines more and more toward it, until it finally becomes parallel to it.

The last few lines of Professor Moulton's letter seem to me to express this very idea. As he says, the body does not stop rotating, its axis of rotation simply turns over. His comparison to a falling body also seems to me to be an apt one. In the case above stated a body would be projected in a horizontal direction. Its horizontal velocity represents the

vector in the axis of x. It is acted on by a vertical acceleration, corresponding to the couple about z, which finally produces a velocity in a nearly vertical direction. This is the vector in the direction of the axis z.

The two cases are not exactly alike, because the uniform horizontal velocity exhibited by the falling body is not maintained in the other case, nor is the acceleration produced by the tidal forces uniform, since it becomes zero when the axis of rotation of the planet becomes parallel to z. A better analogy would be that of a stone projected horizontally through still water. The direction of motion of the stone through the action of gravity gradually becomes vertical.

I think if Professor Moulton will refer to my paper in the *Astronomische Nachrichten*, **164**, 201, he will there find the subject treated from the dynamical standpoint.

William H. Pickering.

November 12, 1905.

NOVA AQUILAE OF 1905

Following is a list of the plates which I made at Mount Wilson, California, with the Bruce telescope covering the region of the *Nova*, to which Professor Frost called attention in the last number of this *Journal* (p. 270).

The given magnitudes are the smallest shown on the plates in that region. These magnitudes are photographic, and were derived from comparison with reflector photographs of the same region, on which Mr. J. A. Parkhurst had kindly marked certain magnitudes for my guidance. In every case the stars were in the region of bad definition, as no plate was centered quite near the place of the *Nova*. All these plates were made with the $6\frac{1}{4}$-inch doublet.

1905	Exposure	Lowest Magnitude
June 5	0 52^{m}	13.5
6	1 43	13.3
July 2	1 58	14.0
7	4 35	14.0
29	0 25	14.0
30	5 30	16.0
Aug. 4	4 30	15.0
23	3 15	*Nova* shown strongly
24	4 0	*Nova* shown strongly

The *Nova* thus appears strongly on the plates of August 23 and 24, but not on any of the other photographs.

E. E. Barnard.

Yerkes Observatory,
November 10, 1905.

INDEX TO VOLUME XXII

SUBJECTS.

INDEX TO VOLUME XXII

AUTHORS